NEUTRINO OSCILLATIONS

(Analysis of Data?)

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IV International Pontecorvo Neutrino Physics School

Plan of Lectures

Introduction: The New Minimal Standard Model

Effects of ν mass: Oscillations in Vaccum and Matter

Atmospheric Neutrinos

Solar Neutrinos

Accelerator and Reactor Neutrinos

Fitting all Together and Subleading effects

Summary

PS:The Near Future Experimental Program and Its Challenges

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The New Minimal Standard Model

- Minimal Extensions to give Mass to the Neutrino:

  ∗ Introduce ν_R AND impose L conservation ⇒ Dirac ν:

  $$\mathcal{L} = \mathcal{L}_{SM} - M_\nu \overline{\nu_R} \nu_L + h.c.$$

  ∗ NOT impose L conservation ⇒ Majorana ν

  $$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{2} M_\nu \overline{\nu_L^C} \nu_L + h.c.$$

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- Charged current and mass for 3 charged leptons ℓ_i and N neutrinos ν_j in weak basis

$$\mathcal{L}_{CC} + \mathcal{L}_M = -\frac{g}{\sqrt{2}} \sum_{i=1}^{3} \overline{\ell_{L,i}^W} \gamma^\mu \nu_i^W W_\mu^+ - \sum_{i,j=1}^{3} \overline{\ell_{L,i}^W} M_{\ell\, ij} \ell_{R,j}^W - \left(\frac{1}{2}\right) \sum_{i,j=1}^{N} \overline{\nu_i^{(C)\,W}} M_{\nu\, ij} \nu_j^W + \text{h.c}$$

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- Changing to mass basis by rotations

$$\ell^W_{L,i} = V^\ell_{L\,ij} \ell_{L,j} \qquad \ell^W_{R,i} = V^\ell_{R\,ij} \ell_{R,j} \qquad \nu_i^W = V^\nu_{ij} \nu_j$$

$$V_L^{\ell\,\dagger} M_\ell V_R^\ell = \text{diag}(m_e, m_\mu, m_\tau) \qquad V^{\nu\,\dagger} M_\nu^\dagger M_\nu V^\nu = \text{diag}(m_1^2, m_2^2, m_3^2, \dots, m_N^2)$$

$V^\ell_{L,R} \equiv$ Unitary 3×3 matrices

$V^\nu \equiv$ Unitary $N \times N$ matrix.

- The charged current in the mass basis

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$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \overline{\ell^i_L} \gamma^\mu U^{ij}_{\rm LEP} \nu_j W^+_\mu$$

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- For example for 3 Dirac ν's : 3 Mixing angles + 1 Dirac Phase

$$U_{\rm LEP} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- For 3 Majorana ν's : 3 Mixing angles + 1 Dirac Phase + 2 Majorana Phases

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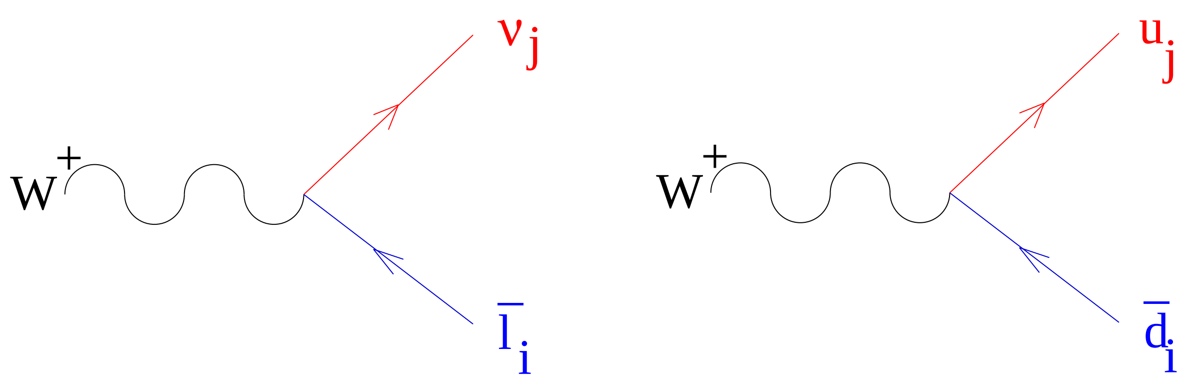
• Also if neutrinos have a mass the charged current interactions of leptons are not diagonal (same as quarks)

$$\frac{g}{\sqrt{2}} W_\mu^+ \sum_{ij} \left(U_{LEP}^{ij}\, \overline{\ell^i}\, \gamma^\mu\, L\, \nu^j + U_{CKM}^{ij}\, \overline{U^i}\, \gamma^\mu\, L\, D^j \right) + h.c.$$



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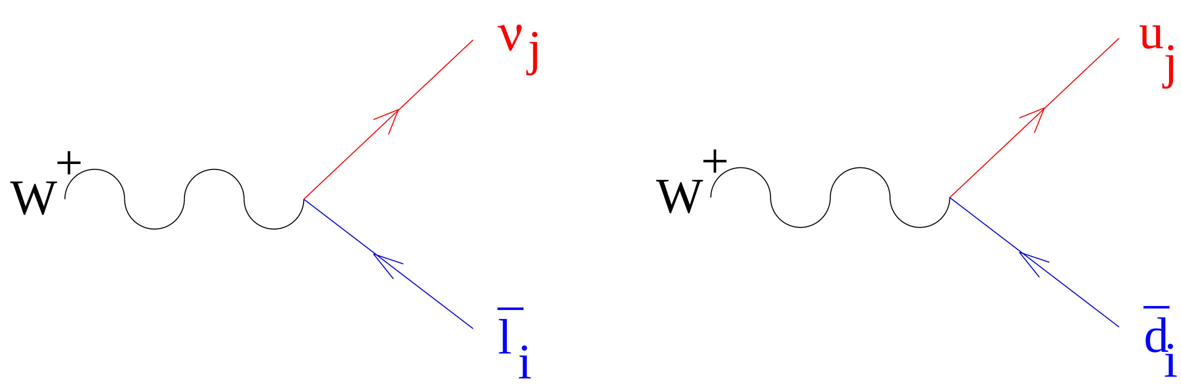

- SM gauge invariance *does not imply* $U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$ symmetry
- Total lepton number $U(1)_L = U(1)_{Le+L_\mu+L_\tau}$ can be or cannot be still a symmetry depending on whether neutrinos are Dirac or Majorana

$$\mathcal{L}_{\rm CC} + \mathcal{L}_M^\nu = -\frac{g}{\sqrt{2}} \sum_{ij} \overline{\ell_L^j}\, \gamma^\mu\, U_{\rm LEP}^{ji}\, \nu_i\, W_\mu^+ + \sum_i m_i \overline{\nu}_i \nu_i$$

- To fully determine the lepton flavour sector we want to know:

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  * How many, N, neutral states ν_i and their masses m_i
  * Their mixing: $\#\ \text{angles} = \frac{N(N-1)}{2} = \begin{cases} 1 \text{ for } N=2 \\ 3 \text{ for } N=3 \\ 6 \text{ for } N=4 \end{cases}$

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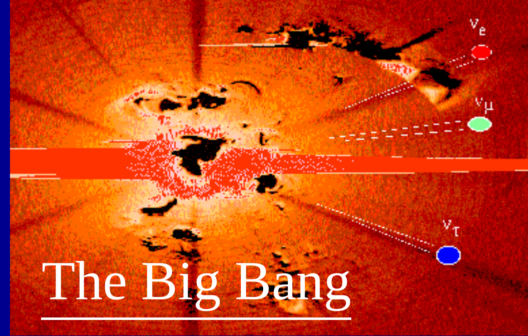
  * Their *nature* and CP properties:

Dirac: $\nu^C \neq \nu$ $\qquad \#$ phases $= \frac{(N-1)(N-2)}{2} = \begin{cases} 0 \text{ for } N=2 \\ 1 \text{ for } N=3 \\ 3 \text{ for } N=4 \end{cases}$

Majorana: $\nu^C = \nu$ $\qquad \#$ extra phases $= (N-1) = \begin{cases} 1 \text{ for } N=2 \\ 2 \text{ for } N=3 \\ 3 \text{ for } N=4 \end{cases}$

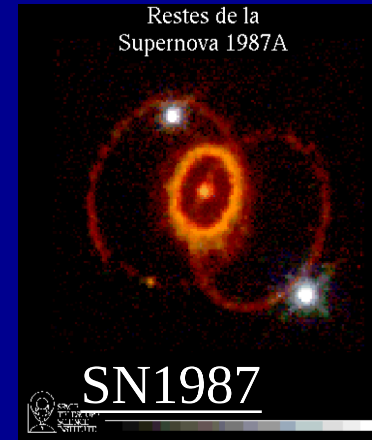
$$U^{\rm Maj}_{\alpha j} = U^{\rm Dir}_{\alpha j} \times {\rm e}^{-i\eta_j}$$

Sources of ν's



$\rho_\nu = 330/\text{cm}^3$

$E_\nu = 0.0004$ eV



$E_\nu \sim$ MeV

The Sun

ν_e

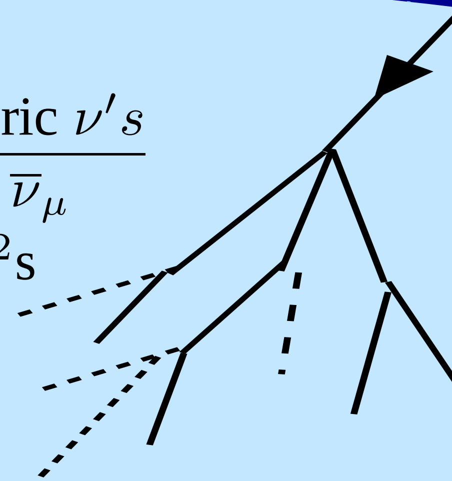
$\Phi_\nu^{Earth} = 6 \times 10^{10} \nu/\text{cm}^2\text{s}$

$E_\nu \sim 0.1$–20 MeV

Atmospheric $\nu's$

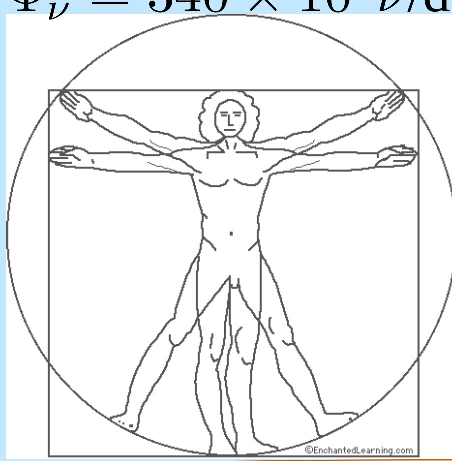
$\nu_e, \nu_\mu, \overline{\nu}_e, \overline{\nu}_\mu$

$\Phi_\nu \sim 1 \nu/\text{cm}^2\text{s}$



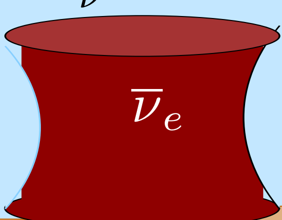
Human Body

$\Phi_\nu = 340 \times 10^6 \nu$/day



Nuclear Reactors

$E_\nu \sim$ few MeV



Fermilab Discovering the Nature of Nature

CERN

～宇宙と物質の起源と構造を探る～ KEK 高エネルギー加速器研究機構

Earth's radioactivity

$\Phi_\nu \sim 6 \times 10^6 \nu/\text{cm}^2\text{s}$

Accelerators

$E_\nu \simeq 0.3$–30 GeV

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$\Longrightarrow$ Need huge detectors with Exposure $\sim$ KTon $\times$ year

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$\Rightarrow$ ν can be detected with different (or same) flavour than produced

- The probability $P_{\alpha\beta}$ of producing neutrino with flavour α and detecting with flavour β has to depend on:
  - Misalignment between interaction and propagation states ($\equiv U$)
  - Difference between propagation eigenvalues
  - Propagation distance

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- Under the approximations:

  $(\mathbf{1})$ $|\nu\rangle$ is a *plane wave* $\Rightarrow$ $|\nu_i(t)\rangle = \mathrm{e}^{\,-i\,E_i t}|\nu_i(0)\rangle$

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{j\neq i}^{n}\mathrm{Re}[U^\star_{\alpha i}U_{\beta i}U_{\alpha j}U^\star_{\beta j}]\sin^2\left(\frac{\Delta_{ij}}{2}\right) + 2\sum_{j\neq i}\mathrm{Im}[U^\star_{\alpha i}U_{\beta i}U_{\alpha j}U^\star_{\beta j}]\sin\left(\Delta_{ij}\right)$$

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$(\mathbf{2})$ *relativistic* ν

$$E_i = \sqrt{p_i^2 + m_i^2} \simeq p_i + \frac{m_i^2}{2E_i}$$

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$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{j \neq i}^{n} \mathrm{Re}[U^\star_{\alpha i} U_{\beta i} U_{\alpha j} U^\star_{\beta j}] \sin^2\left(\frac{\Delta_{ij}}{2}\right) + 2 \sum_{j \neq i} \mathrm{Im}[U^\star_{\alpha i} U_{\beta i} U_{\alpha j} U^\star_{\beta j}] \sin\left(\Delta_{ij}\right)$$

with $\Delta_{ij} = (E_i - E_j)t$

(**2**) *relativistic* ν

$$E_i = \sqrt{p_i^2 + m_i^2} \simeq p_i + \frac{m_i^2}{2E_i}$$

(**3**) Lowest order in mass $p_i \simeq p_j = p \simeq E$

$$\frac{\Delta_{ij}}{2} = 1.27 \frac{m_i^2 - m_j^2}{\mathrm{eV}^2} \frac{L/E}{\mathrm{Km/GeV}}$$

Vacuum Oscillations

- The oscillation probability:

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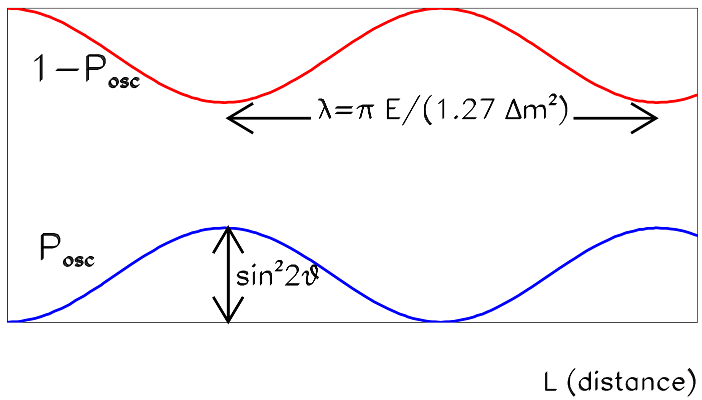

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$$\left(\alpha_x \{ E^2 - m_i^2 \}^{1/2} + \beta\, m_i \right) \phi_i = E\phi_i \qquad (1)$$

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- Using (1) in Dirac Eq. we can factorize ϕ_i and α_x and get:

$$-i\frac{\partial\nu_1(x)}{\partial x} = \left\{E^2 - m_1^2\right\}^{1/2}\nu_1(x)$$
$$-i\frac{\partial\nu_2(x)}{\partial x} = \left\{E^2 - m_2^2\right\}^{1/2}\nu_2(x)$$

- In the relativistic limit $\sqrt{E^2 - m_i^2} \simeq E - \frac{m_i^2}{2E}$

$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \begin{pmatrix} E - \frac{m_1^2}{2E} & 0 \\ 0 & \frac{E - m_2^2}{2E} \end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

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• And the flavour transition probability

$$P(\nu_\alpha \to \nu_\beta) = |\nu_\beta(L)|^2 = B_1^2 + B_2^2 + 2B_1B_2\cos(2\omega L) = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

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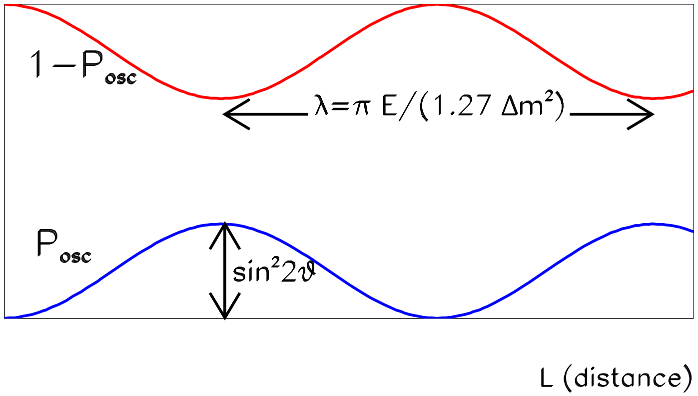

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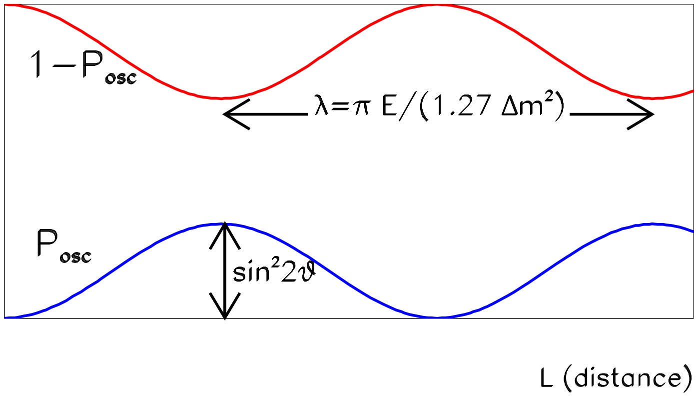

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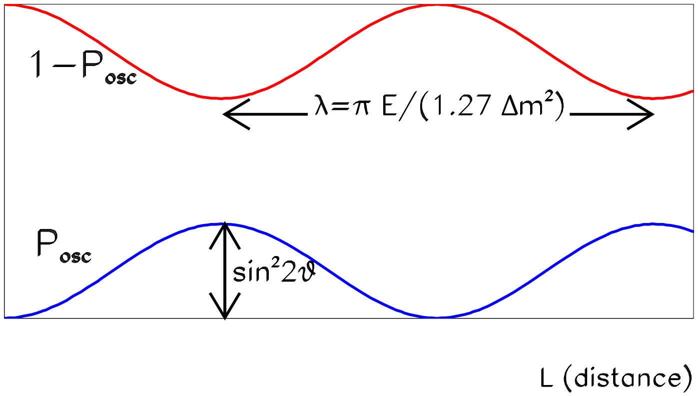

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- Moreover P_{osc} is symmetric under $\Delta m^2 \to -\Delta m^2$ *or* $\theta \to -\theta + \frac{\pi}{2}$

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  This only happens for 2ν vacuum oscillations

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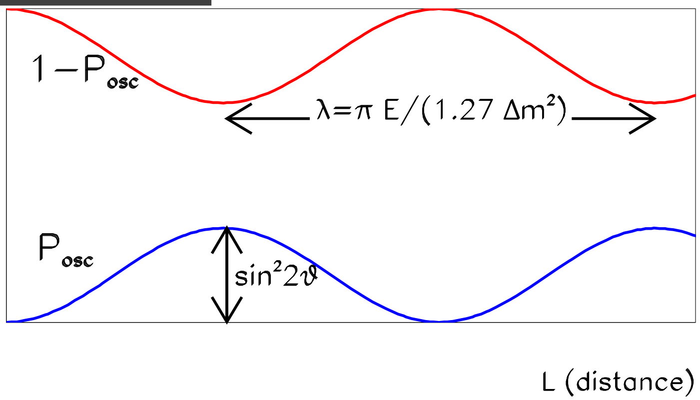

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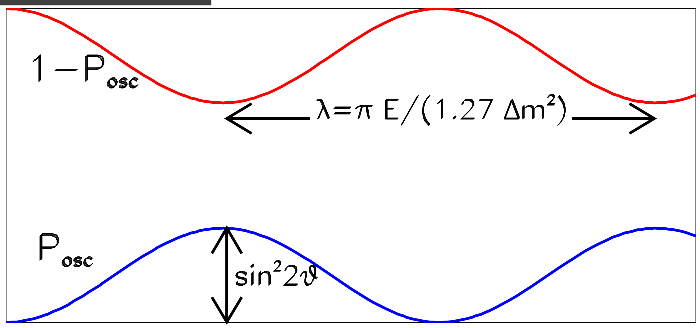

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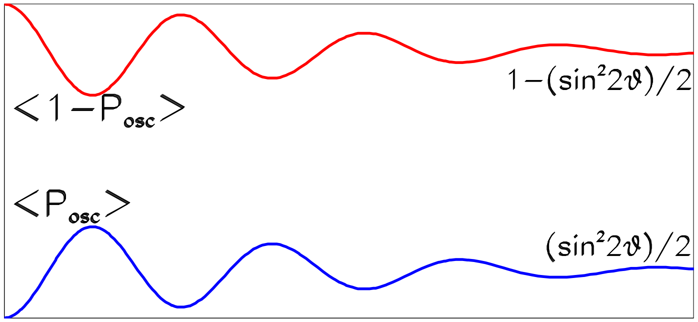

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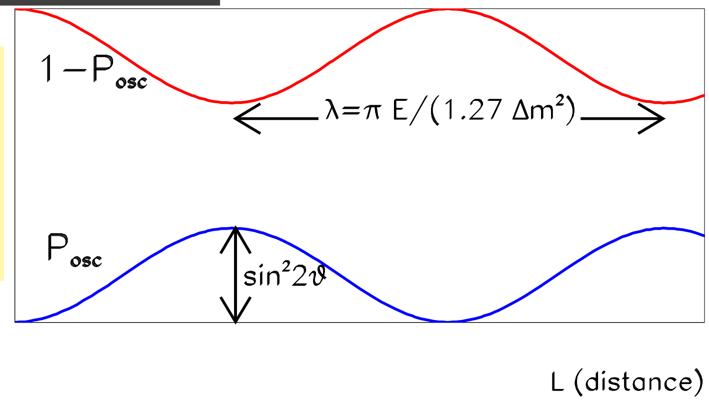

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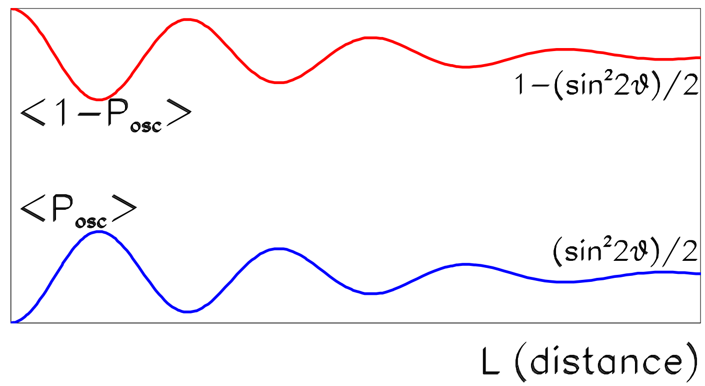

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2-ν Oscillations

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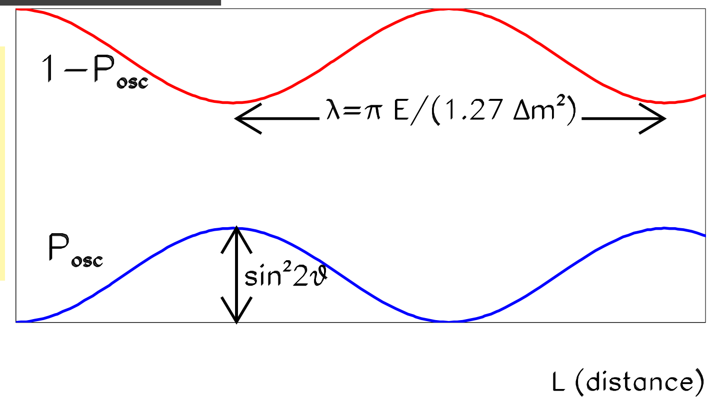

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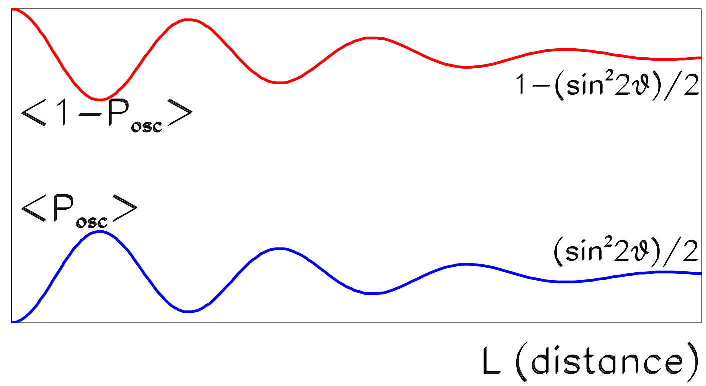

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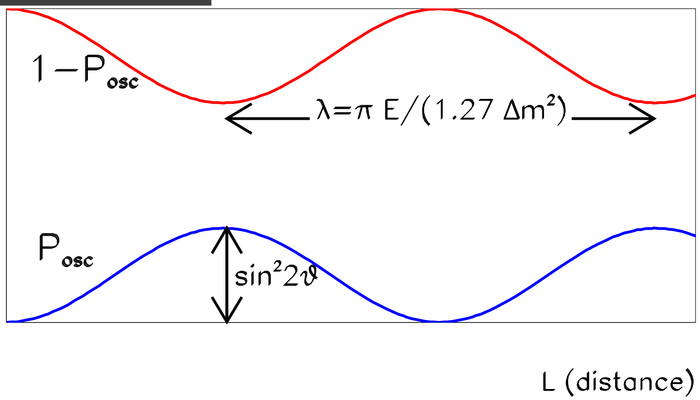

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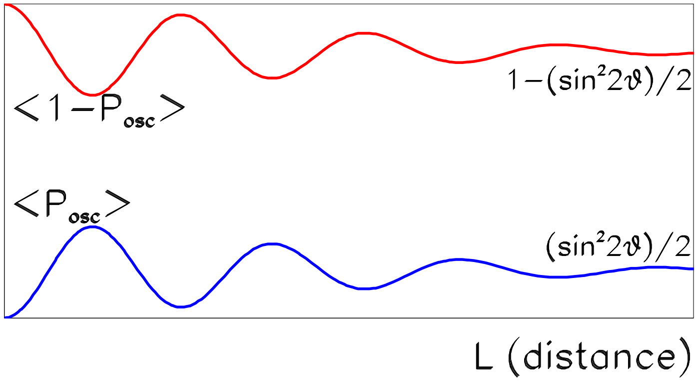

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Neutrinos in Matter:Effective Potentials

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- In *coherent* interactions $\Rightarrow$ ν and medium remain unchanged

  Interference of scattered and unscattered ν waves

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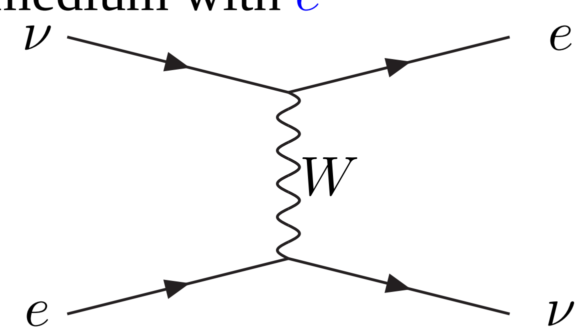

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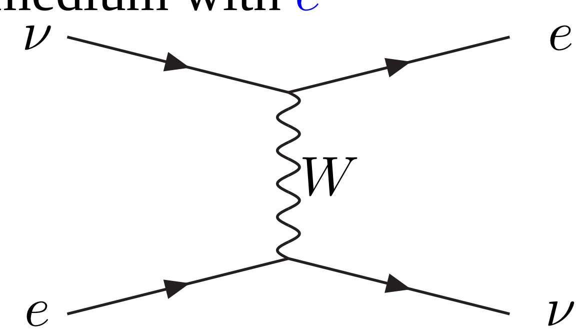

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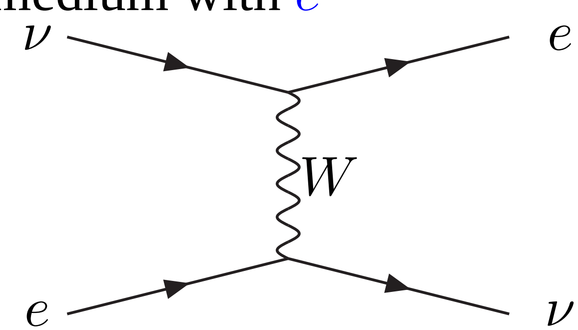

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• Other potentials for ν_e $(\overline{\nu}_e)$ due to different particles in medium

medium	V_C	V_N
e^+ and e^-	$\pm\sqrt{2}G_F(N_e - N_{\bar{e}})$	$\mp\frac{G_F}{\sqrt{2}}(N_e - N_{\bar{e}})(1 - 4\sin^2\theta_W)$
p and $\bar{p}$	0	$\pm\frac{G_F}{\sqrt{2}}(N_p - N_{\bar{p}})(1 - 4\sin^2\theta_W)$
n and $\bar{n}$	0	$\mp\frac{G_F}{\sqrt{2}}(N_n - N_{\bar{n}})$
Neutral $(N_e = N_p)$	$\pm\sqrt{2}G_F N_e$	$\mp\frac{G_F}{\sqrt{2}}N_n$

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Evolution Eq. for $|\nu\rangle = \nu_1|\nu_1\rangle + \nu_2|\nu_2\rangle \equiv \nu_e|\nu_e\rangle + \nu_X|\nu_X\rangle$ $(X = \mu, \tau, \mathrm{sterile})$

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(a) In vacuum in the weak basis

$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_e \\ \nu_X\end{pmatrix} = E - \frac{m_1^2+m_2^2}{2E} - \begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}\nu_e \\ \nu_X\end{pmatrix}$$

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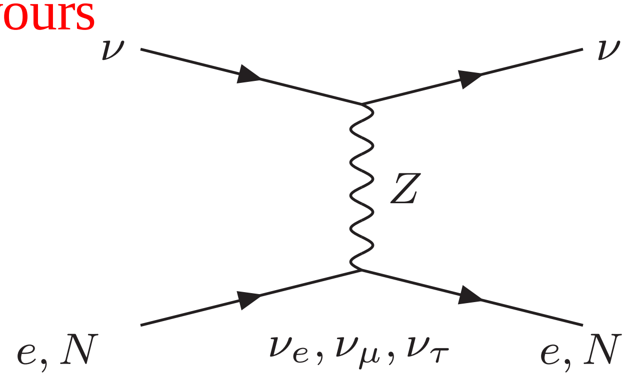
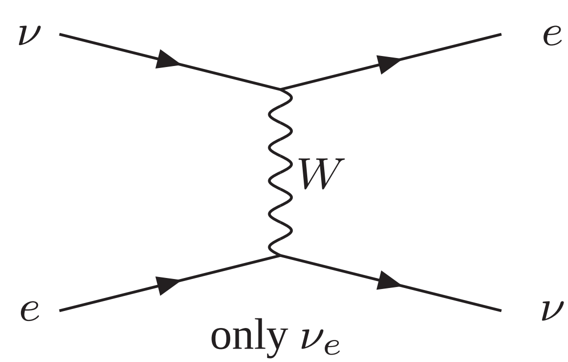
(a)$\neq$ (b) because different flavours have different interactions

For example $X = \mu, \tau$:

$$V_{CC} = V_e - V_X = \sqrt{2}G_F N_e$$

(opposite sign for $\overline{\nu}$)





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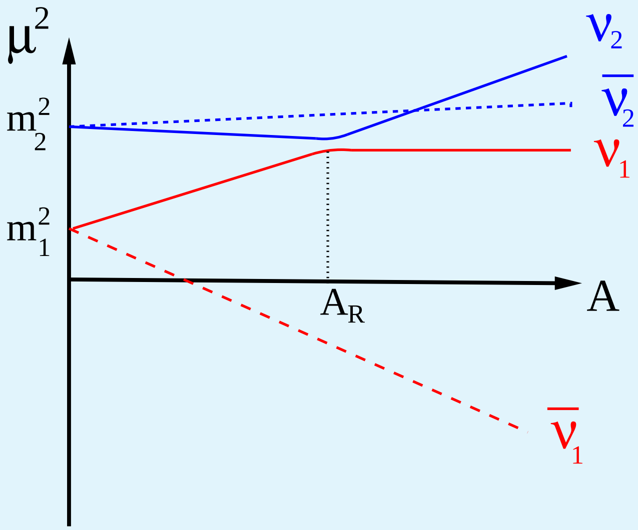
The effective masses: $(A = 2E(V_e - V_X))$

$$\mu_{1,2}(x) = \frac{m_1^2 + m_2^2}{2} + E(V_e + V_X)$$
$$\pm \frac{1}{2}\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}$$



At *resonant* potential: $A_R = \Delta m^2 \cos 2\theta$

Minimum $\Delta\mu^2 = \mu_2^2 - \mu_1^2$

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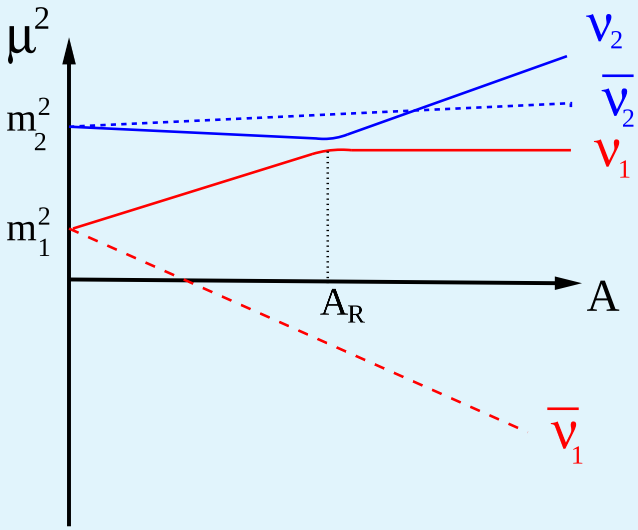

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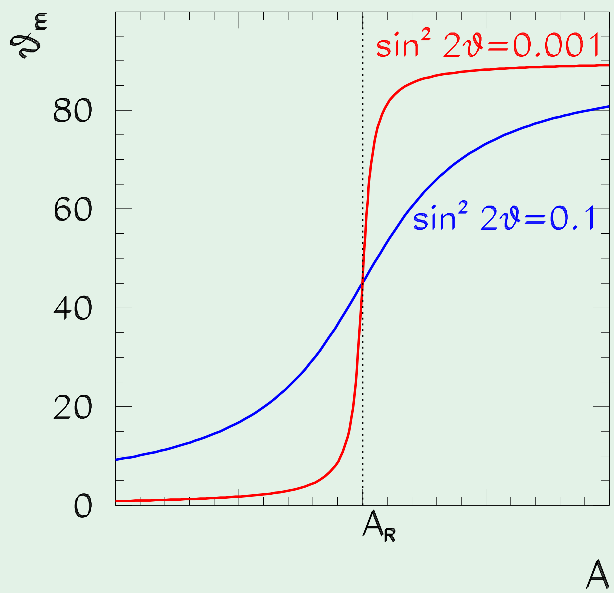
The mixing angle in matter

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A}$$



∗ At $A = 0$ (vacuum) ⇒ $\theta_m = \theta$

∗ At $A = A_R$ ⇒ $\theta_m = \frac{\pi}{4}$

∗ At $A >> A_R$ ⇒ $\theta_m = \frac{\pi}{2}$

The oscillation length in vacuum

$$L_0^{osc} = \frac{4\pi E}{\Delta m^2}$$

The oscillation length in matter

$$L^{osc} = \frac{L_0^{osc}}{\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}} \equiv \frac{4\pi E}{\Delta \mu^2}$$

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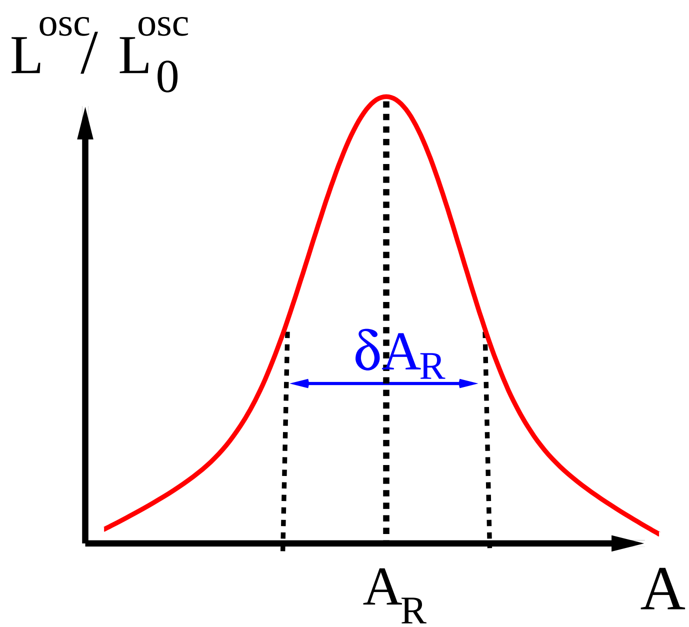
L^{osc} presents a resonant behaviour



At the resonant point

$$L_R^{osc} = \frac{L_0^{osc}}{\sin 2\theta}$$

The width of the resonance in potential:

$$\delta V_R = \frac{\Delta m^2 \sin 2\theta}{E}$$

The width of the resonance in distance:

$$\delta r_R = \frac{\delta V_R}{|\frac{dV}{dr}|_R}$$

• In terms of the mass eigenstates in matter: $$\begin{pmatrix} \nu_e \\ \nu_X \end{pmatrix} = U[\theta_m(x)] \begin{pmatrix} \nu_1^m(x) \\ \nu_2^m(x) \end{pmatrix}$$

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$\Rightarrow$ the evolution equation in flavour basis (removing diagonal part)

$$i \begin{pmatrix} \dot\nu_e \\ \dot\nu_X \end{pmatrix} = \frac{1}{2E} \begin{pmatrix} A - \frac{\Delta m^2}{2}\cos 2\theta & \frac{\Delta m^2}{2}\sin 2\theta \\ \frac{\Delta m^2}{2}\sin 2\theta & \frac{\Delta m^2}{2}\cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_X \end{pmatrix}$$

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$$\Rightarrow\; i\begin{pmatrix}\dot\nu_1^m\\ \dot\nu_2^m\end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\Delta\mu^2(x) & -4\,i\,E\,\dot\theta_m(x) \\ 4\,i\,E\,\dot\theta_m(x) & \Delta\mu^2(x)\end{pmatrix}\begin{pmatrix}\nu_1^m\\ \nu_2^m\end{pmatrix}$$

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$\Rightarrow$ ν_i^m behave approximately as *evolution eigenstates*

$\Rightarrow$ ν_i^m do not mix in the evolution This is the *adiabatic* transition approximation

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$\Rightarrow$ Transitions $\nu_1^m \rightarrow \nu_2^m$ can occur $\equiv$ *Non adiabaticity*

• For $\Delta\mu^2(x) \gg 4\,E\,\dot{\theta}_m(x)$ $\left[\frac{1}{V}\frac{dV}{dx}\Big|_R \ll \frac{\Delta m^2}{2E}\frac{\sin^2 2\theta}{\cos 2\theta}\right]$ $\equiv$ Slowly varying matter potent

$\Rightarrow$ ν_i^m behave approximately as *evolution eigenstates*

$\Rightarrow$ ν_i^m do not mix in the evolution This is the *adiabatic* transition approximation

The adiabaticity condition

$$\frac{1}{V}\frac{dV}{dx}\Big|_R \ll \frac{\Delta m^2}{2E}\frac{\sin^2 2\theta}{\cos 2\theta} \equiv 2\pi\,\delta r_R \gg L_R^{osc}$$

$\Rightarrow$ Many oscillations take place in the resonant region

ν Oscillations: Experimental Probes

- Generically there are two types of experiments to search for ν oscillations :

ν Oscillations: Experimental Probes

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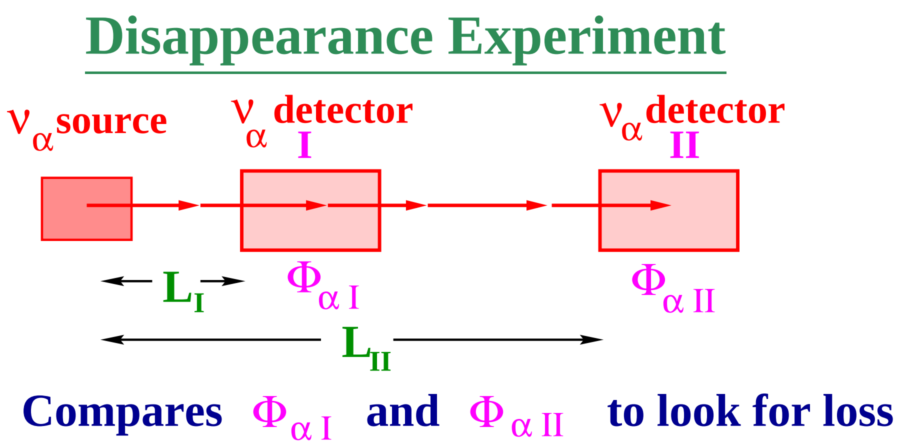
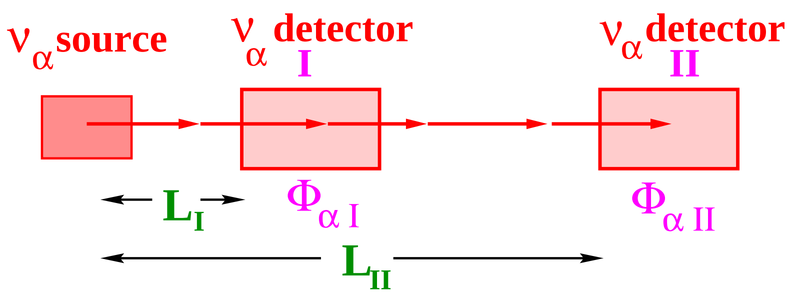


ν Oscillations: Experimental Probes

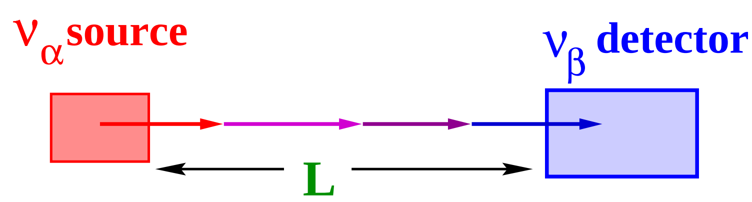
- Generically there are two types of experiments to search for ν oscillations :

Disappearance Experiment



Compares $\Phi_{\alpha\, I}$ **and** $\Phi_{\alpha\, II}$ **to look for loss**

Appearance Experiment



Searches for β diff α

To allow observation of neutrino oscillations:

– Nature has to be good: $\theta \not\ll 0$

– Need the right set up ($\equiv$right $\langle \frac{L}{E} \rangle$) for Δm^2

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Source	E (GeV)	L (Km)	Δm^2 (eV^2)
Solar	10^{-3}	10^7	10^{-10}
Atmospheric	0.1–10^2	10–10^3	10^{-1}–10^{-4}
Reactor	10^{-3}	SBL: 0.1–1	10^{-2}–10^{-3}
		LBL: 10–10^2	10^{-4}–10^{-5}
Accelerator	10	SBL: 0.1	$\gtrsim 0.01$
		LBL: 10^2– 10^3	10^{-2}–10^{-3}

Plan of Lectures

Introduction: The New Minimal Standard Model

Effects of ν mass: Oscillations in Vaccum and Matter

Atmospheric Neutrinos

Solar Neutrinos

Accelerator and Reactor Neutrinos

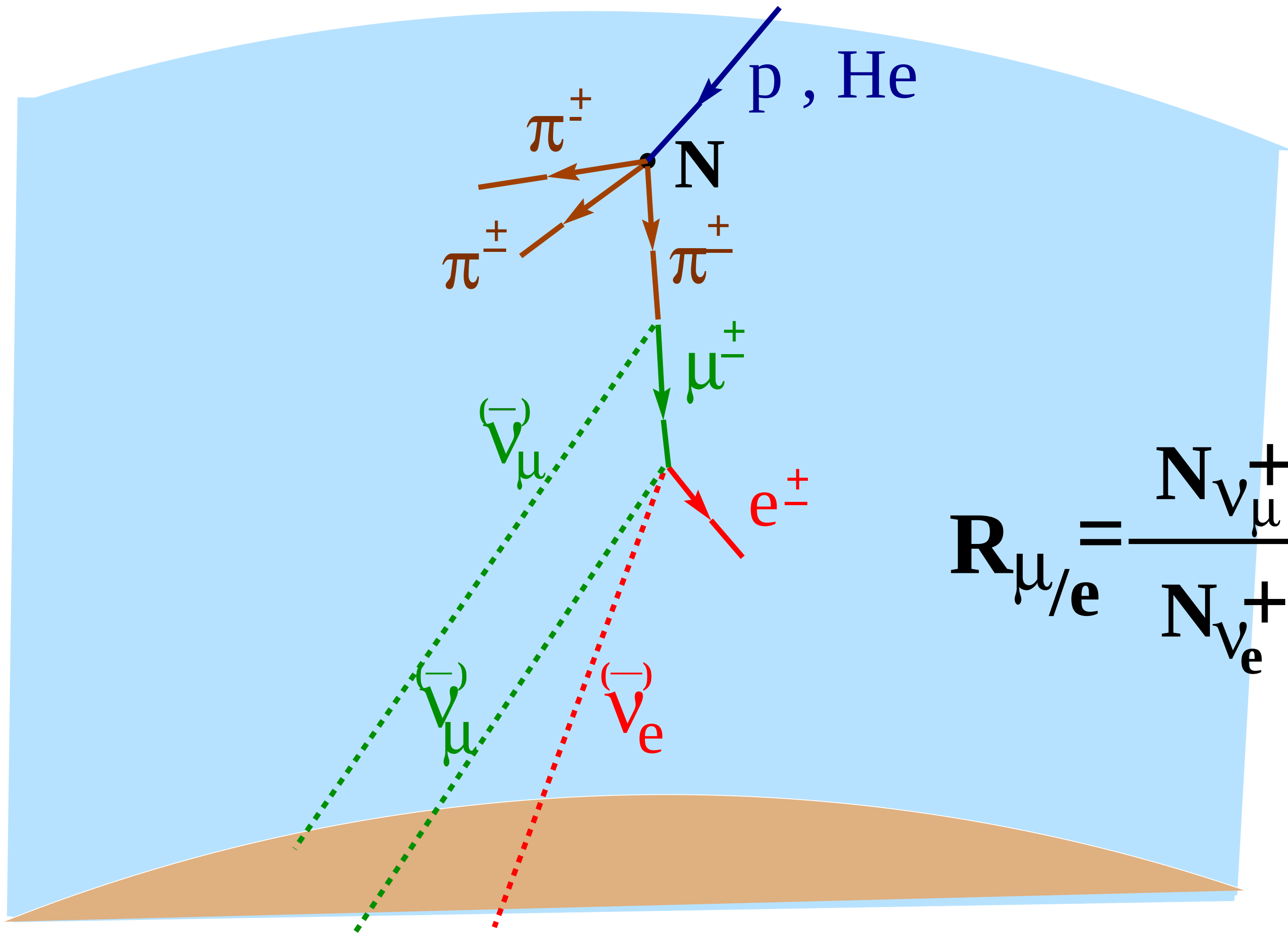
Fitting all Together and Subleading effects

Summary

PS:The Near Future Experimental Program and Its Challenges

Atmospheric Neutrinos

Atmospheric $\nu_{e,\mu}$ are produced by the interaction of cosmic rays (p, He . . .) with the atmosphere



$$R_{\mu/e} = \frac{N_{\nu_\mu} + N_{\bar{\nu}_\mu}}{N_{\nu_e} + N_{\bar{\nu}_e}} \sim 2$$

Atmospheric Neutrinos: Data

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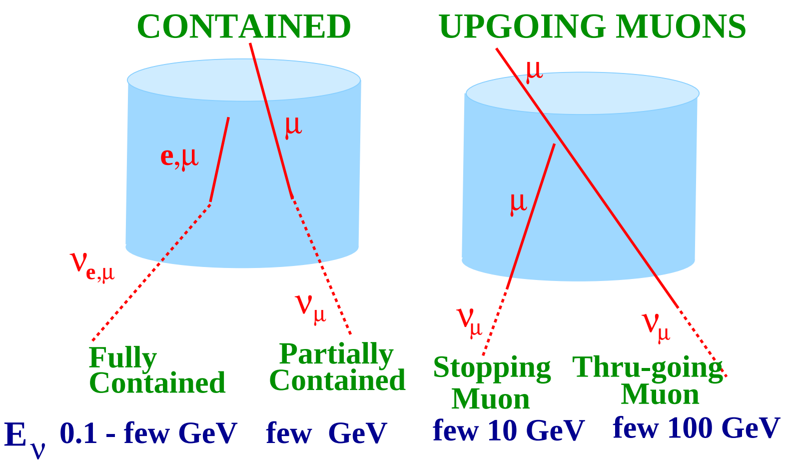
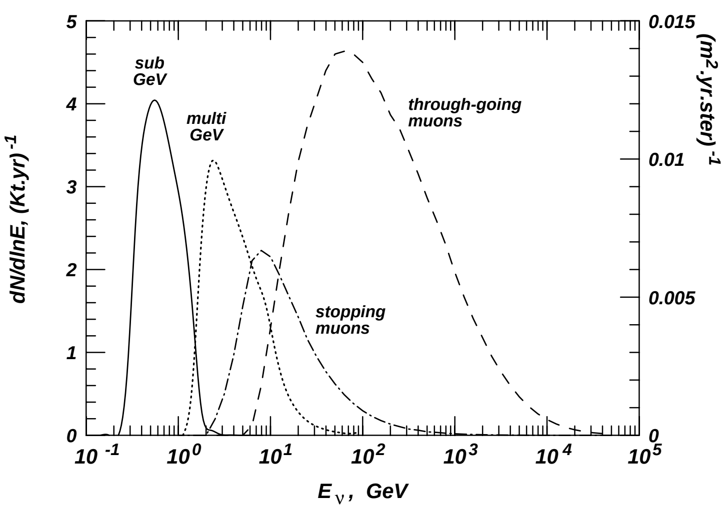
Atmospheric Neutrinos: Data

EVENT CLASSIFICATION





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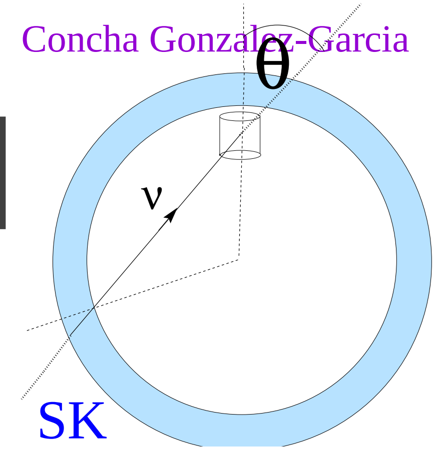

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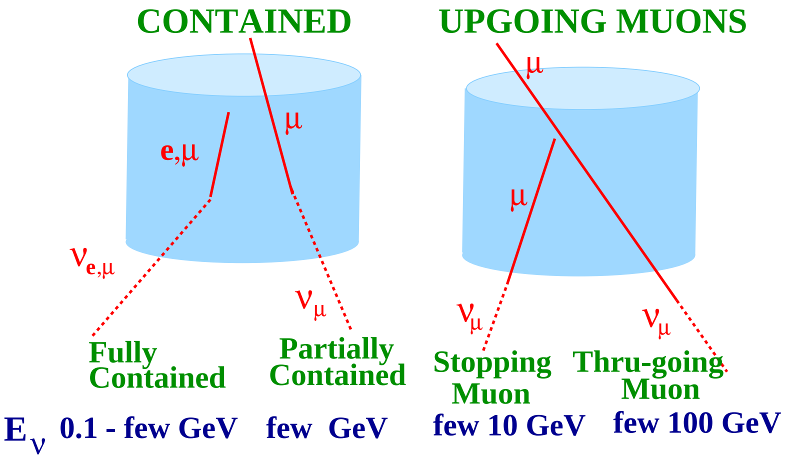
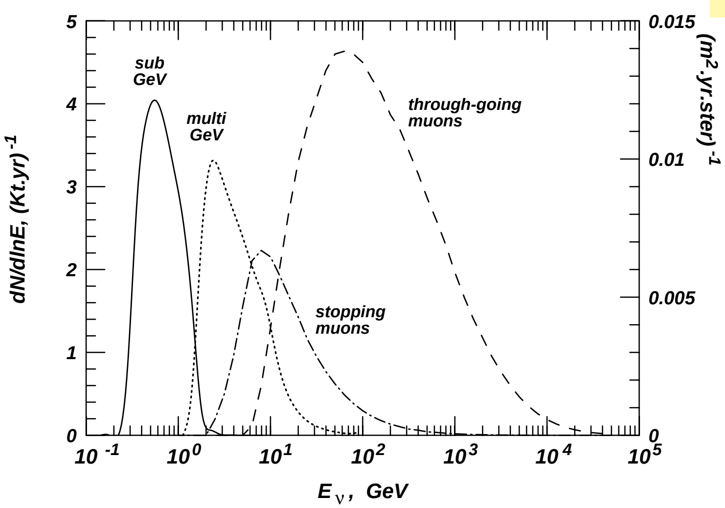
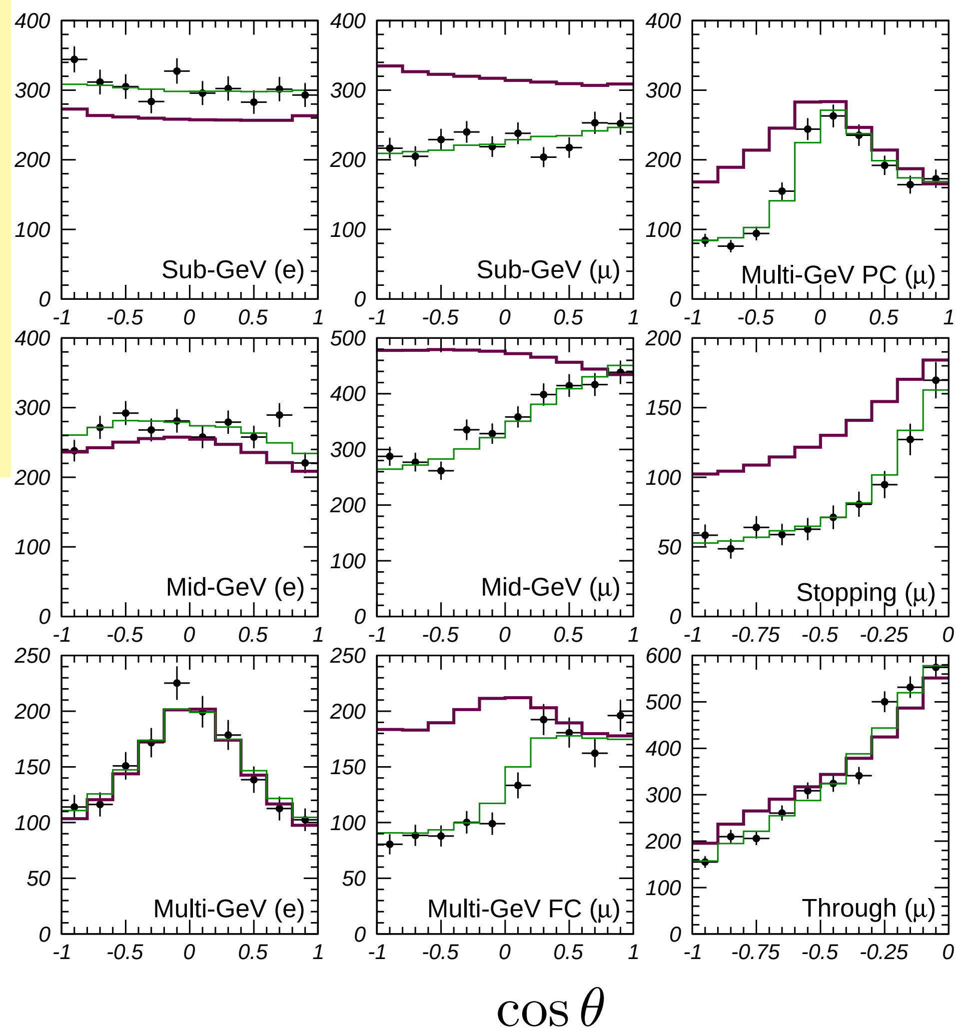




ν_e in agreement with SM

- Angular Distribution at SK



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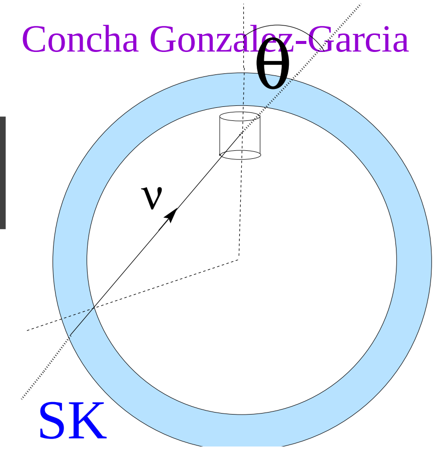

Atmospheric Neutrinos: Data

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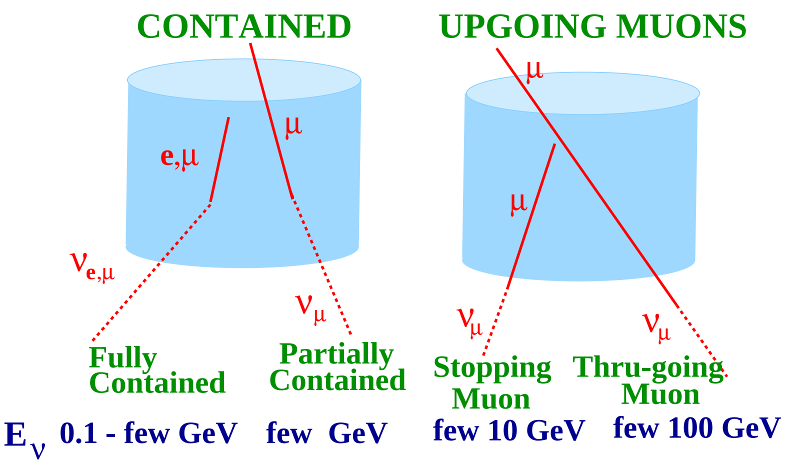

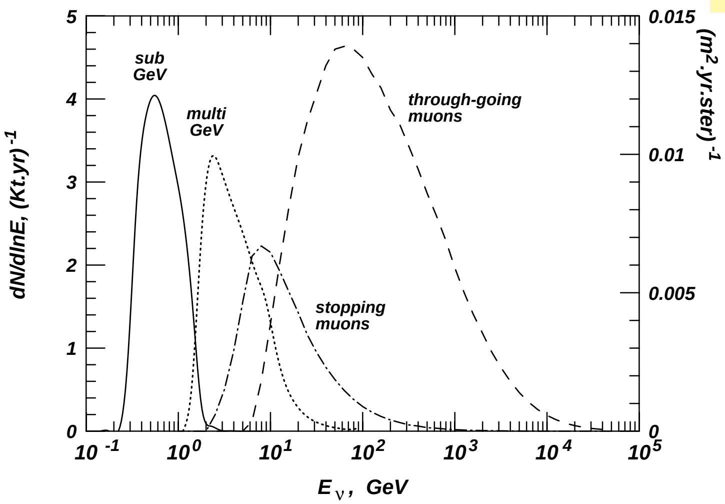

ν_e in agreement with SM

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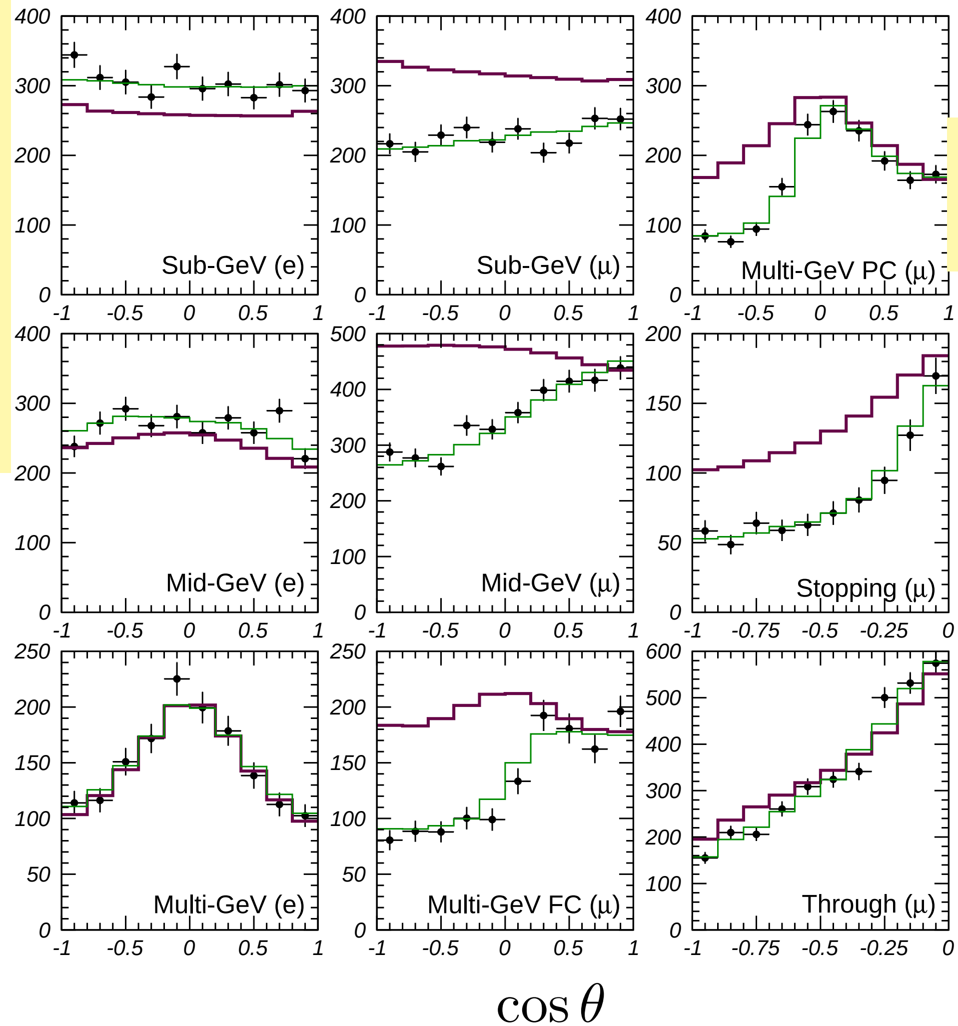

$\rightarrow \nu_\mu$ Deficit grows with L

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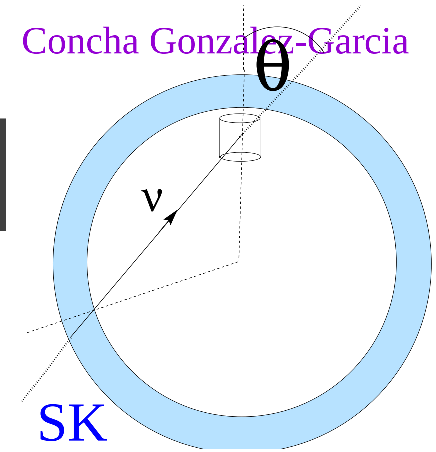
Atmospheric Neutrinos: Data



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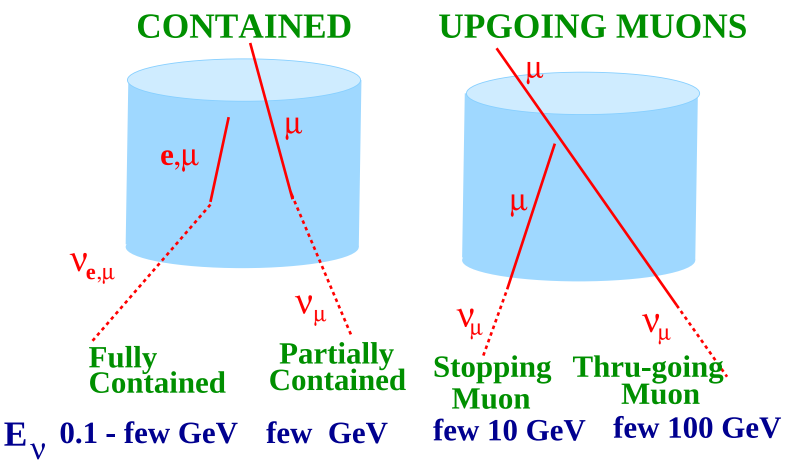
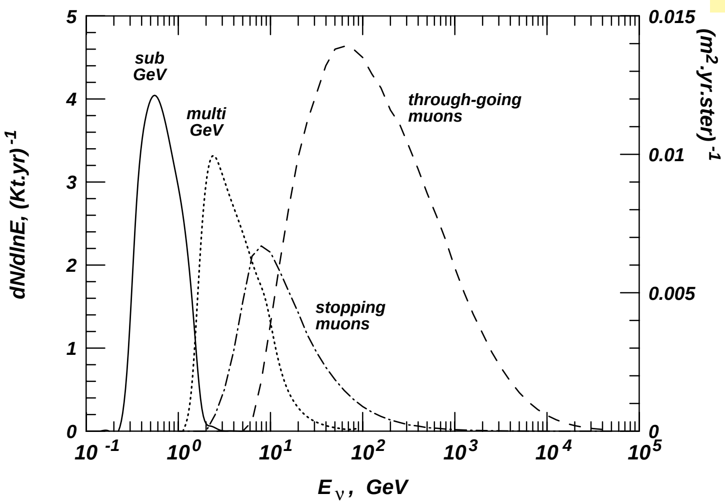
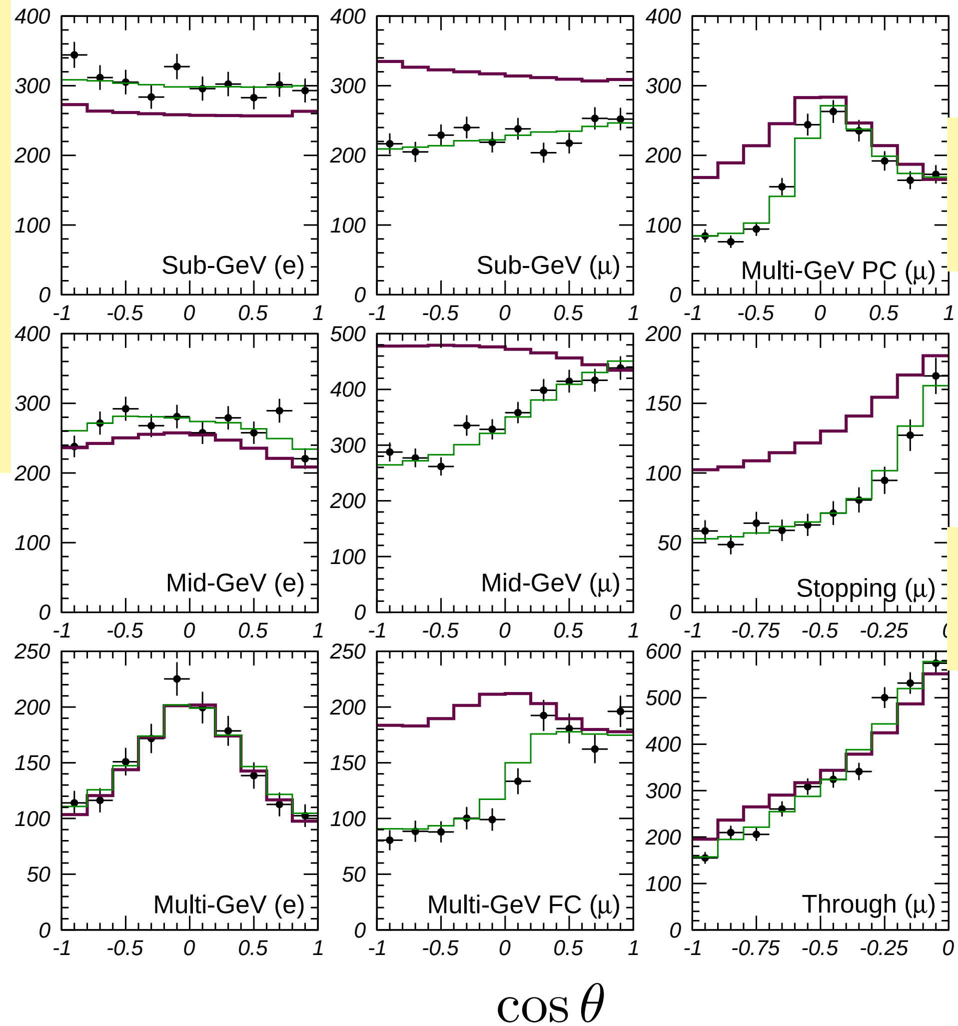




ν_e in agreement with SM

- Angular Distribution at SK



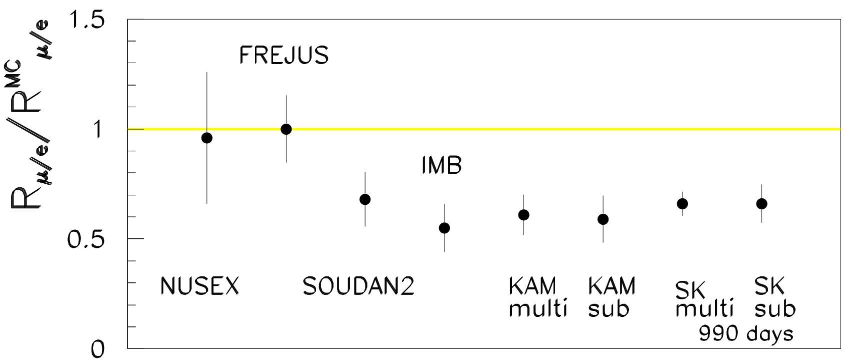
→ ν_μ Deficit grows with L

→ Decreases with E

ATM $\nu_\mu \to \nu_\tau$ Oscillations: Parameter Estimate

ATM $\nu_\mu \to \nu_\tau$ Oscillations: Parameter Estimate

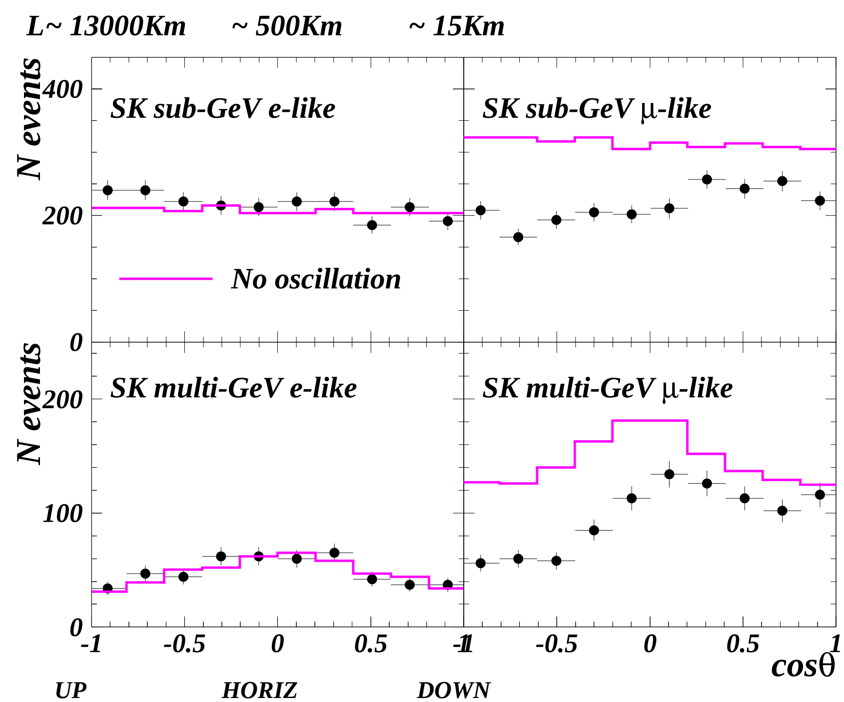
- From Total Contained Event Rates:



$$\langle P_{\mu\mu} \rangle = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{2E}$$
$$\sim 0.5 - 0.7$$

$$\Rightarrow \sin^2 2\theta \gtrsim 0.6$$

- From Angular Distribution:



For $E \sim 1$ GeV deficit at $L \sim 10^2 - 10^4$ Km

$$\frac{\Delta m^2(\mathrm{eV}^2) L(\mathrm{km})}{2E(\mathrm{GeV})} \sim 1$$

$$\Rightarrow \Delta m^2 \sim 10^{-4} - 10^{-2} \mathrm{eV}^2$$

Atmospheric ν Oscillation Analysis

(1) Theoretical Predictions:

– The expected number of contained events

$$R_\alpha(\theta) = \sum_\beta n_t T \int \frac{d^2\Phi_\beta}{dE_\nu d\cos\theta_\nu} P_{\beta\alpha}(E_\nu)\kappa_\beta(h)\frac{d\sigma}{dE_\alpha}\varepsilon(E_\alpha)dE_\nu dE_\alpha d\cos\theta_\nu dh$$

$\Phi_\beta \equiv$ Neutrino Flux $\qquad \kappa_\alpha \equiv$ Neutrino Production Point Distribution

$\frac{d\sigma}{dE_\alpha} \equiv$ Neutrino Interaction Cross Section $\qquad \varepsilon(E_\alpha) \equiv$ Detection Efficiency

– The expected upgoing-μ events:

$$R_\mu(\theta)_{S,T} = \int \frac{d\Phi_\mu(E_\mu, \cos\theta)}{dE_\mu d\cos\theta} A_{S,T}(E_\mu,\theta)dE_\mu$$

$$\frac{d\Phi_\mu}{dE_\mu d\cos\theta} = \int_0^\infty \frac{d\Phi_{\nu_\mu}}{dE_\nu d\cos\theta} P_{\mu\mu}(E_\nu)\frac{d\sigma}{dE_{\mu 0}} F_{rock}(E_{\mu 0}, E_\mu, X) N_A dE_\nu dE_{\nu 0} dX$$

$A_{S,T}(E_\mu,\theta) \equiv$ Detector Effective Area $\qquad F_{rock}(E_{\mu 0}, E_\mu, X) \equiv$ Muon Energy Loss in Rock

Atmospheric ν Oscillation Analysis

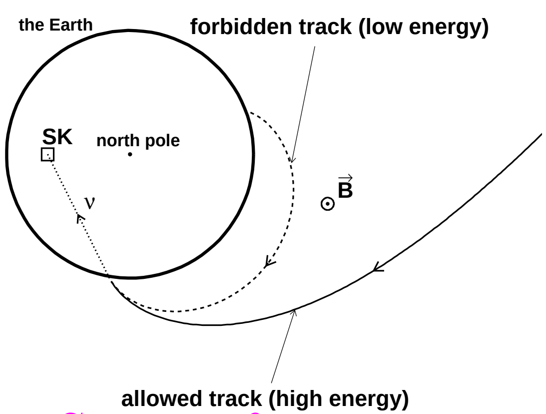
Atmospheric Fluxes

- Several calculations in literature
- Schematically all do:

$$\Phi_\nu = \sum_A \Phi_A \otimes R_A \otimes Y_{A\to\nu}$$

$\Phi_A \equiv$ Cosmic ray spectrum (fit from data)

$R_A \equiv$ Geomagnetic Cutoff (modeled)



$Y_{A\to\nu} \equiv$ Cross Section for

$$A + A_{atmos} \to \nu + X$$

(Main difference between calculations)

Interaction Cross Section

- **QEL**: $\nu + (A, Z) \to l^\pm + (A, Z \pm 1)$
  Dominant at $E_\nu < 1$ GeV
- **1** π: $\nu + N \to l^\pm + N' + \pi \;\; N, N' \equiv p, n$
  Important at $E_\nu \sim 1$ GeV
- **DIS**: $\nu + N \to l^\pm +$ hadrons
  Dominant at $E_\nu >$ few GeV

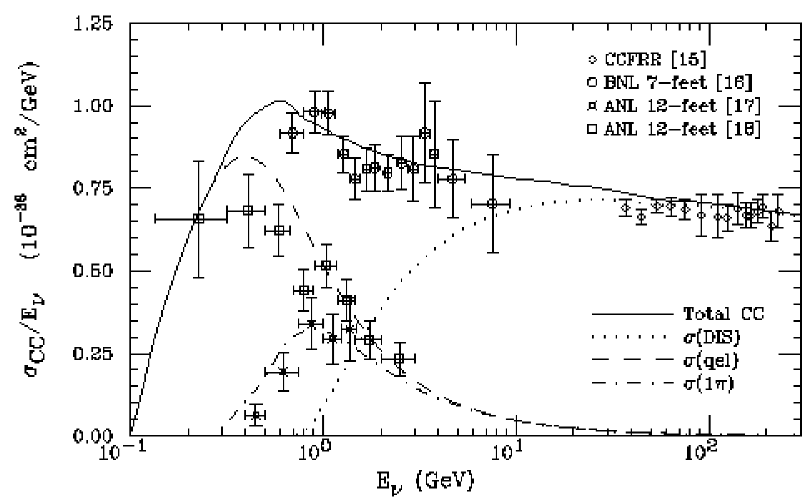


Figure 1

Atmospheric ν Oscillation Analysis

(2) Statistical Analysis:

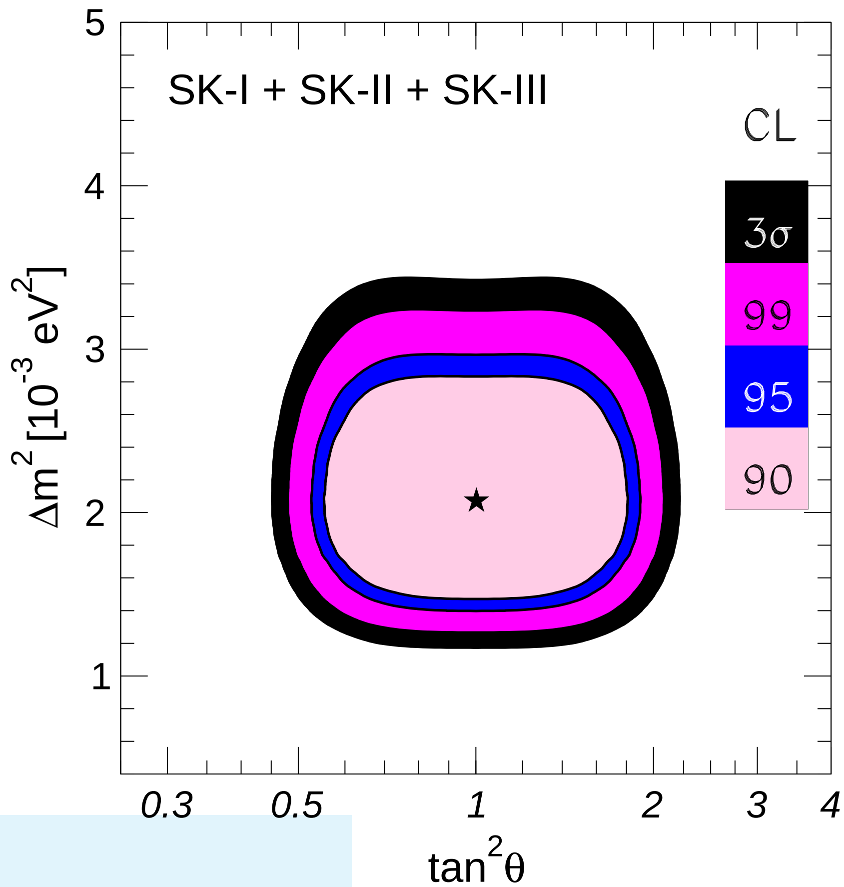
Including 90 data points SKI+II+III data:

- Sub-GeV e-like and μ-like: 10+10 points
- Mid-GeV e-like and μ-like: 10+10 points
- Multi-GeV e-like:10 points
- Multi-GeV FC and PC μ-like: 10+10 points
- Stopping and Thrugoing μ's: 10+10 points

Using 3-dim atmospheric fluxes from Honda

Use "pull" approach for theoretical and systematic errors

$\nu_\mu \to \nu_\tau$



$$\chi^2 = \min_{\xi_i} \left[\sum_{n=1}^{90} \left(\frac{R_n^{\rm theo} - \sum_i \xi_i \sigma_n^i - R_n^{\rm exp}}{\sigma_n^{\rm stat}} \right)^2 + \sum_{i,theory} \xi_i^2 + \sum_{i,syt} \xi_i^2 \right]$$

Include *all* sources of theoretical and systematic uncertainties

Best fit:

$\Delta m^2 \;=\; 2.1 \times 10^{-3}\ {\rm eV}^2$

$\tan^2\theta = 1$

Some New Physics in ATM ν -Oscillations

- Oscillations are due to:
  - Misalignment between CC-int and propagation states: Mixing $\Rightarrow$ Amplitude
  - Difference phases of propagation states $\Rightarrow$ Wavelength. For Δm^2-OSC $\lambda = \frac{4\pi E}{\Delta m^2}$

Some New Physics in ATM ν -Oscillations

- Oscillations are due to:
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- ν masses are not the only mechanism for oscillations

Violation of Equivalence Principle (VEP): Gasperini 88, Halprin,Leung 01

Non universal coupling of neutrinos $\gamma_1 \neq \gamma_2$ *to gravitational potential* ϕ

$$\lambda = \frac{\pi}{E|\phi|\delta\gamma}$$

Violation of Lorentz Invariance (VLI): Coleman, Glashow 97

Non universal asymptotic velocity of neutrinos $c_1 \neq c_2 \Rightarrow E_i = \frac{m_i^2}{2p} + c_i p$

$$\lambda = \frac{2\pi}{E\Delta c}$$

Interactions with space-time torsion: Sabbata, Gasperini 81

Non universal couplings of neutrinos $k_1 \neq k_2$ *to torsion strength* Q

$$\lambda = \frac{2\pi}{Q\Delta k}$$

Violation of Lorentz Invariance (VLI) Colladay, Kostelecky 97; Coleman, Glashow 99

due to CPT violating terms: $\bar{\nu}_L^\alpha b_\mu^{\alpha\beta}\gamma_\mu \nu_L^\beta \Rightarrow E_i = \frac{m_i^2}{2p} \pm b_i$

$$\lambda = \pm\frac{2\pi}{\Delta b}$$

Non-standard ν interactions in matter: Wolfenstein 78

$G_F \varepsilon_{\alpha\beta}(\overline{\nu_\alpha}\gamma^\mu \nu_\beta)(\overline{f}\gamma_\mu f)$

$$\lambda = \frac{2\pi}{2\sqrt{2}G_f\ N_f\ \sqrt{\varepsilon_{\alpha\beta}^2 + (\varepsilon_{\alpha\alpha} - \varepsilon_{\beta\beta})^2/4}}$$

ATM ν's: Subdominant NP Effects

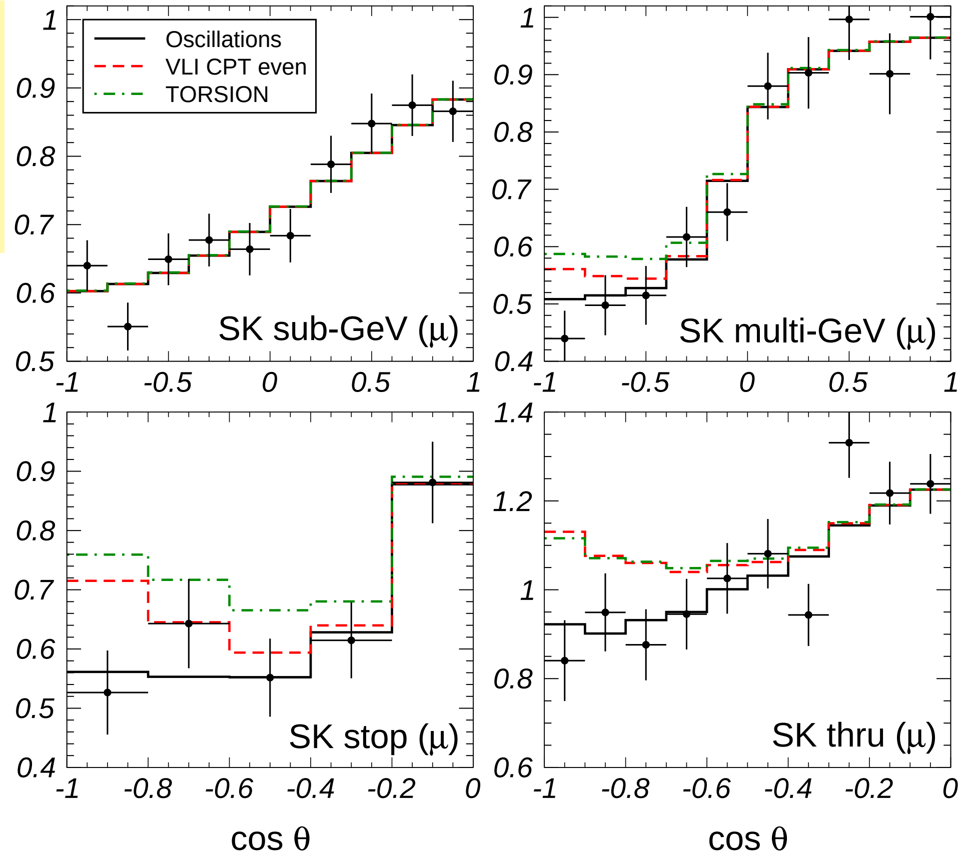
$\lambda_{NP} \propto \frac{1}{E^n \Delta\delta_n}$ with $n \neq -1$

$$P_{\nu_\mu \to \nu_\mu} = 1 - \sin^2 2\Theta \, \sin^2\left(\frac{\Delta m^2 L}{4E} \mathcal{R}\right)$$

$$\mathcal{R} \cos 2\Theta = \cos 2\theta + \sum_n R_n \cos 2\xi_n$$

$$\mathcal{R} \sin 2\Theta = \sin 2\theta + \sum_n R_n \sin 2\xi_n \, e^{i\eta_n}$$

$$R_n = \sigma_n^+ \frac{\Delta\delta_n E^n}{2} \frac{4E}{\Delta m^2}$$



- Questions:
  - Do these effects affect our determination of oscillation parameters?
  - Can we limit these effects?

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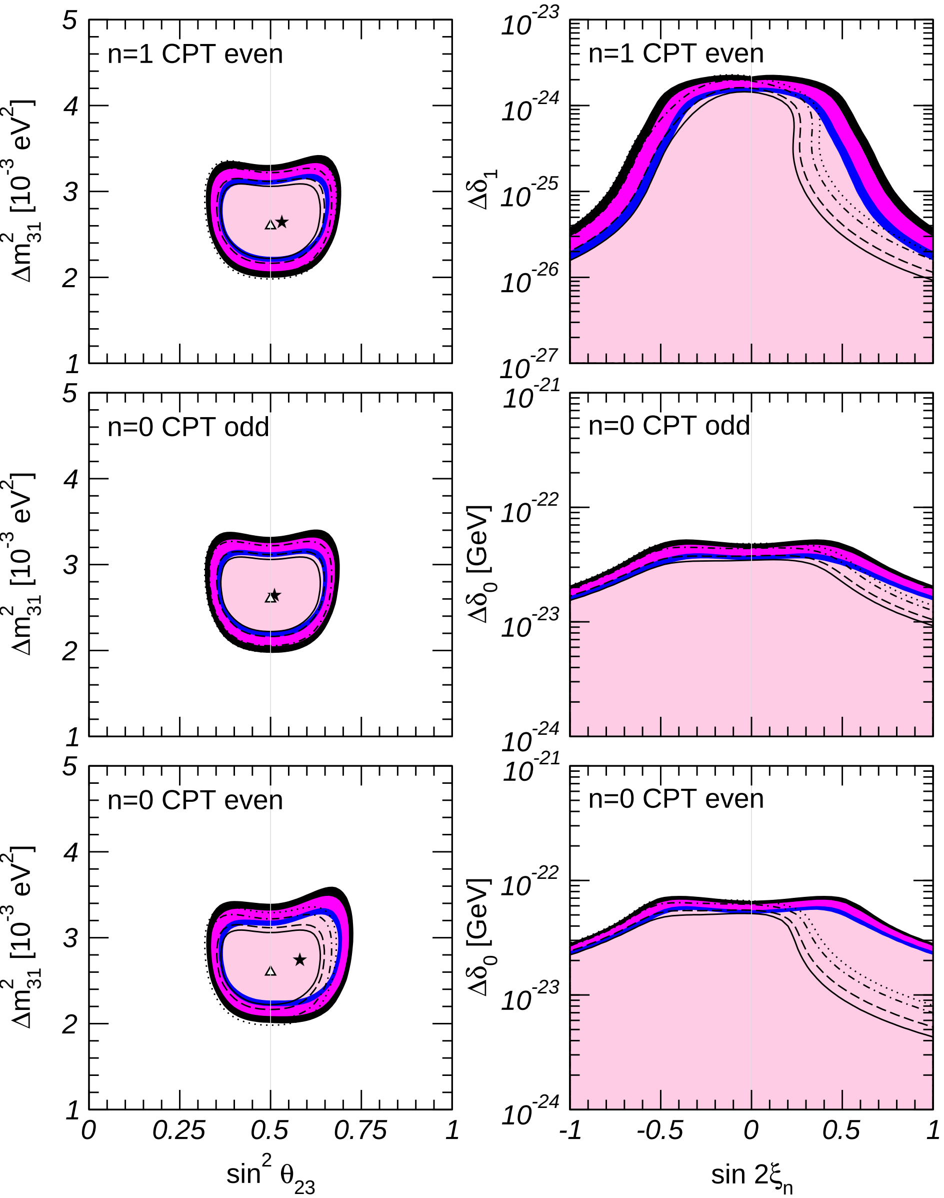
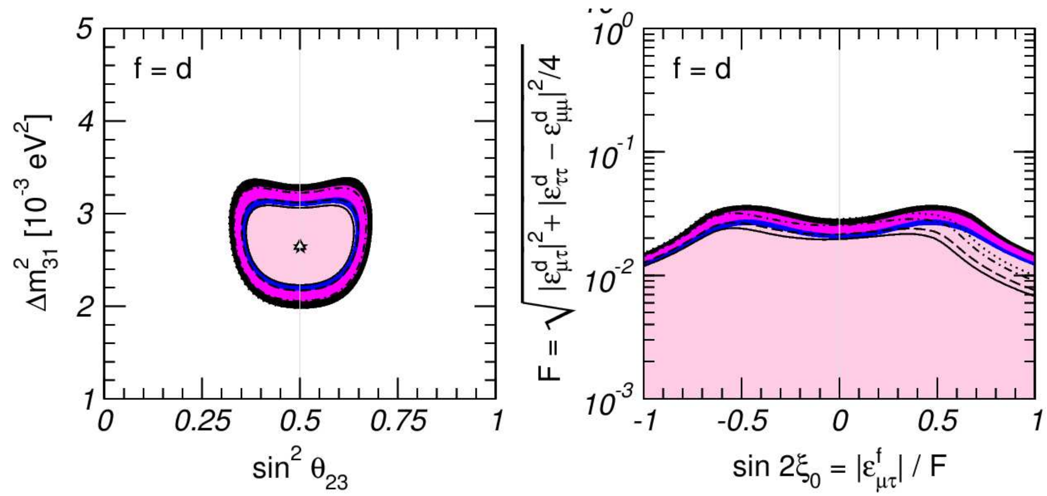
ATM ν's: Subdominant NP Effects





At 90% CL:

$$\frac{|\Delta c|}{c} \leq 1.2 \times 10^{-24}$$

$$|\phi\,\Delta\gamma| \leq 5.9 \times 10^{-25}$$

$$|\Delta b| \leq 3.0 \times 10^{-23}\ \mathrm{GeV}$$

$$|Q\,\Delta k| \leq 4.8 \times 10^{-23}\ \mathrm{GeV}$$

$$|\varepsilon^d_{\mu\mu} - \varepsilon^d_{\tau\tau}| \leq 0.012$$

$$|\varepsilon^d_{\mu\tau}| \leq 0.038$$

Learning About ATM Fluxes

$$R_\beta = n_t T \sum_\alpha \int \frac{d^2\Phi_\alpha}{dE_\nu d\cos\theta_\nu} \kappa_\alpha(h) \frac{d\sigma}{dE_{l,\beta}} \varepsilon(E_\nu, E_{l,\beta}) dE_\nu dE_{l,\beta} P_{\alpha\beta}(E_\nu, \cos\theta)) d\cos\theta_\nu dh$$

- Question? Can we Extract ($\equiv$ *Deconvolute*) Φ_α from ATM ν data?
  - ∗ Answer : Yes, but you need:
    - – Independent knowledge of oscillation parameters (OK)
    - – General enough analytical parametrization of fluxes (MISSING)

Learning About ATM Fluxes

$$R_\beta = n_t T \sum_\alpha \int \frac{d^2\Phi_\alpha}{dE_\nu d\cos\theta_\nu} \kappa_\alpha(h) \frac{d\sigma}{dE_{l,\beta}} \varepsilon(E_\nu, E_{l,\beta}) dE_\nu dE_{l,\beta} P_{\alpha\beta}(E_\nu, \cos\theta)) d\cos\theta_\nu dh$$

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  - $*$ Answer : Yes, but you need:
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    - – General enough analytical parametrization of fluxes (MISSING)

      Or Neural Network parametrization of fluxes

- Our First Attempt: Extract only E dependence using SK data

$$\Rightarrow \Phi_{\alpha,net}(E_\nu, \cos\theta) = F_{net}(E_\nu) \Phi_{\alpha,calc}(E_\nu, \cos\theta)$$

Procedure:

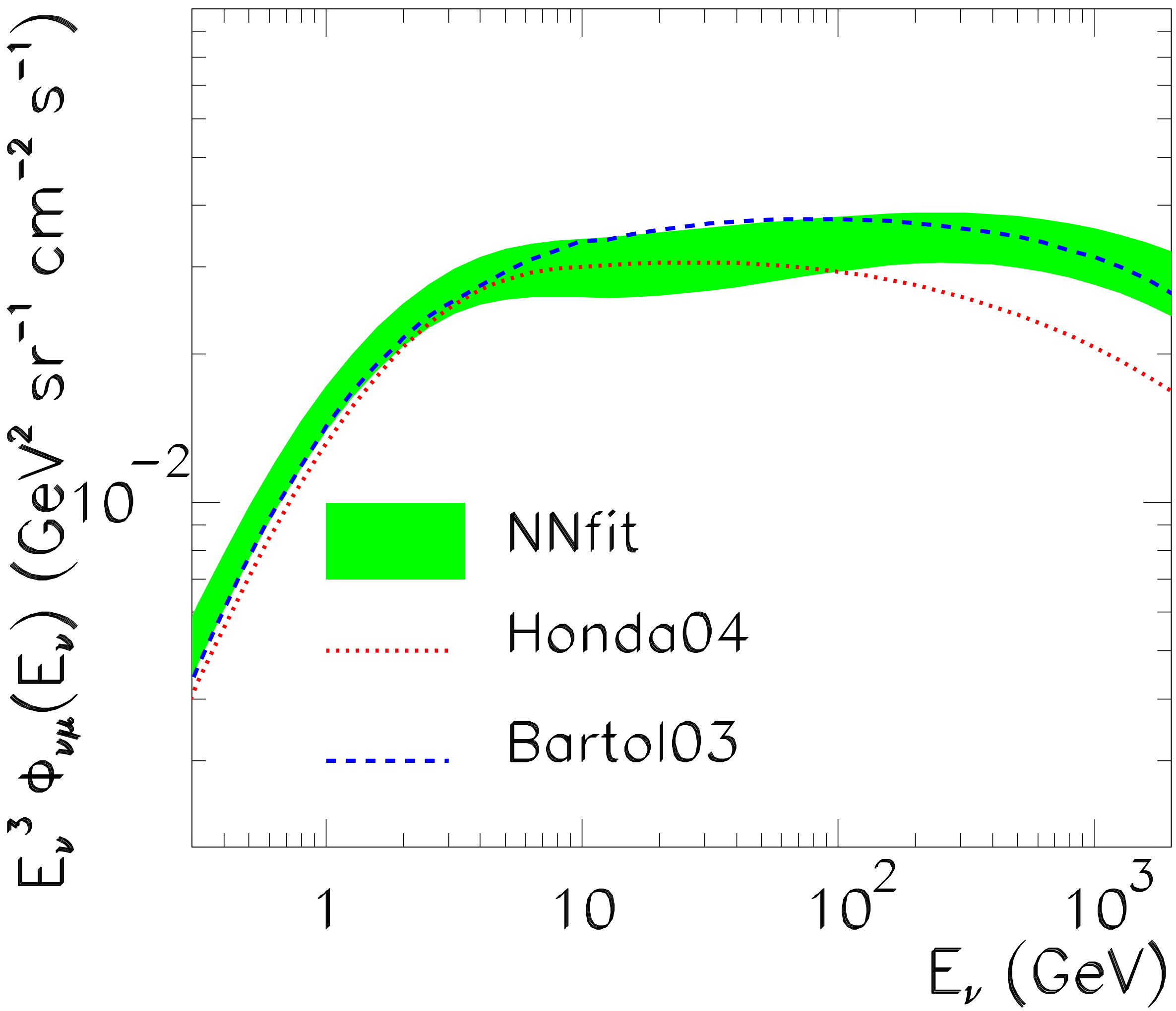
(1) Generate N_R Replicas of Data according to all uncertainties:
Statistical, Systematic, Theo from Cross Section ...

(2) Train Network to each Replica k to get best fit flux $\Phi_\alpha^{(\mathrm{net})(k)}(E_\nu, \cos\theta)$
$\Rightarrow$ Chose some statistical criterion to define "best fit" avoiding *overlearning*

(3) Define average and range of fluxes:

$$\left\langle \Phi_\alpha^{(\mathrm{net})} \right\rangle_{\mathrm{rep}} (E_\nu, \cos\theta) = \frac{1}{N_{\mathrm{rep}}} \sum_{k=1}^{N_{\mathrm{rep}}} \Phi_\alpha^{(\mathrm{net})(k)} \qquad \sigma_\Phi^2 = \frac{1}{N_{\mathrm{rep}}} \sum_{k=1}^{N_{\mathrm{rep}}} \Phi^{(\mathrm{net})(k)\,2} - \left\langle \Phi^{(\mathrm{net})} \right\rangle_{\mathrm{rep}}^2$$

Extracted ATM Fluxes from SK Data



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Solar Neutrinos: The Solar Models

- Sun=Main sequence star 4.6×10^9 yr old
- Solar Models describes the Sun based on:

Surface Luminosity:	$L_\odot = 3.9 \times 10^{33}$ erg/sec
Surface Temperature:	$T_{s\odot} = 5.8 \times 10^3$ K
Solar Mass:	$M_\odot = 2 \times 10^{33}$ gr
Solar Radius:	$R_\odot = 7 \times 10^5$ km

- Basic assumptions:

The Sun is spherically symmetric

Hydrostatic and Thermal equilibrium

Equation of state of an ideal gas

- The total energy emited per nucleon

$$\frac{(4.6 \times 10^9 \text{ yr}) \times (3.86 \times 10^{33} \text{ erg/sec})}{(2 \times 10^{33} \text{ gr}) \times N_{AV}} \simeq 3 \times 10^5; \text{eV}$$

$$\Downarrow$$

Energy is Produced by Nuclear Reactions

- Composition:∼70.5 % p, 27.5 % ^{4}He and 2% heavier elements
- The Sun consists of 3 zones:

– **Core**: $R \lesssim 0.3 R_\odot$ where nuclear reactions

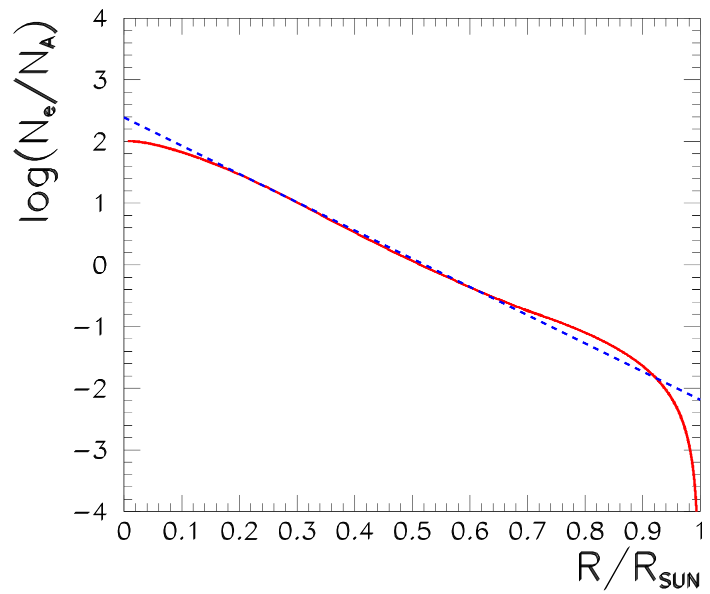
– **Radiation Zone**: $0.3 R_\odot \lesssim R \lesssim 0.7 R_\odot$

– **Convective Zone**: $0.7 R_\odot \lesssim R$

- Heavier elements in the core

$$N_e/N_n \simeq 2(core) - 6(surface)$$

- The Solar density distribution



Solar Neutrinos: Fluxes

Solar Neutrinos: Fluxes

- The Sun shines converting protons into α, e^+ and $\nu' s$

  $4\,p \;\longrightarrow\; {}^4He \;+\; 2\,e^+ \;+\; 2\,\nu_e \;+\; \gamma$

  $4m_p - m_{^4He} - 2m_e \simeq 26$ MeV Thermal energy mostly in γ
- Two major chains of nuclear reactions

Solar Neutrinos: Fluxes

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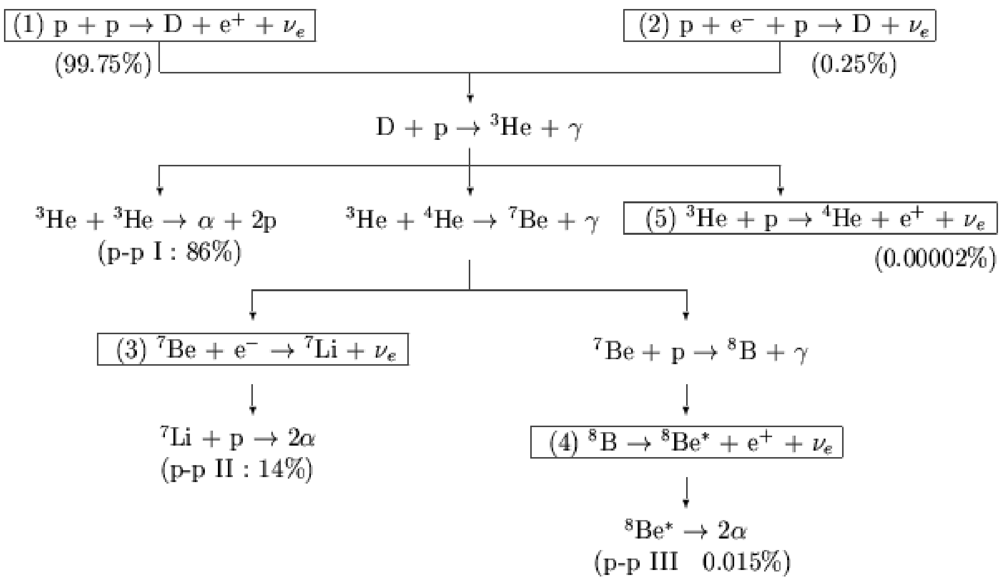
  pp chain:



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Solar Neutrinos: Fluxes

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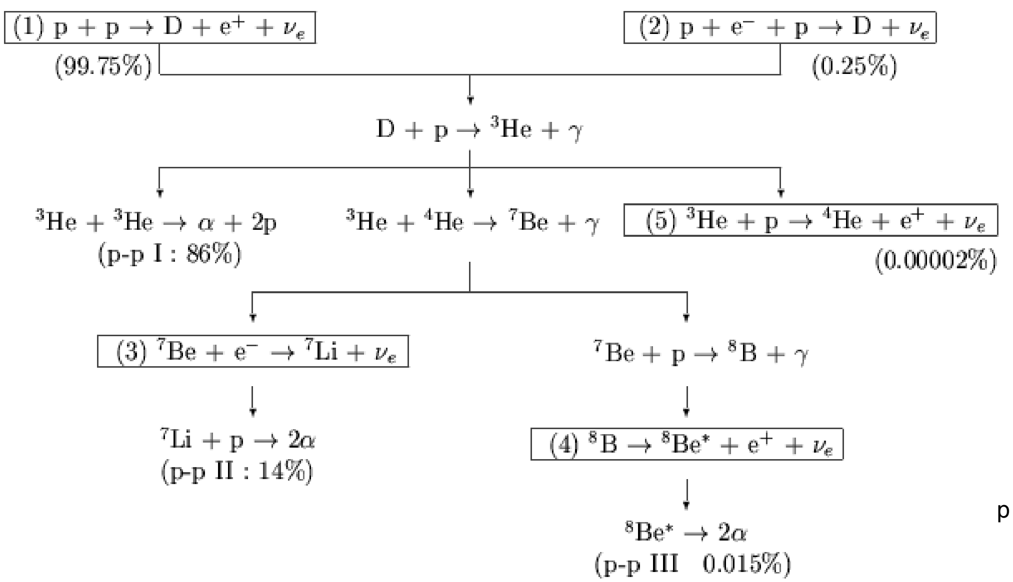
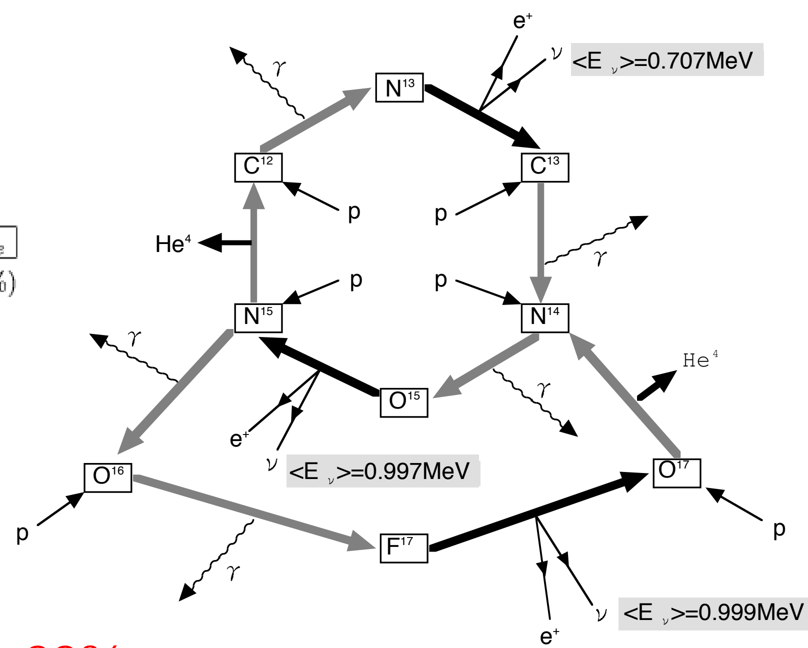
pp chain: CNO cycle:





Present Solar Model ⇒ pp-chain dominates by 99%

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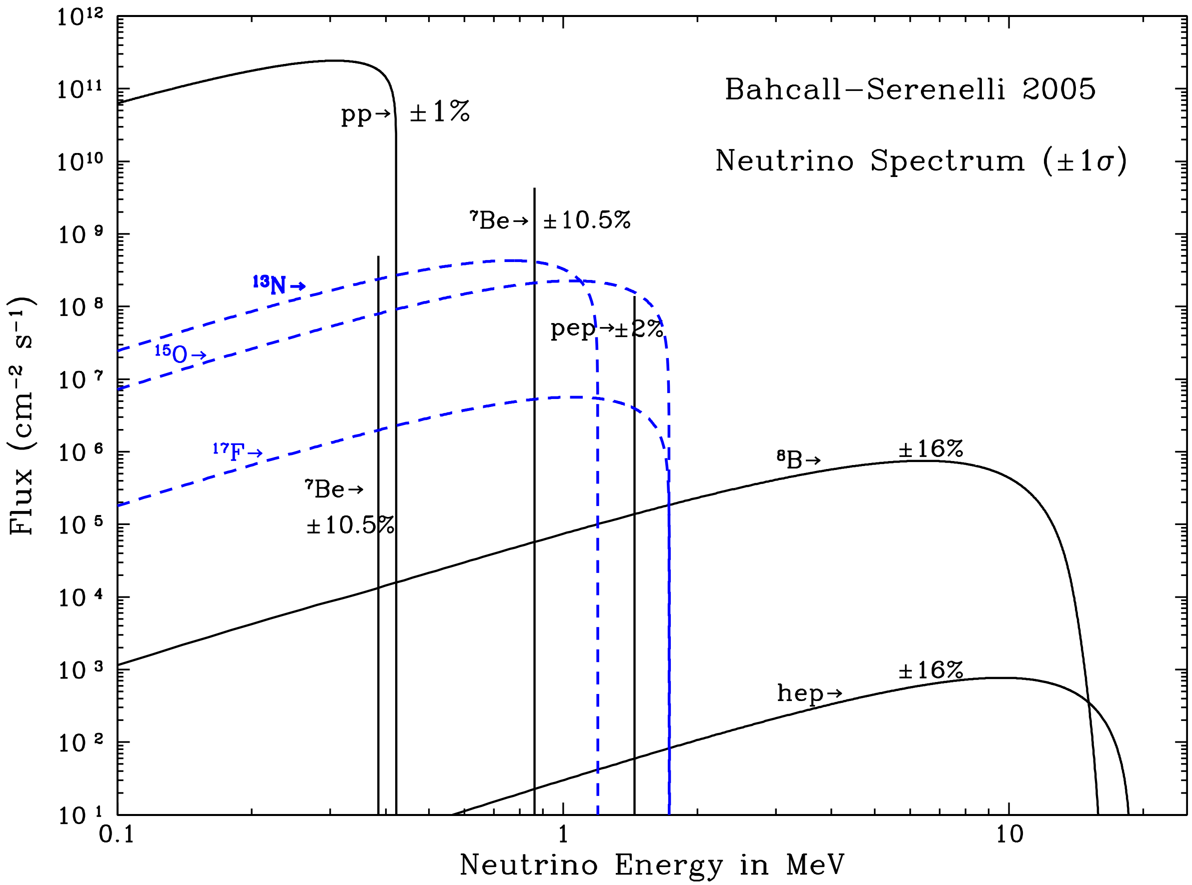
Solar Neutrinos: Fluxes

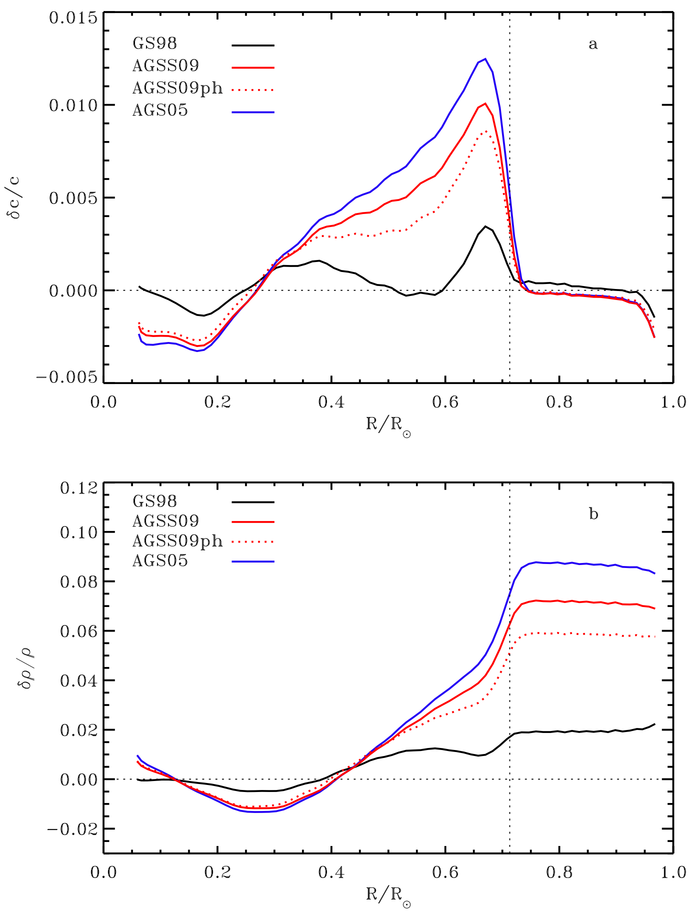


PP CHAIN	E_ν (MeV)
(pp)	
$p + p \rightarrow^2 H + e^+ + \nu_e$	≤ 0.42
(pep)	
$p + e^- + p \rightarrow^2 H + \nu_e$	1.552
(^{7}Be)	
$^7Be + e^- \rightarrow^7 Li + \nu_e$	$0.862(90\%)$
	0.384 (10%)
(hep)	
$^2He + p \rightarrow^4 He + e^+ + \nu_e$	≤ 18.77
(^{8}B)	
$^8B \rightarrow^8 Be^* + e^+ + \nu_e$	≤ 15
CNO CHAIN	E_ν (MeV)
(^{13}N)	
$^{13}N \rightarrow^{13} C + e^+ + \nu_e$	≤ 1.199
(^{15}O)	
$^{15}O \rightarrow^{15} N + e^+ + \nu_e$	≤ 1.732
(^{17}F)	
$^{17}F \rightarrow^{17} O + e^+ + \nu_e$	≤ 1.74

Solar Neutrinos:the Solar Composition Problem

– Newer determination of abundance of heavy elements in solar surface give lower values

– Solar Models with these lower metalicities fail in reproducing helioseismology data



– Two sets of SSM:

Starting from Bahcall *etal* 05,now Serenelli *etal* 0909.2

GS98 uses older metalicities

AGSXX uses newer metalicities

Flux cm^{-2} s^{-1}	GS98	AGSS09
$pp/10^{10}$	$5.97\ (1 \pm 0.006)$	$6.03\ (1 \pm 0.005)$
$pep/10^{8}$	$1.41\ (1 \pm 0.011)$	$1.44\ (1 \pm 0.010)$
$hep/10^{3}$	$7.91 (1 \pm 0.15)$	$8.18\ (1 \pm 0.15)$
$^{7}Be/10^{9}$	$5.08\ (1 \pm 0.06)$	$4.64\ (1 \pm 0.06)$
$^{8}B/10^{6}$	$5.88\ (1 \pm 0.11)$	$4.85\ (1 \pm 0.12)$
$^{13}N/10^{8}$	$2.82\ (1 \pm 0.14)$	$2.07 (1^{+0.14}_{-0.13})$
$^{15}O/10^{8}$	$2.09\ (1^{+0.16}_{-0.15})$	$1.47\ (1^{+0.16}_{-0.15})$
$^{17}F/10^{16}$	$5.65\ (1^{+0.17}_{-0.16})$	$3.48\ (1^{+0.17}_{-0.16})$

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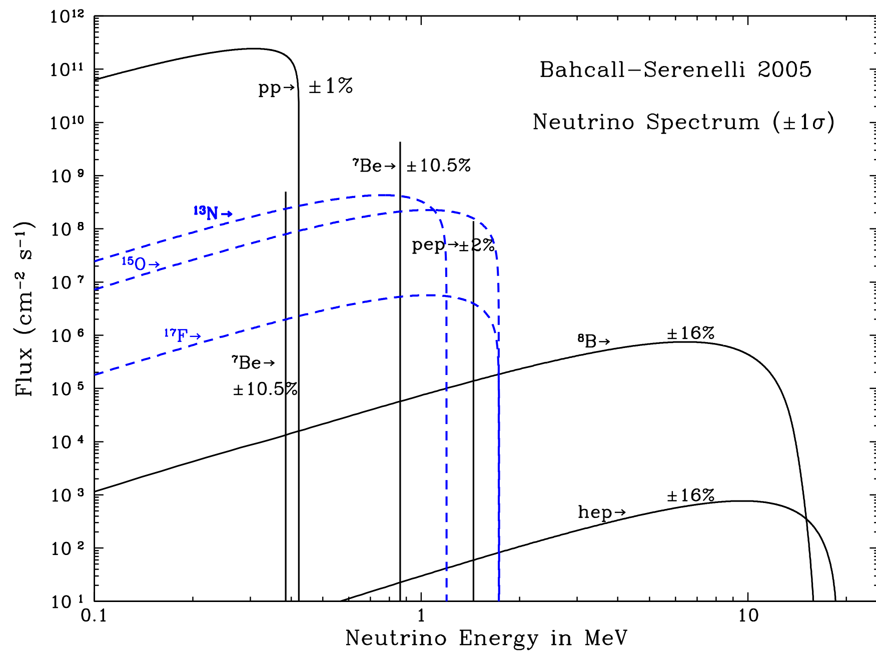
Solar Neutrinos: Data

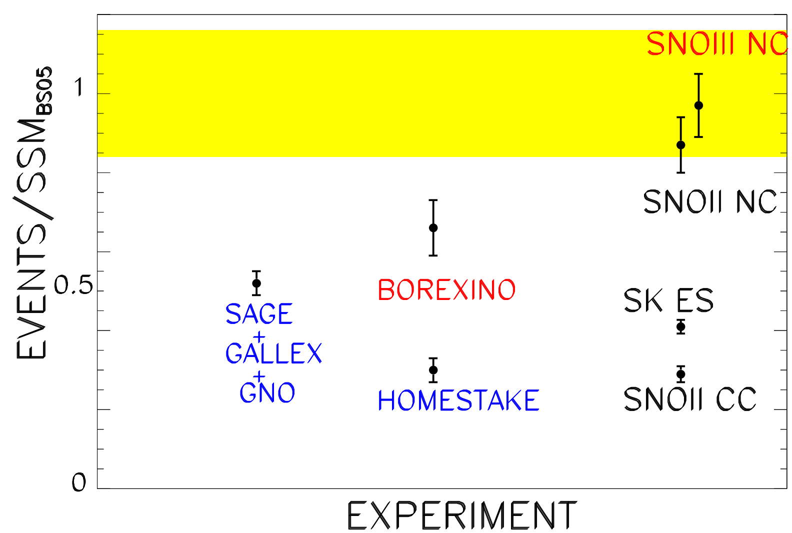
	Experiment	Detection	Flavour	$E_{\rm th}$ (MeV)
radio-chemical	Homestake	$^{37}\mathrm{Cl}(\nu,e^-)^{37}\mathrm{Ar}$	ν_e	$E_\nu > 0.81$
	Sage + Gallex+GNO	$^{71}\mathrm{Ga}(\nu,e^-)^{71}\mathrm{Ge}$	ν_e	$E_\nu > 0.23$
real time	Kam ⇒ SK	ES $\nu_x e^- \to \nu_x e^-$	$\nu_e, \nu_{\mu/\tau}$ $\left(\frac{\sigma_{\mu\tau}}{\sigma_e} \simeq \frac{1}{6}\right)$	$E_e > 5$
	SNO	CC $\nu_e d \to p p e^-$	ν_e	$T_e > 5$
		NC $\nu_x d \to \nu_x p\, n$	$\nu_e, \nu_{\mu/\tau}$	$T_\gamma > 5$
		ES $\nu_x e^- \to \nu_x e^-$	$\nu_e, \nu_{\mu/\tau}$	$T_e > 5$
	Borexino	$\nu_x e^- \to \nu_x e^-$	$\nu_e, \nu_{\mu/\tau}$	$E_\nu = 0.862$



Experiments measuring ν_e found deficit

Deficit is energy dependent

Deficit disappears in NC



- Real Time experiments can also give information on Energy and Direction of $\nu' s$ and can search for Energy and Time variations of the effect

- From SK (also from SNO)

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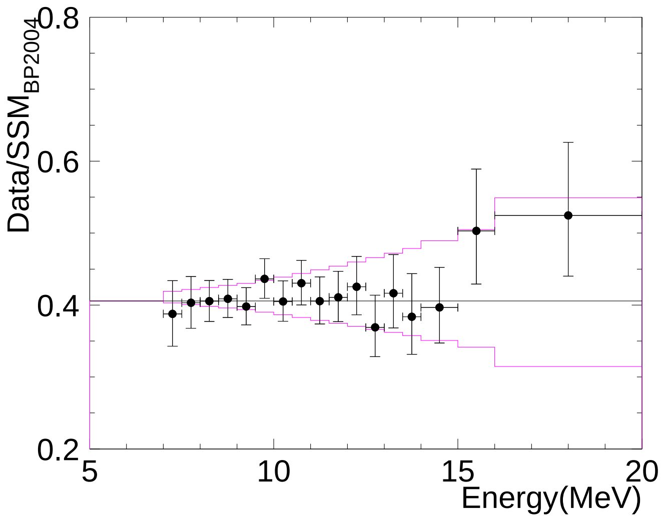
Energy Dependence



Deficit indep $E_\nu \gtrsim 5$ MeV

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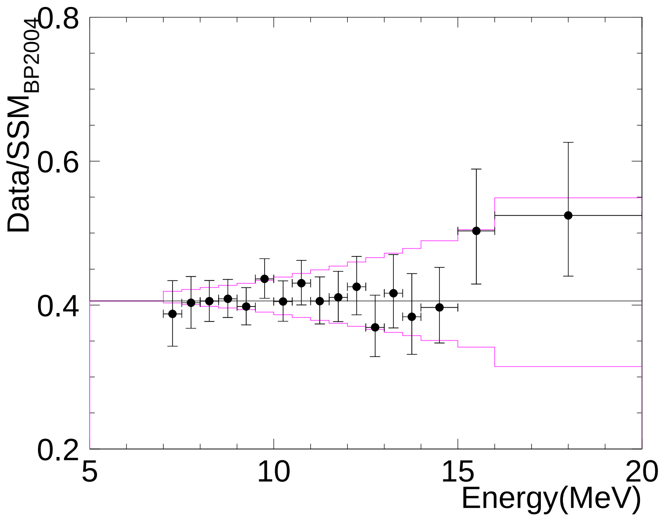
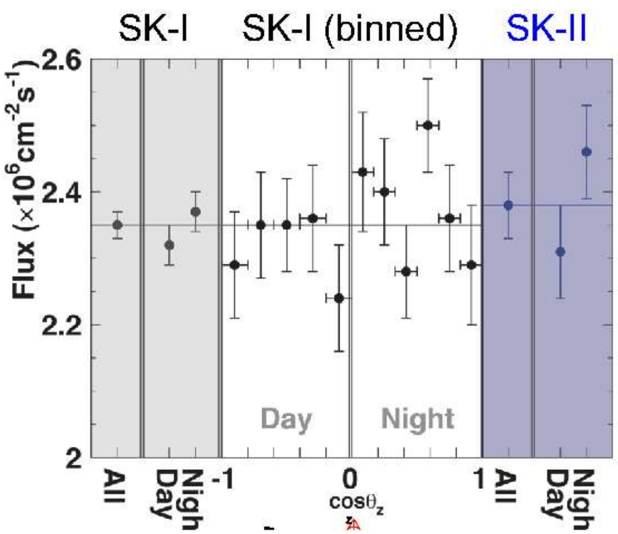


Day-Night Variation



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Not significant

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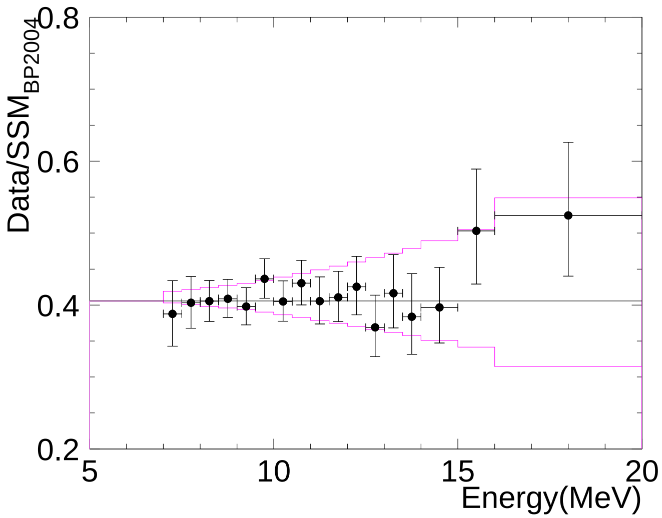

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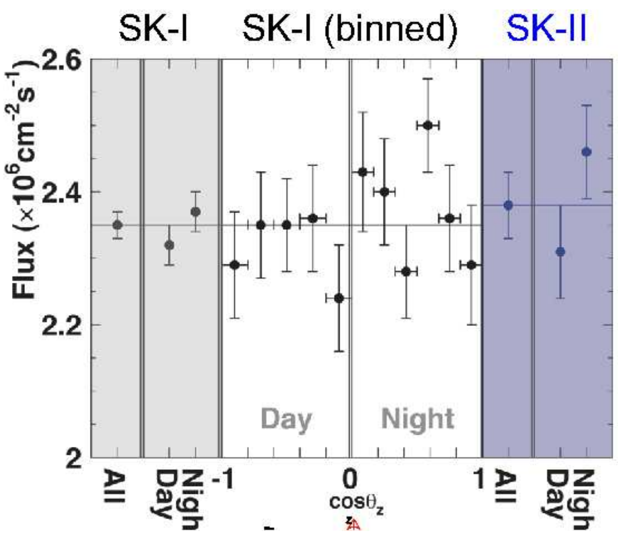
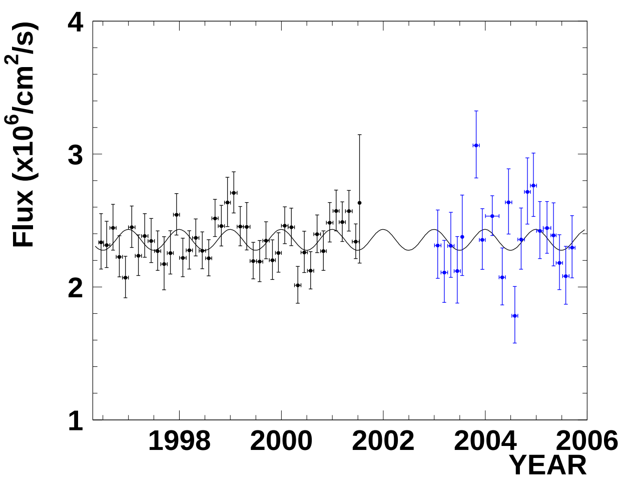


Seasonal Variation



Deficit indep $E_\nu \gtrsim 5$ MeV

Not significant

Nothing beyond $\frac{1}{R^2}$

Neutrinos in The Sun : MSW Effect

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- Solar neutrinos are ν_e produced in the core ($R \lesssim 0.3 R_\odot$) of the Sun

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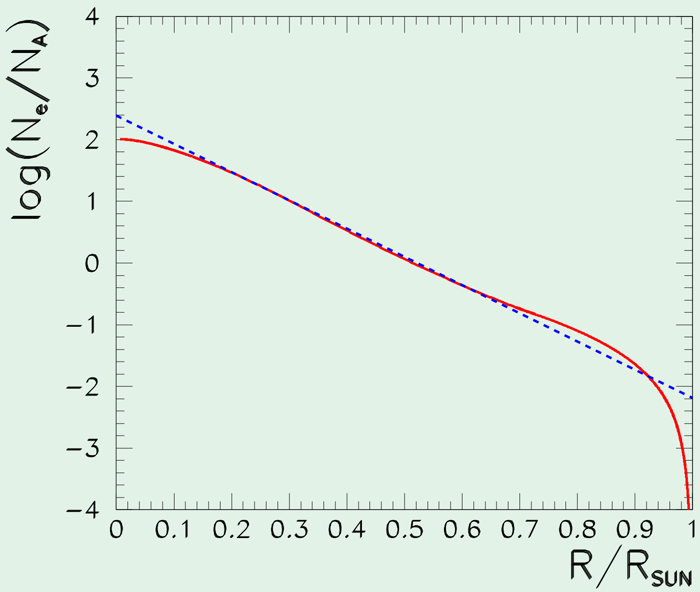
The solar matter density



$V_{CC} = \sqrt{2} G_F N_e \sim 10^{-14} \frac{N_e}{N_A}$ eV

At core: $V_{CC,0} \sim 10^{-14}$–10^{-12} eV

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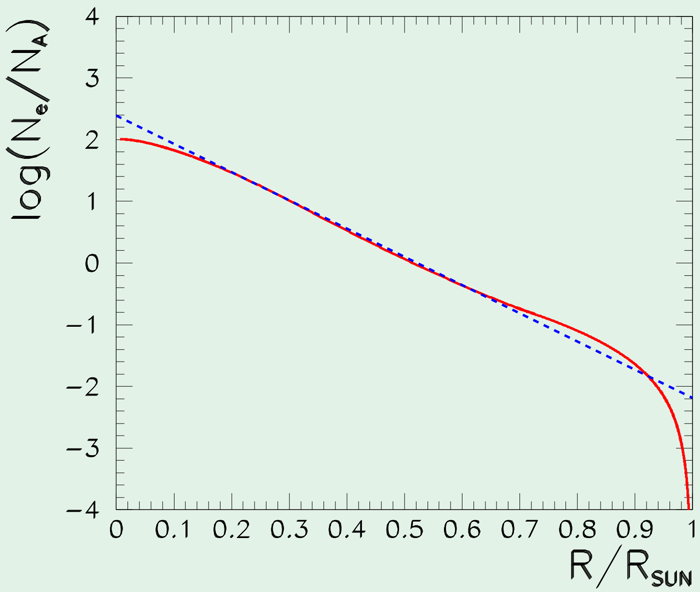

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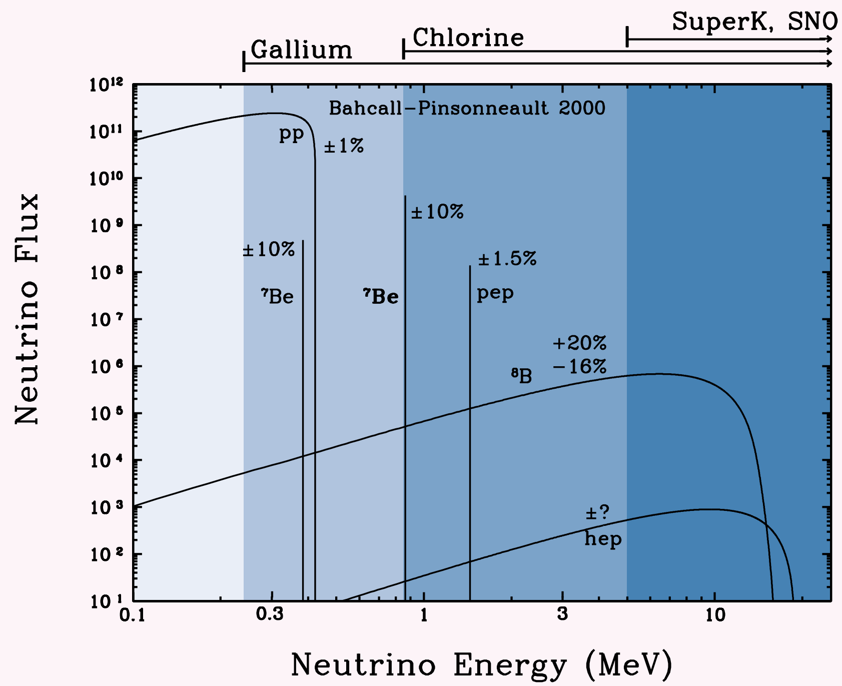
The energy spectrum of solar $\nu_e' s$



$E_\nu \sim 0.1$–10 MeV

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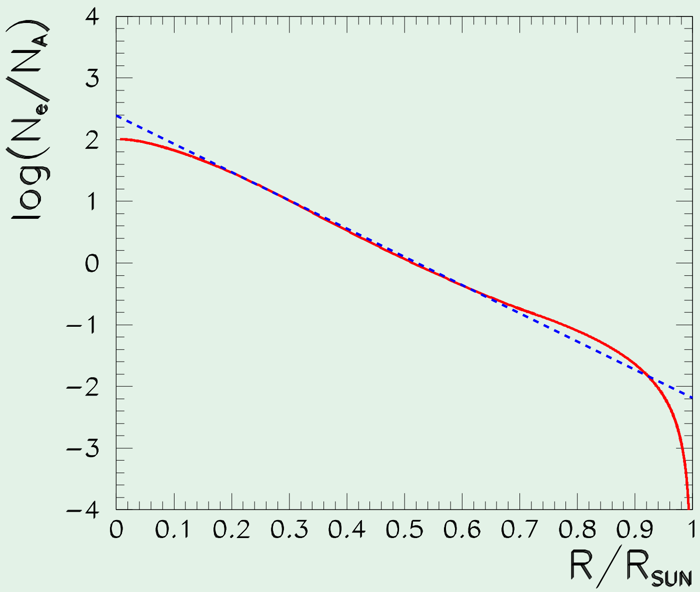

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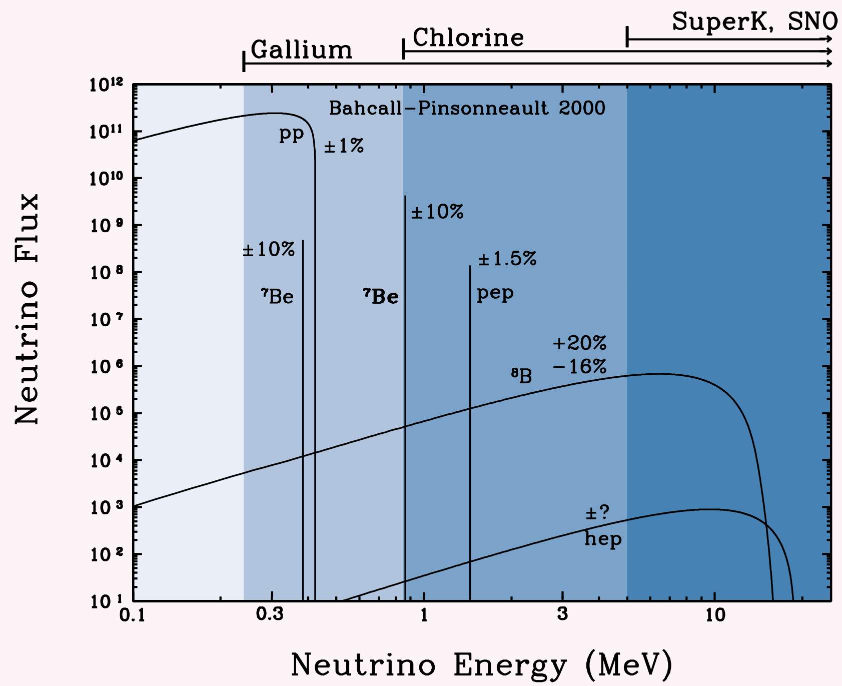

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- For $\nu_e \leftrightarrow \nu_{\mu(\tau)}$, in vacuum $\nu_e = \cos\theta\, \nu_1 + \sin\theta\, \nu_2$
- For 10^{-9} eV$^2 \lesssim \Delta m^2 \lesssim 10^{-4}$ eV^2 $\Rightarrow$ $2E_\nu V_{CC,0} > \Delta m^2 \cos 2\theta$

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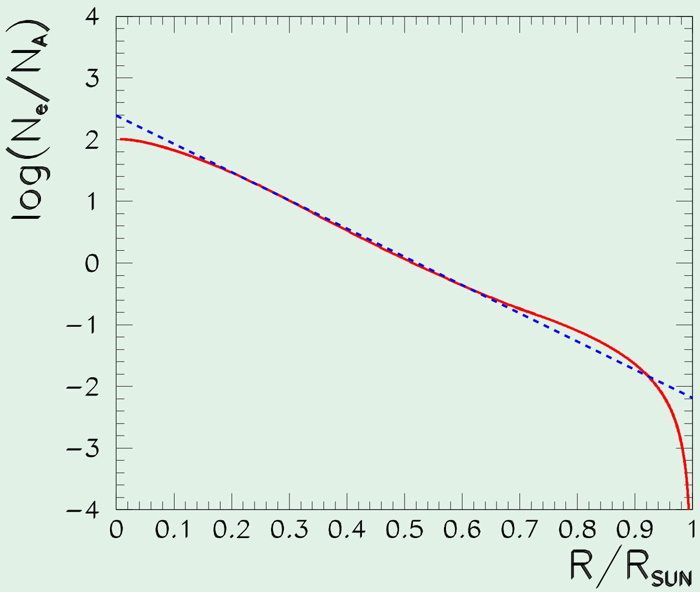

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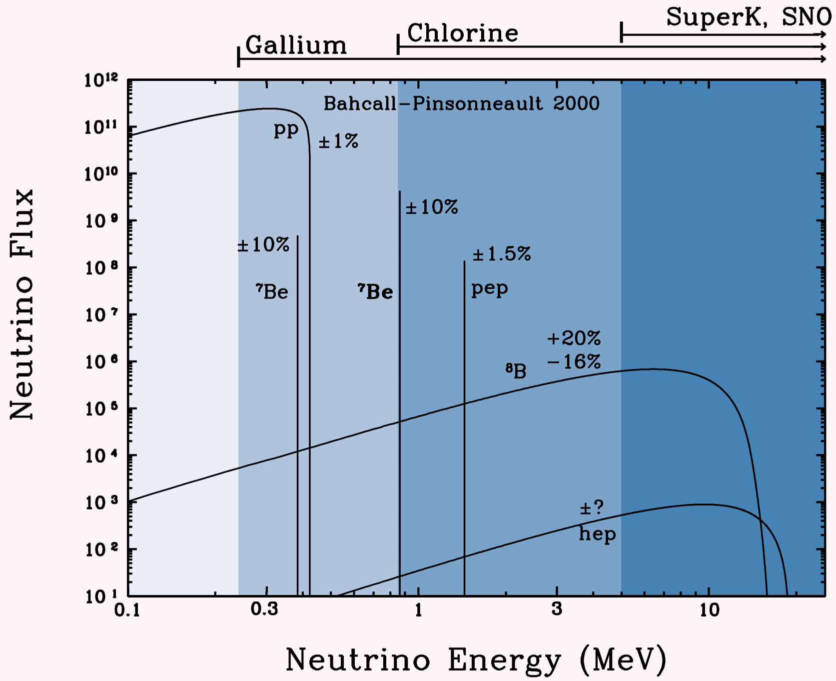

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  $\Rightarrow$ ν can cross resonance condition in its way out of the Sun

For $\theta \ll \frac{\pi}{4}$: In vacuum $\nu_e = \cos\theta\,\nu_1 + \sin\theta\,\nu_2$ is mostly ν_1

In Sun core $\nu_e = \cos\theta_{m,0}\,\nu_1 + \sin\theta_{m,0}\,\nu_2$ is mostly ν_2

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If $\frac{(\Delta m^2/\text{eV}^2)\sin^2 2\theta}{(E/\text{MeV})\cos 2\theta} \gg 3\times 10^{-9}$

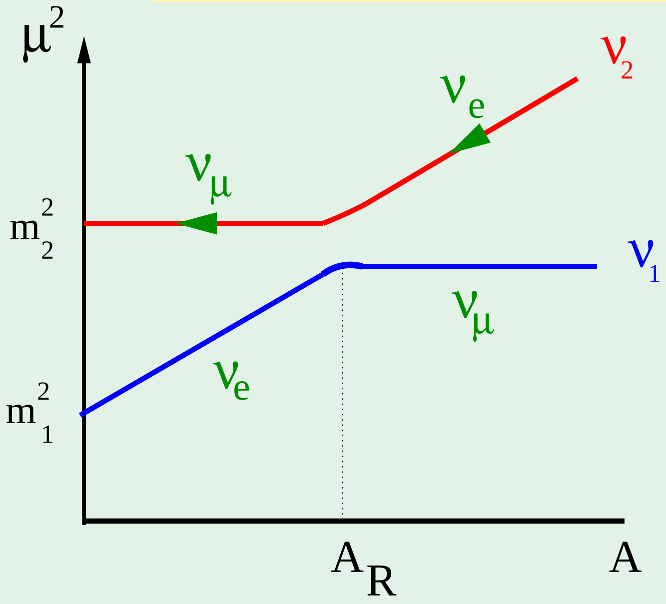
$\Rightarrow$ Adiabatic transition

$*$ ν is mostly ν_2 before and after resonance

$*$ $\theta_m \downarrow$ *dramatically* at resonance

$\Rightarrow$ ν_e component $\downarrow \Rightarrow P_{ee} \downarrow$

This is the MSW effect



$$P_{ee} = \frac{1}{2}\left[1 + \cos 2\theta_{m,0}\cos 2\theta\right] \simeq \sin^2\theta$$

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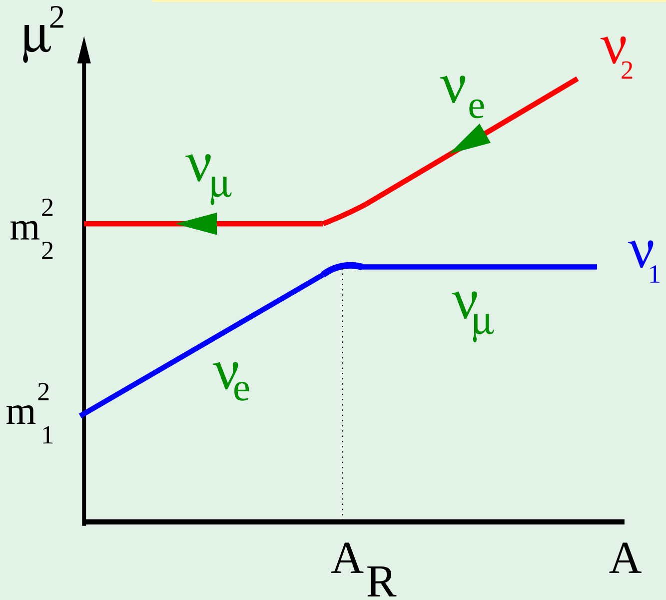


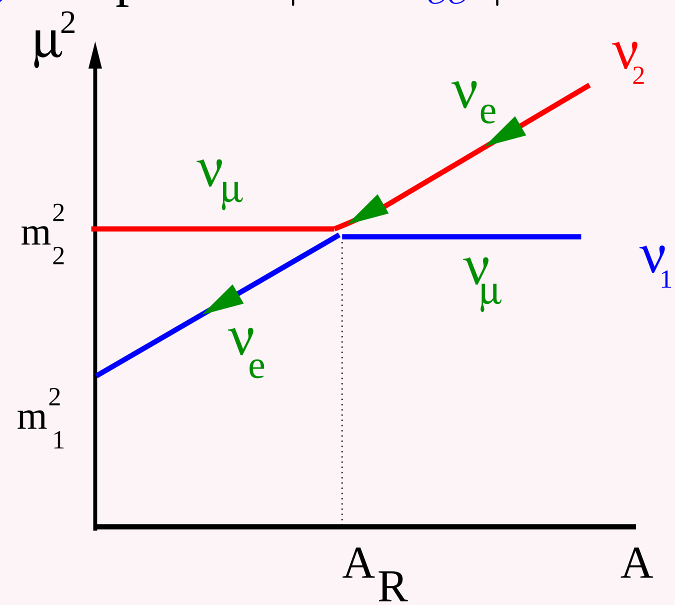
$$P_{ee} = \frac{1}{2}\left[1 + \cos 2\theta_{m,0}\cos 2\theta\right] \simeq \sin^2\theta$$

If $\frac{(\Delta m^2/\mathrm{eV}^2)\sin^2 2\theta}{(E/\mathrm{MeV})\cos 2\theta} \lesssim 3\times 10^{-9}$

$\Rightarrow$ Non-Adiabatic transition

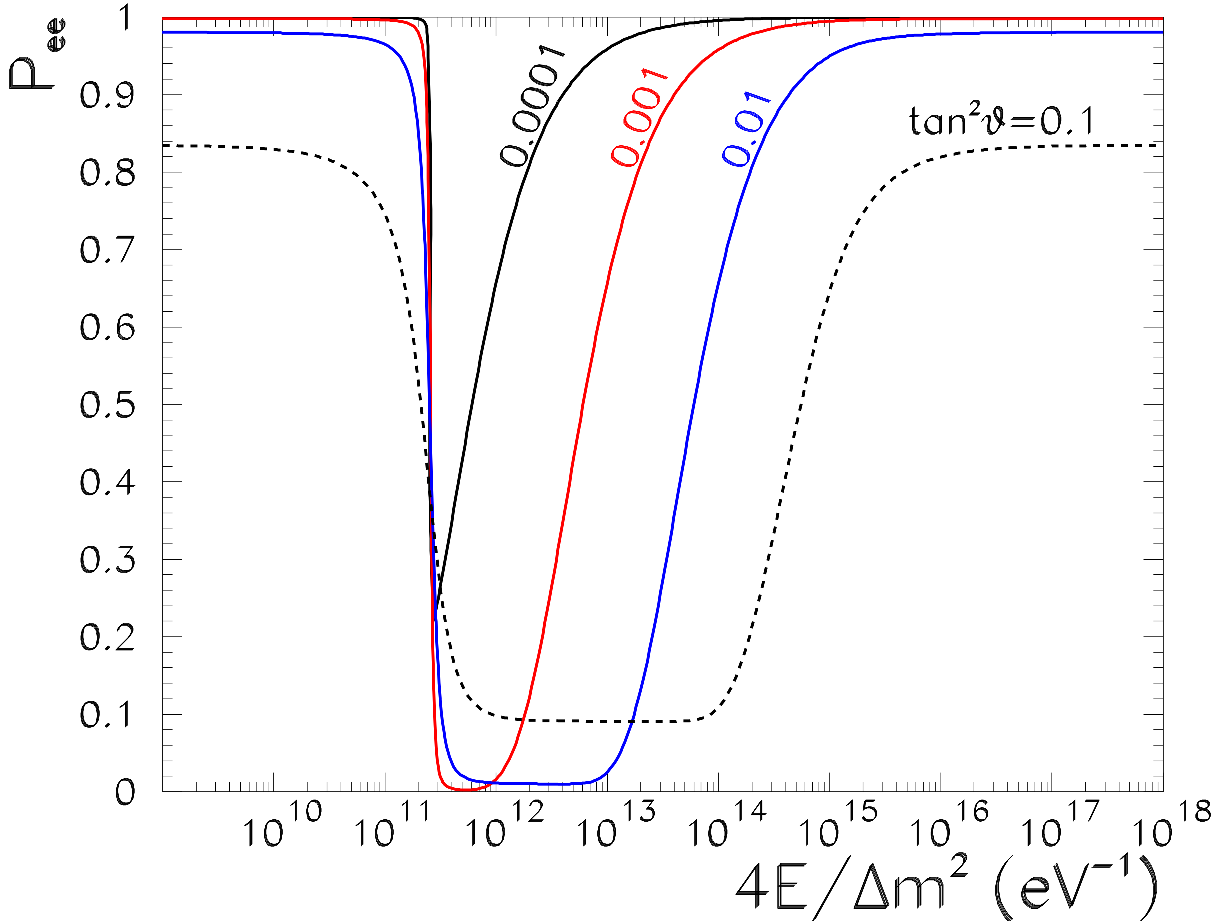
* ν is mostly ν_2 till the resonance
* At resonance the state can jump into ν_1 (with probability P_{LZ})

$\Rightarrow \nu_e$ component $\uparrow \Rightarrow P_{ee} \uparrow$



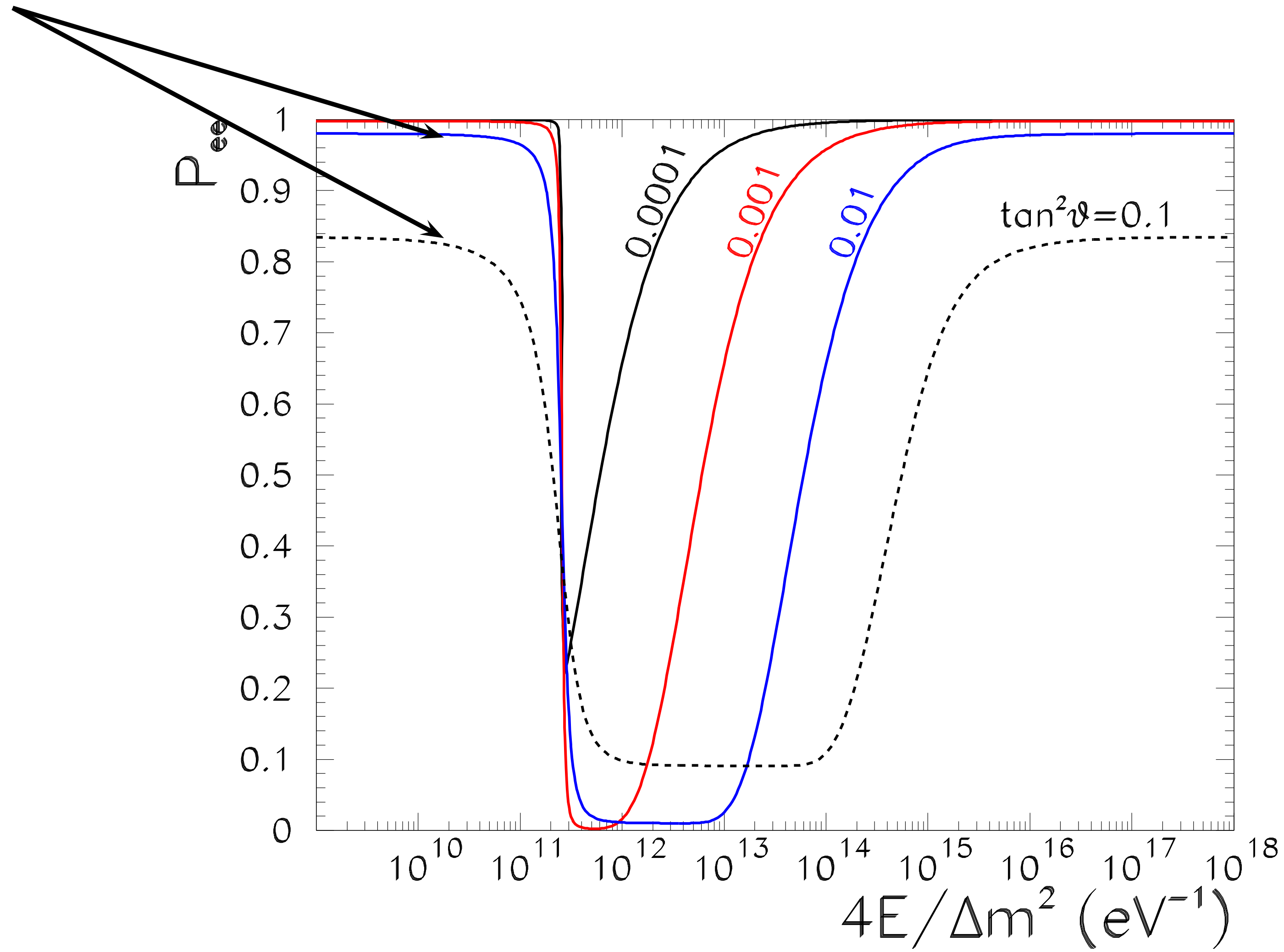
$$P_{ee} = \frac{1}{2}\left[1 + (1 - 2P_{LZ})\cos 2\theta_{m,0}\cos 2\theta\right]$$

Neutrinos in The Sun : MSW Effect



Neutrinos in The Sun : MSW Effect

ν does not cross resonance: $P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta > \frac{1}{2}$



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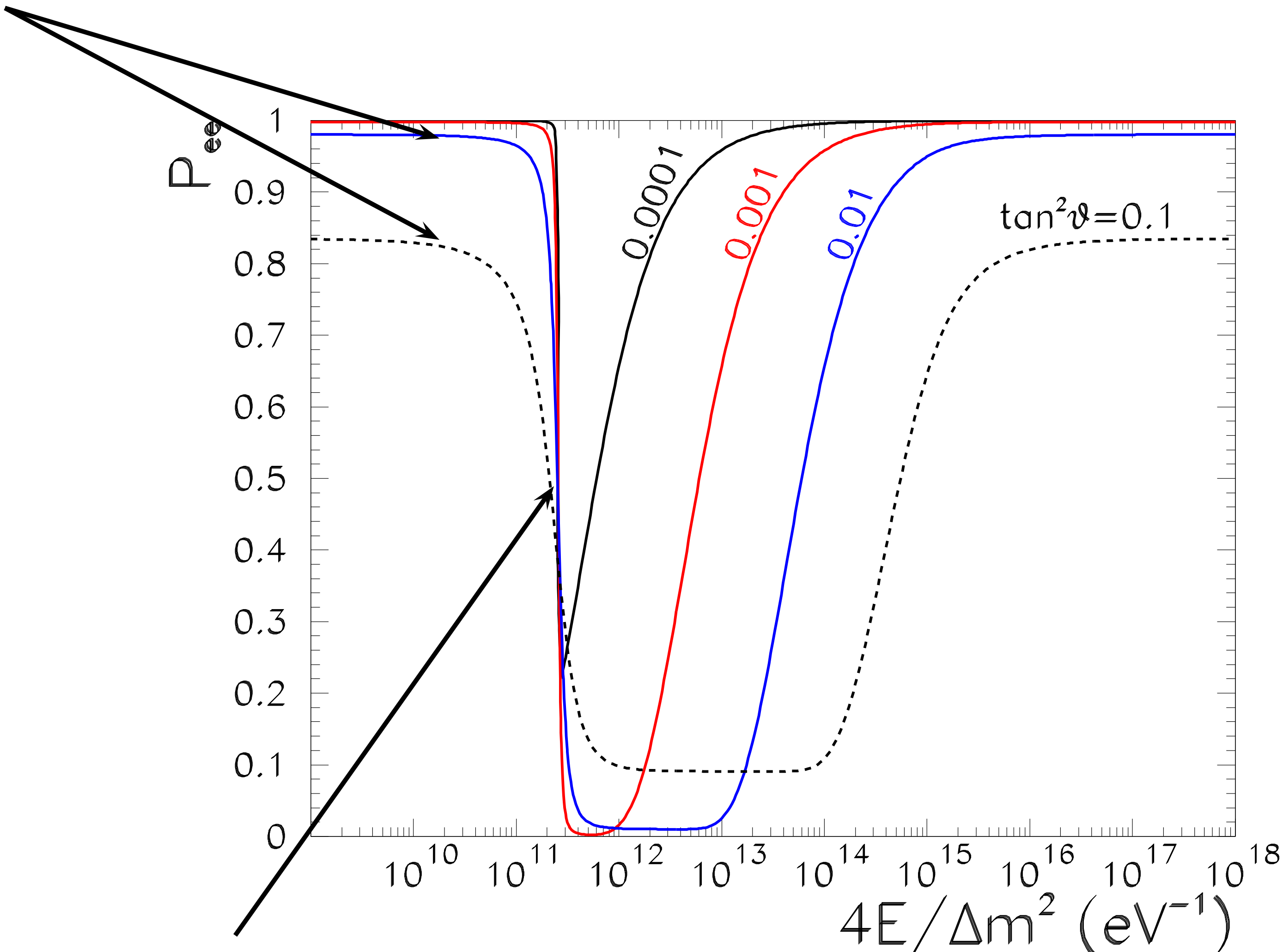

ν crosses resonance

MSW effect

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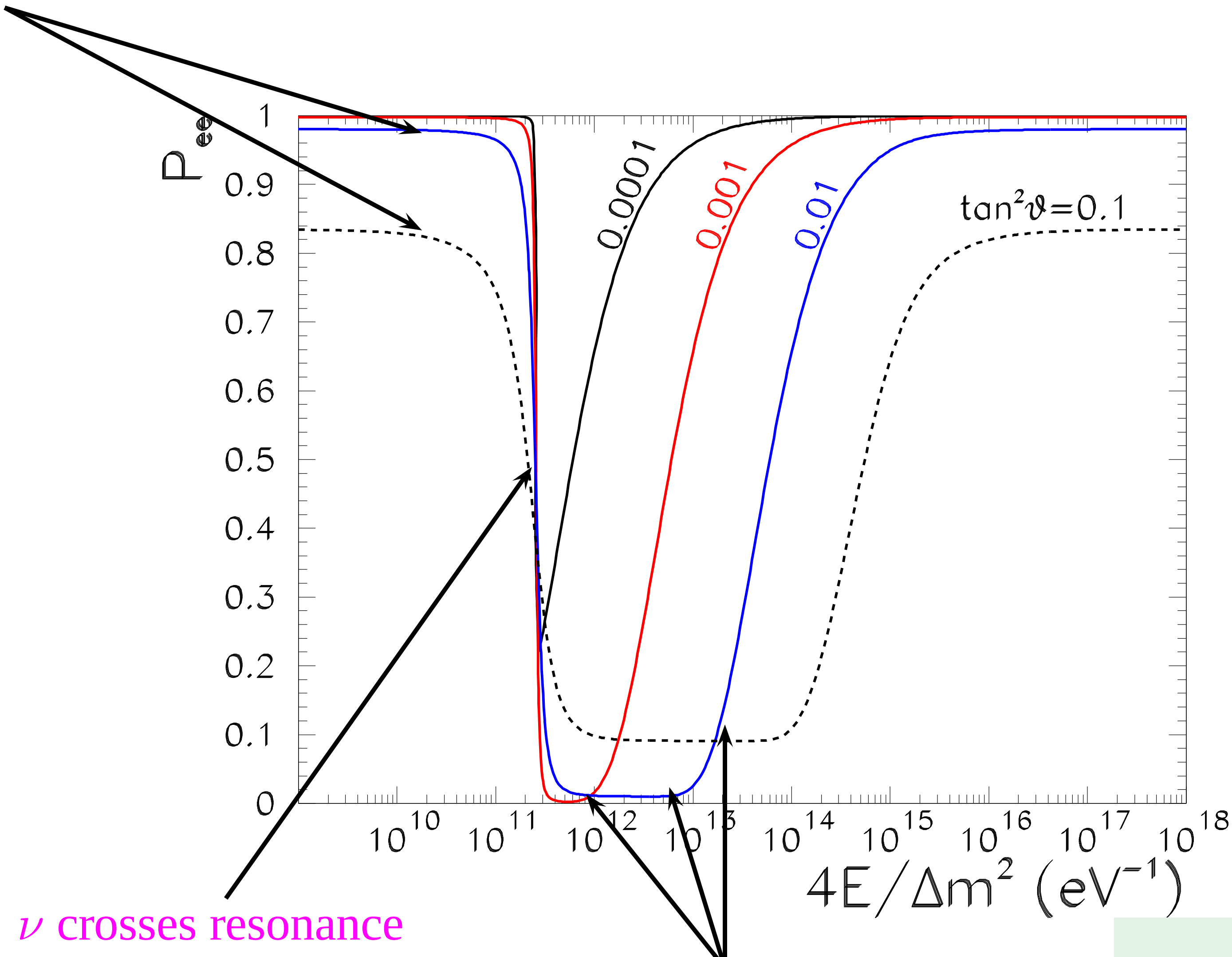

ν crosses resonance

MSW effect

Adiabatic MSW transition $P_{ee} = \sin^2\theta < \frac{1}{2}$

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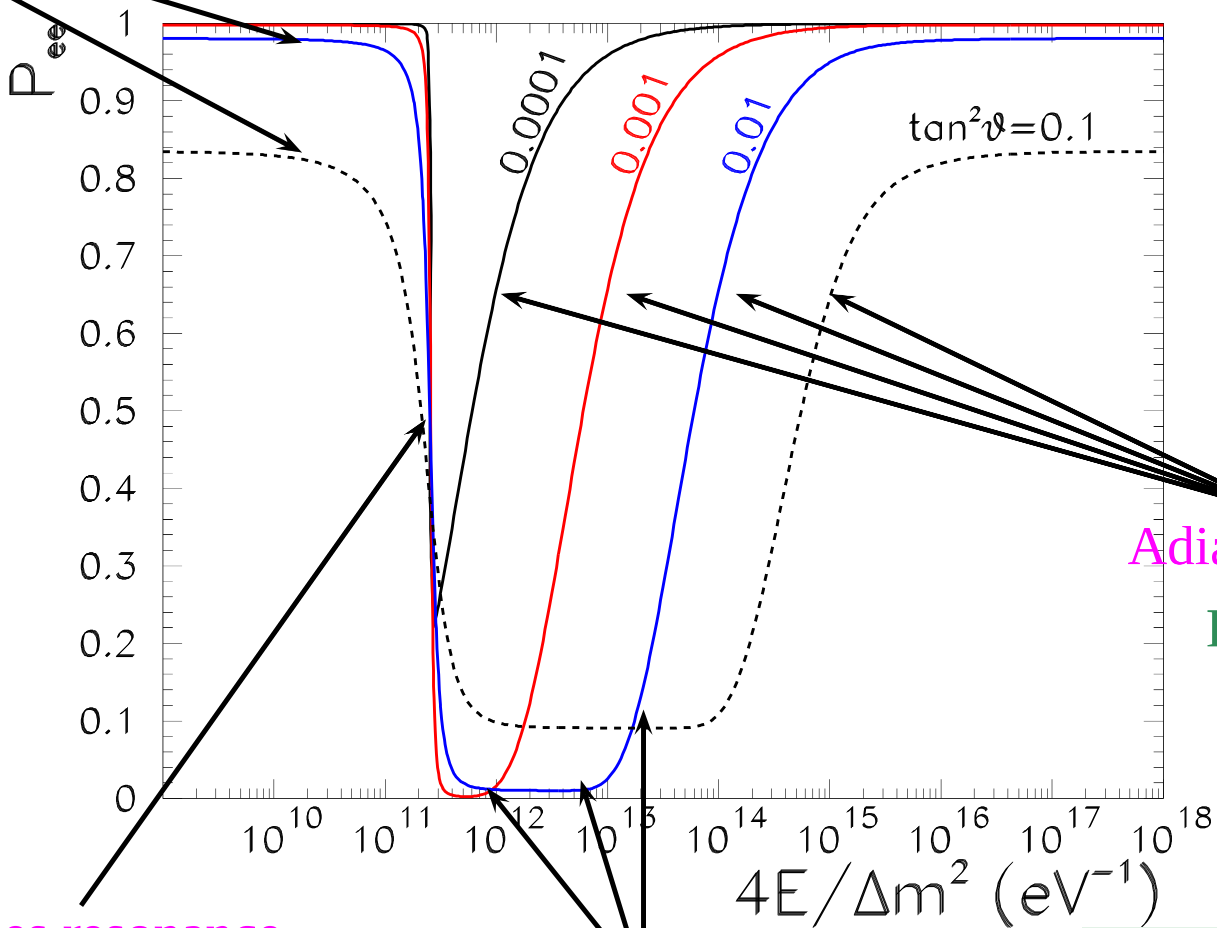


Adiabacity breaking

Effect of P_{LZ}

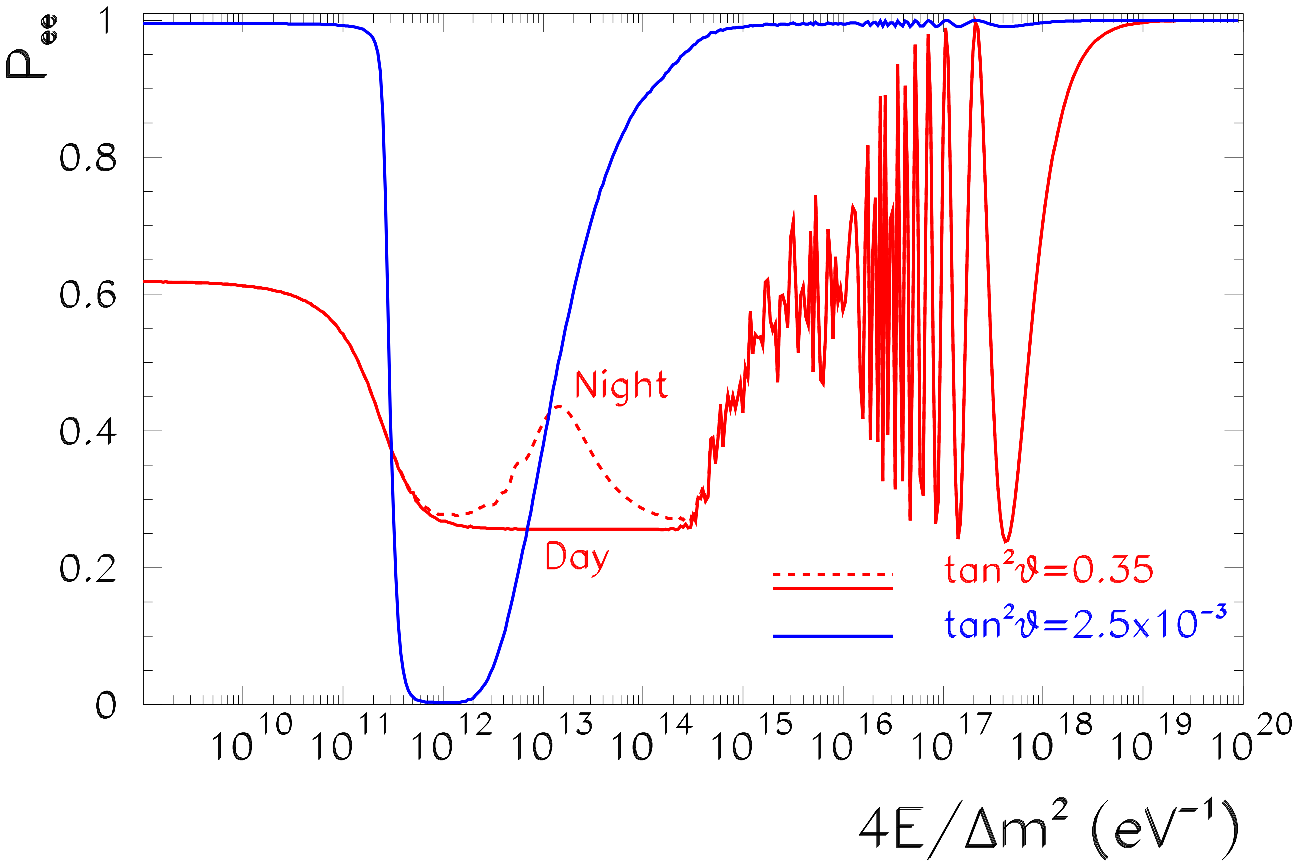
ν crosses resonance

MSW effect

Adiabatic MSW transition $P_{ee} = \sin^2\theta < \frac{1}{2}$

Neutrinos from The Sun : The Full Story

$$A(\nu_e \to \nu_e) = A_{Sun}(\nu_e \to \nu_1) \times A_{\rm vac}(\nu_1 \to \nu_1) \times A_{Earth}(\nu_1 \to \nu_e)$$
$$+ A_{Sun}(\nu_e \to \nu_2) \times A_{\rm vac}(\nu_2 \to \nu_2) \times A_{Earth}(\nu_2 \to \nu_e)$$



Solar Neutrinos: Oscillation Analysis

• With all the measured event rates:

$$\chi^2_R = \sum_i (R_i^{th} - R_i^{exp})\sigma_{ij}^{-2}(R_j^{th} - R_j^{exp})$$

σ_{ij} contains theoretical uncertainties and the experimental systematic and statistical errors

– For Cl, Ga, and SNO CC

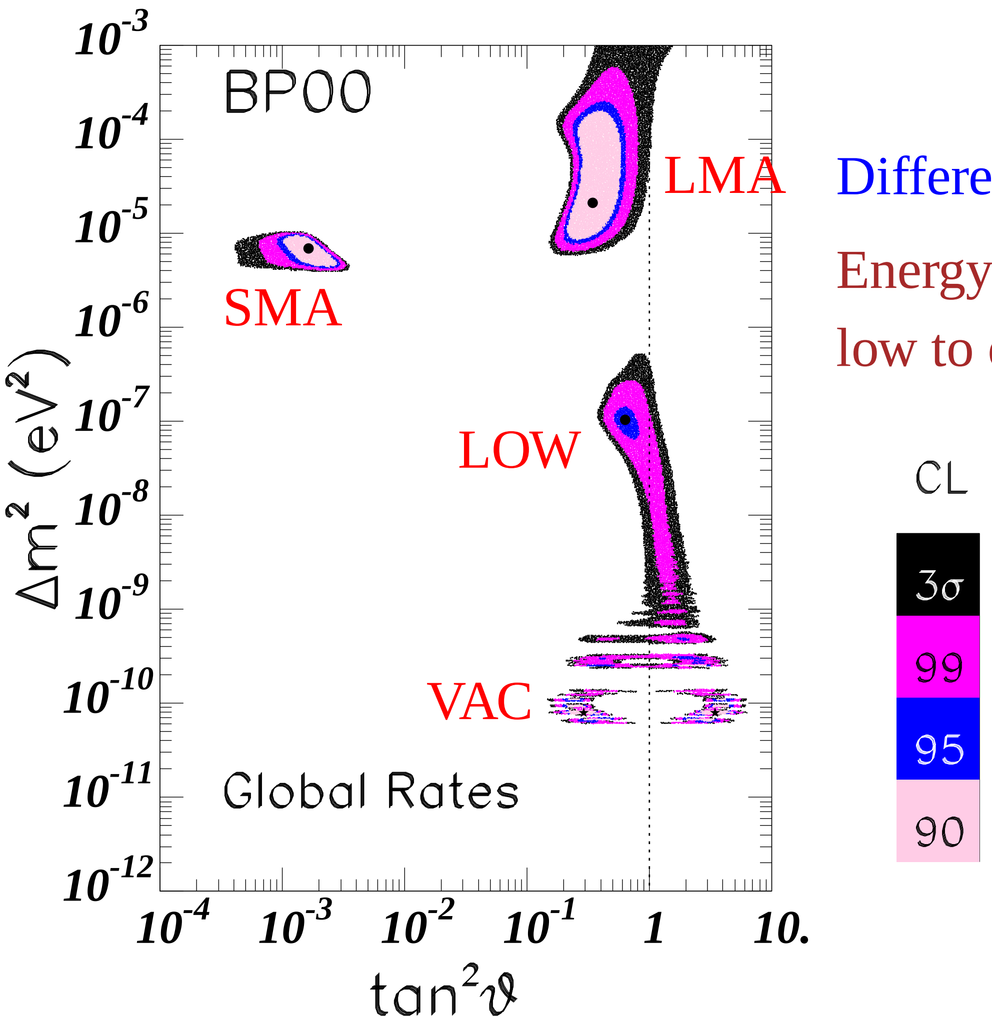
$$R_i^{th} = \sum_{k=1,8} \phi_k \int dE_\nu \, \lambda_k(E_\nu) \times \Big[\sigma_{e,i}(E_\nu)\langle P_{ee}(E_\nu,t)\rangle\Big]$$

– For SK, SNO and Borexino ES

$$\begin{aligned} R_i^{th} \quad &= \quad \sum_{k=1,8} \phi_k \int dE_\nu \, \lambda_k(E_\nu) \times \Big[\sigma_{e,i}(E_\nu)\langle P_{ee}(E_\nu,t)\rangle \\ &\quad + \sigma_{x,i}(E_\nu)(1-\langle P_{ee}(E_\nu,t)\rangle)\Big] \end{aligned}$$

Solar Neutrinos: $\nu_e \rightarrow \nu_{\mu,\tau}$ Oscillations

Allowed regions by Fit to Total Rates: Cl, Ga, SK and SNO CC



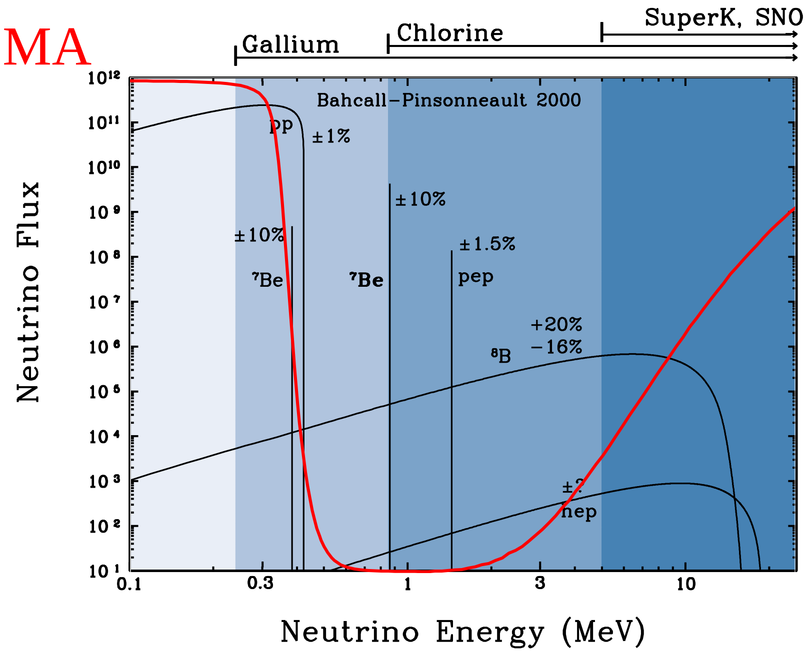
$\nu_e \rightarrow \nu_{\mu,\tau}$

Different regimes can explain the Total Rates

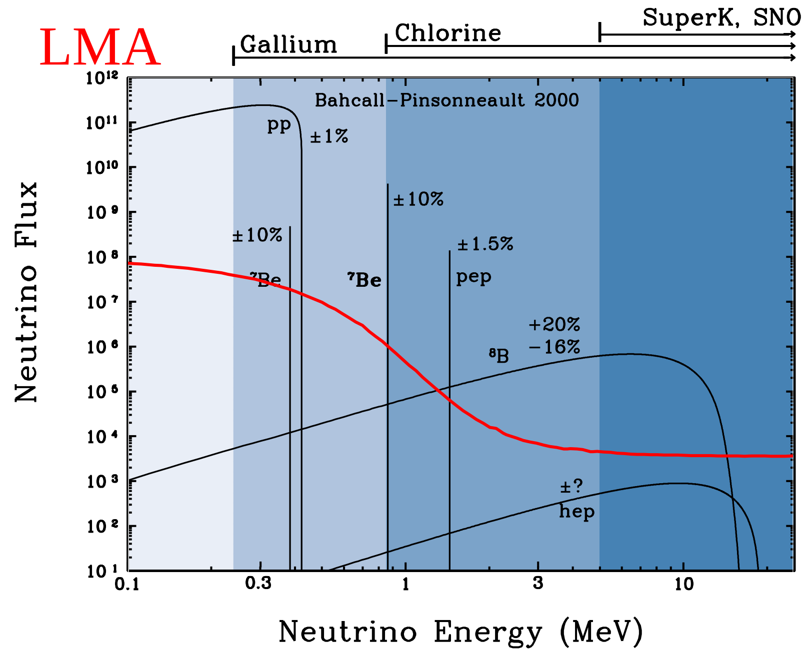
Energy and Time dependence of Observables allow to discriminate

Energy Dependence of P_{ee} for Different Solutions

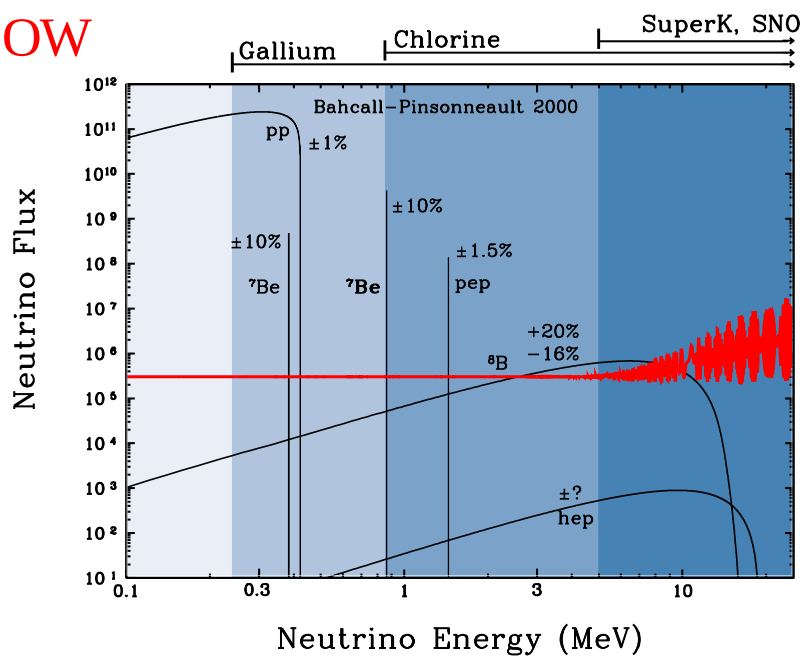
SMA



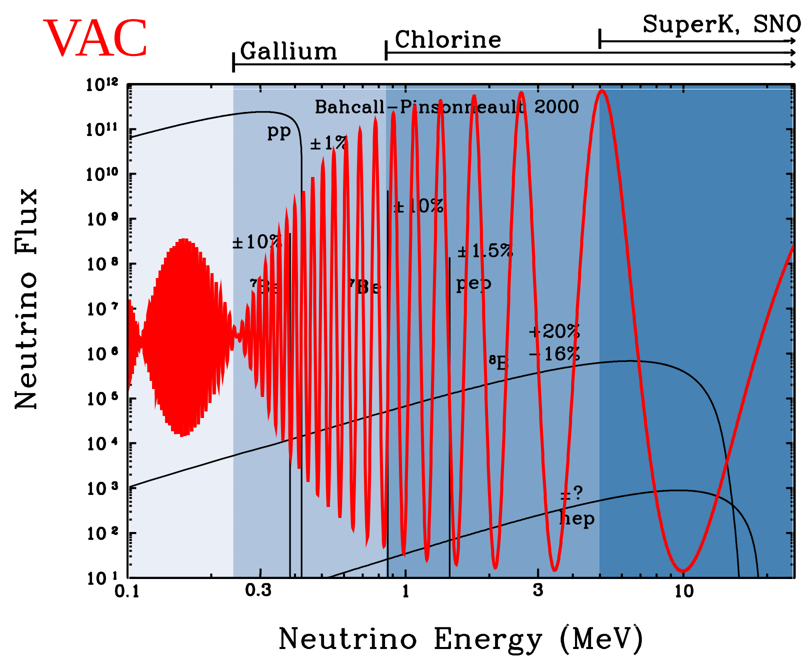
LMA



LOW



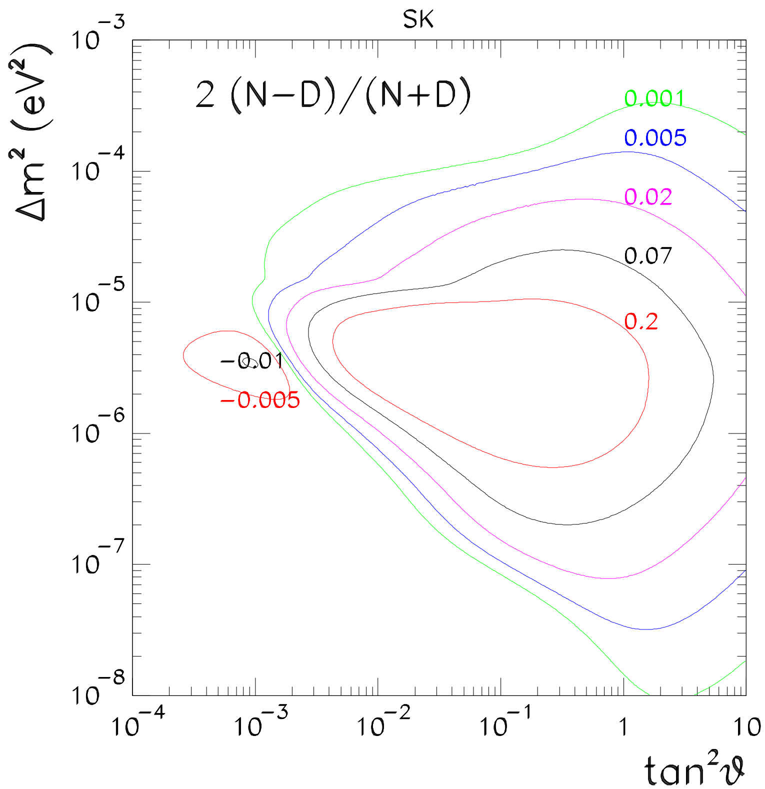
VAC



Expected Day-Night variation

- Matter effects in the Earth are maximal when $\frac{\Delta m^2 \cos(2\theta)/2E}{10^{-13}\text{eV}} \sim \frac{\text{gcm}^{-3}}{\rho_{Earth} Y_e}$
- Since $\langle \rho_{Earth} Y_e \rangle = 2 - 5$ g cm^{-3}

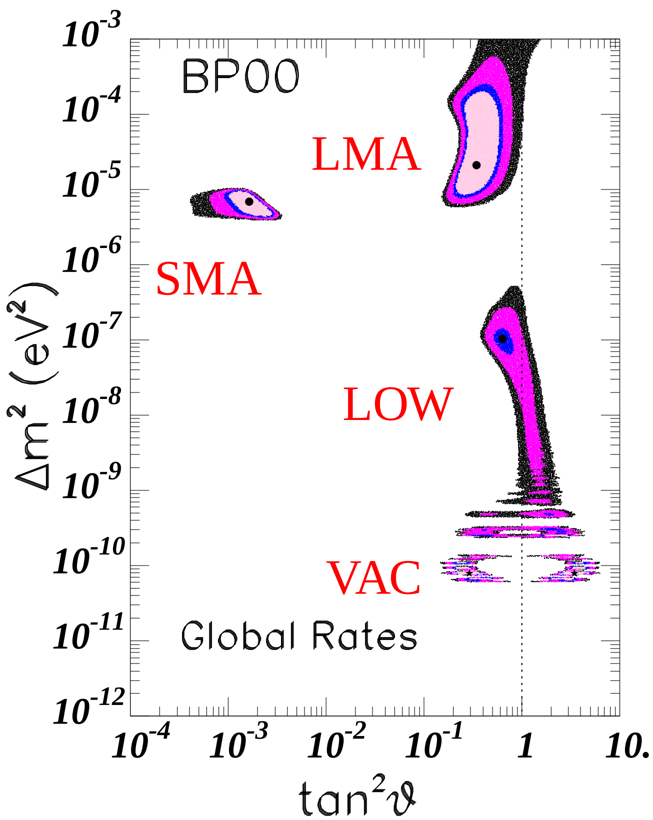
$\Rightarrow$ For example AT SK effect is most important for $\Delta m^2 \cos(2\theta) \sim 10^{-6} - 10^{-7}$ eV^2

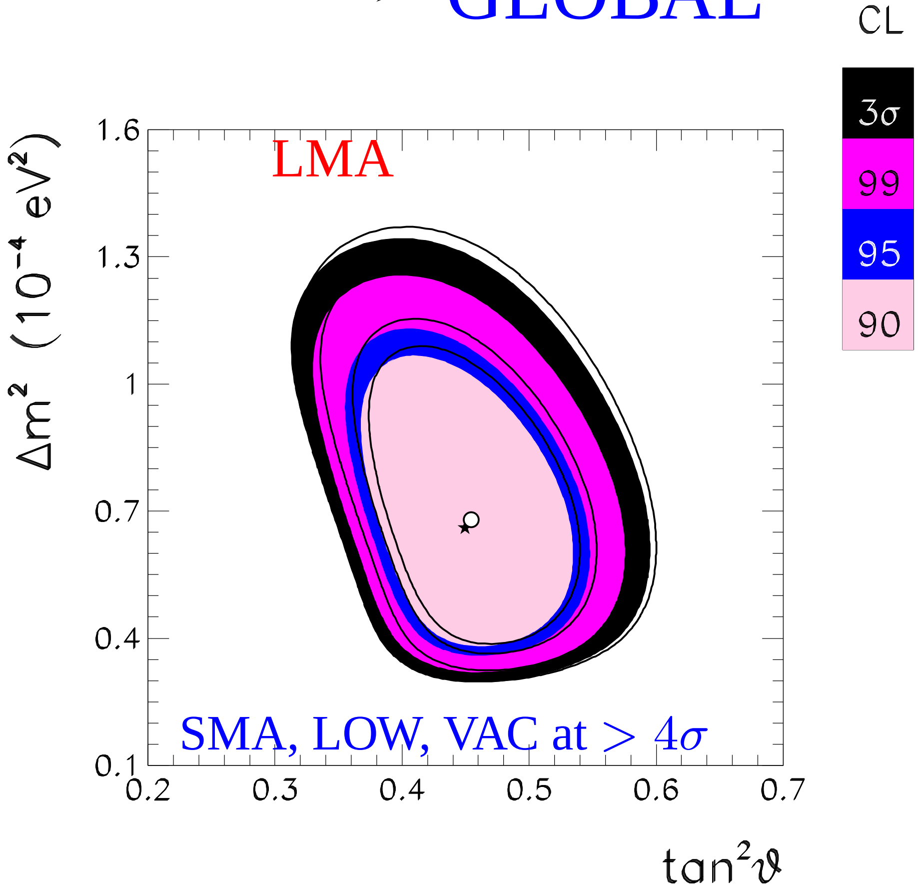


Solar Neutrinos: $\nu_e \rightarrow \nu_{\mu,\tau}$ Oscillations

RATES ONLY → GLOBAL

SK and SNO E and t dependence





Best fit

$\Delta m^2 = 6.8 \times 10^{-5}$ eV^2

$\tan^2 \theta = 0.45$

Learning How the Sun Shines

– Solar ν experiments measure a convolution $Obs_\odot = P_{eX}^{\rm sun} \otimes {\rm Sun\ Properties}$

– We will see that now Non-solar exp determine independently $P_{eX}({\rm osc\ param})$

$\Rightarrow$ Back to study Sun Properties from $Obs_\odot$

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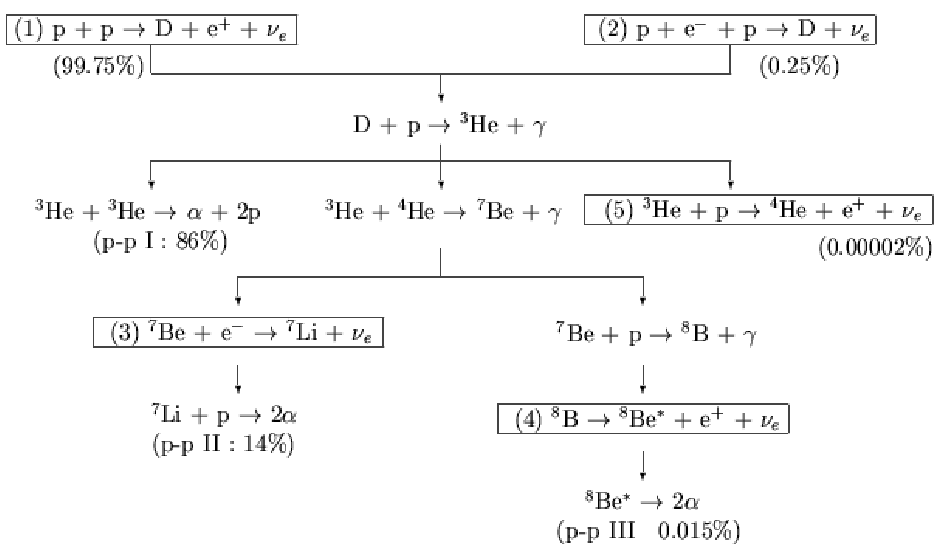
- The Sun shines converting protons into α, e^+ and $\nu' s$

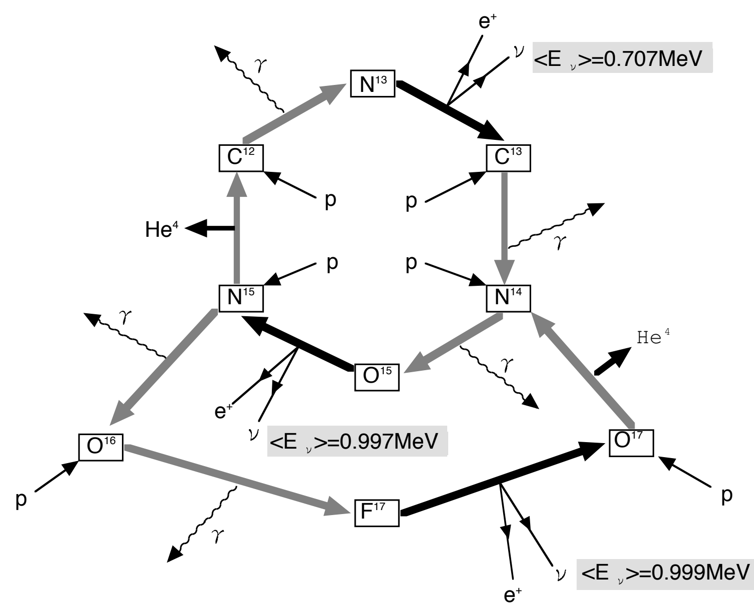
$$4\,p \;\rightarrow\; {}^4He \;+\; 2\,e^+ \;+\; 2\,\nu_e \;+\; \gamma$$

$4m_p - m_{^4He} - 2m_e \simeq 26$ MeV Thermal energy mostly in γ

pp chain:



CNO cycle:



- First proposal by Bethe (1939) was that CNO dominated

“ It is shown that the most important source of energy in ordinary stars is the reactions of carbon and nitrogen with protons.”

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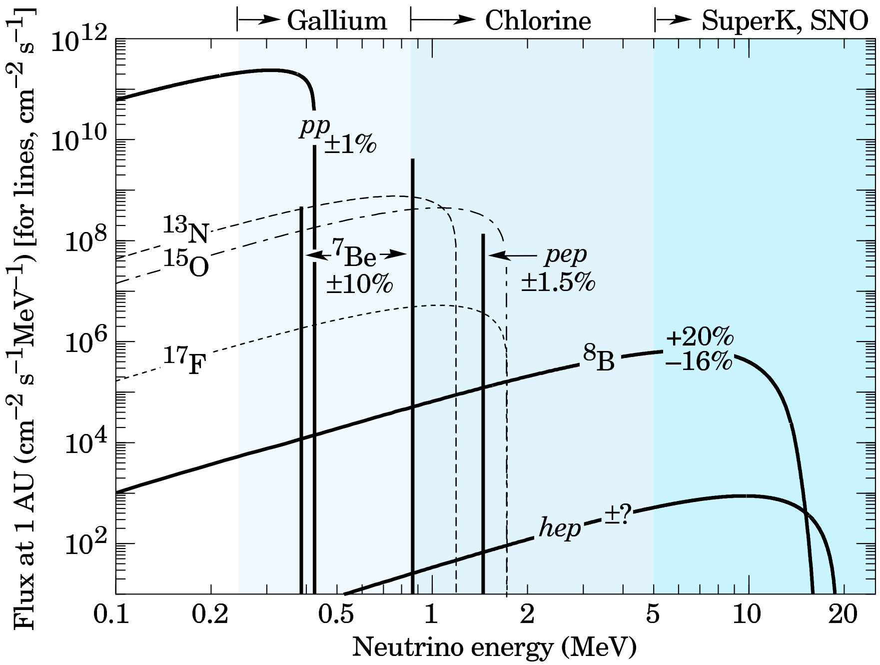
- Improved Solar Model& nuclear reaction data $\Rightarrow$ Sun shines primarily by p-p

- SSM Fluxes



$$\frac{L_{CNO}}{L_\odot} = 1.5\%$$

$$\frac{L_{p-p}}{L_\odot} = 98.5\%$$

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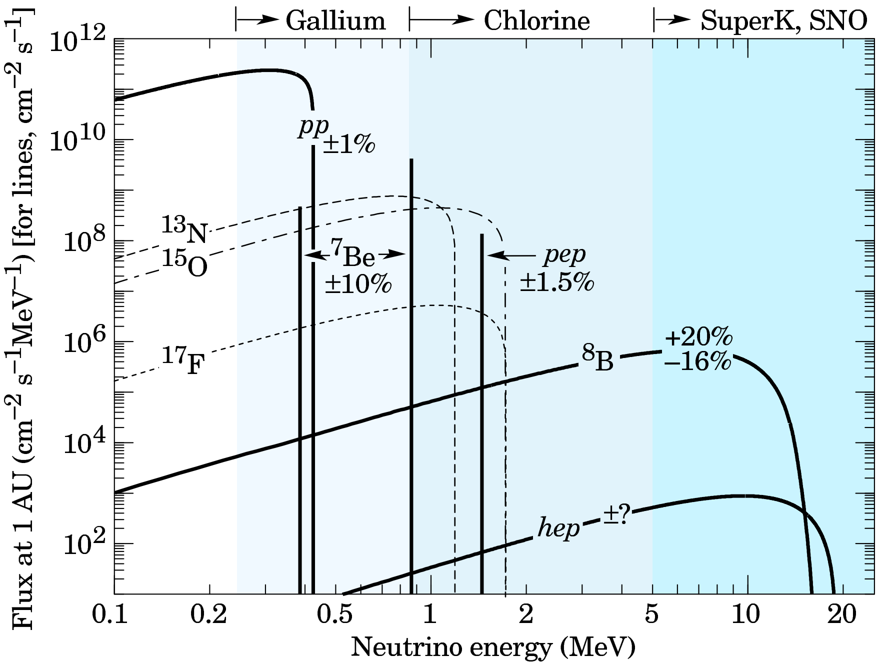

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$$\frac{L_{p-p}}{L_\odot} = 98.5\%$$

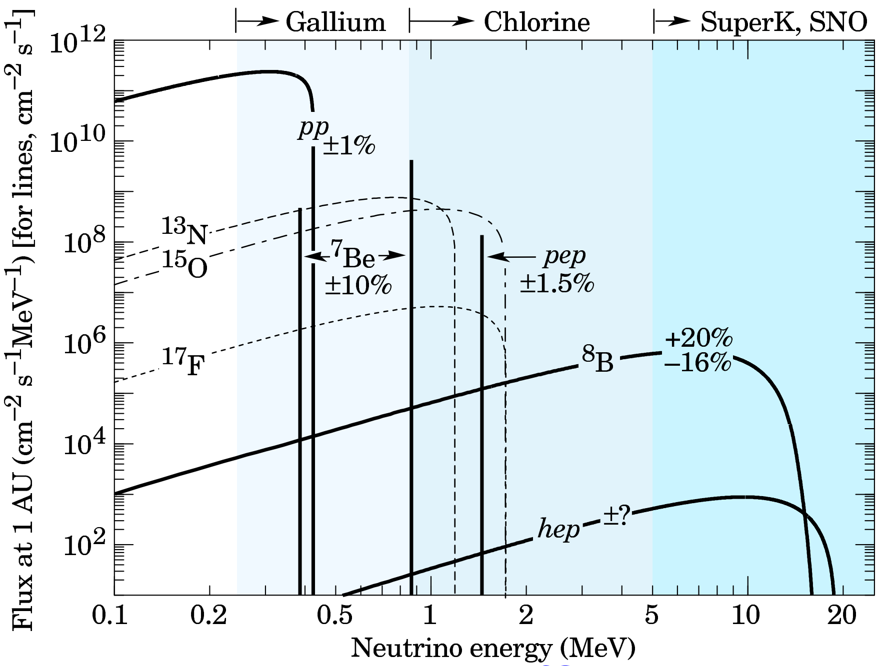
- Can this be tested experimentally?

- First proposal by Bethe (1939) was that CNO dominated

" It is shown that the most important source of energy in ordinary stars is the reactions of carbon and nitrogen with protons."

- Improved Solar Model& nuclear reaction data $\Rightarrow$ Sun shines primarily by p-p

- SSM Fluxes



$$\frac{L_{CNO}}{L_\odot} = 1.5\%$$

$$\frac{L_{p-p}}{L_\odot} = 98.5\%$$

- Can this be tested experimentally? Difficult in past

– Radiochemical experiments sensitive to CNO fluxes

But do not measure $E \Rightarrow$ only integrated flux above E_{th}

– Oscillations modify the E dependence of detected fluxes

$\Rightarrow$ Possible suppression of CNO fluxes $\Rightarrow$ no experimental limit

How the Sun Shines? Older Answer

- Before SK and SNO large CNO solutions allowed

  Bahcall, Fukugita, Krastev PLB (1996)

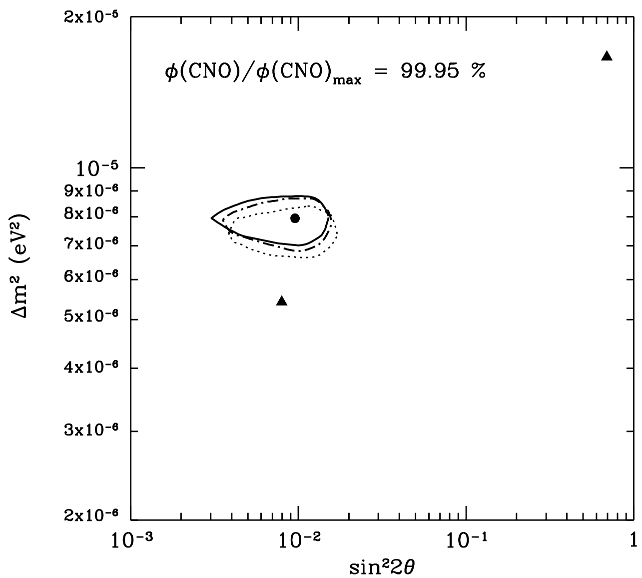
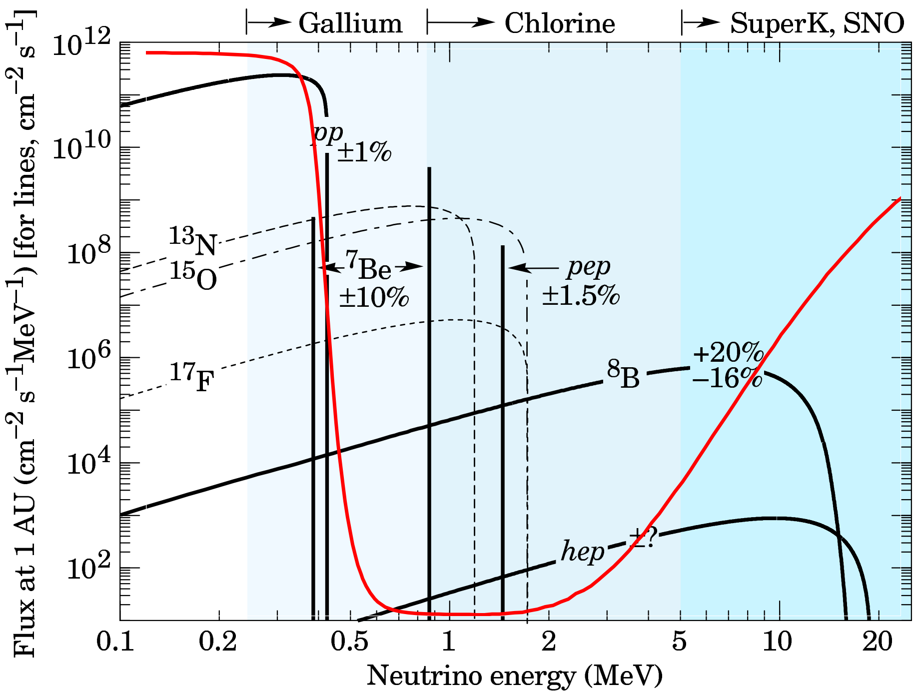


Fig.2



How the Sun Shines? Present Answer

Global Fit to solar+LBL+ATM+reactor:

– 3ν oscillations+8 free solar ν fluxes: $f_i \equiv \frac{\Phi_i}{\Phi_i(\mathrm{GS98})}$

Under conditions:

* Nuclear Physics inequalities:

– From pp-chain:

$$\Phi_{^7\mathrm{Be}} + \Phi_{^8\mathrm{B}} \leq \Phi_{\mathrm{pp}} + \Phi_{\mathrm{pep}}$$

$$\Rightarrow 8.49 \times 10^{-2} f_{^7\mathrm{Be}} + 9.95 \times 10^{-5} f_{^8\mathrm{B}} \leq f_{\mathrm{pp}} + 2.36 \times 10^{-3} f_{\mathrm{pep}}$$

– The $^{14}\mathrm{N}(p,\gamma)^{15}\mathrm{O}$ slowest of the CNO-cycle: $\Phi_{^{15}\mathrm{O}} \leq \Phi_{^{13}\mathrm{N}} \Rightarrow f_{^{15}\mathrm{O}} \leq 1.34 f_{^{13}\mathrm{N}}$

– CNO-II branch subdominant $\Phi_{^{17}\mathrm{F}} \leq \Phi_{^{15}\mathrm{O}} \Rightarrow f_{^{17}\mathrm{F}} \leq 37 f_{^{15}\mathrm{O}}$

– pep flux related to pp flux (same matrix elements): $\frac{f_{pep}}{f_{pp}} = 1.008 \pm 0.010$

* Luminosity constraint: $$\frac{L_\odot}{4\pi(A.U.)^2} = \sum_{i=1}^{8} \alpha_i \Phi_i$$

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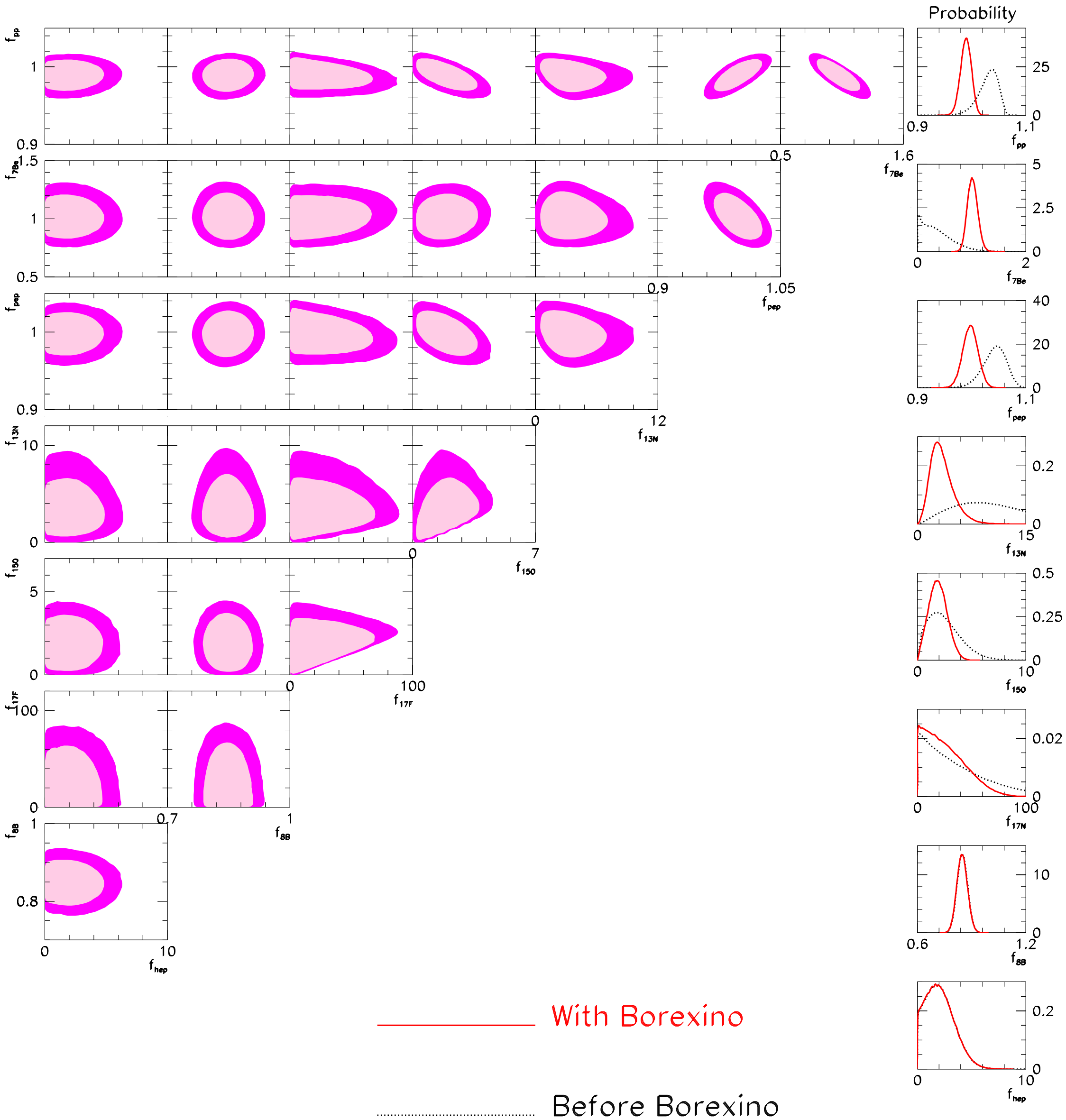
How the Sun Shines? Present Answer

Results of Oscillation fit with solar flux normalizations free:



Present limit on CNO:

$$\frac{L_{\rm CNO}}{L_\odot} < 3.2\,\%\ (3\sigma)$$

Test of Lum Constraint:

$$\frac{L_\odot(\nu - \text{inferred})}{L_\odot} = 1.0 \pm 0.14\ (1\sigma$$

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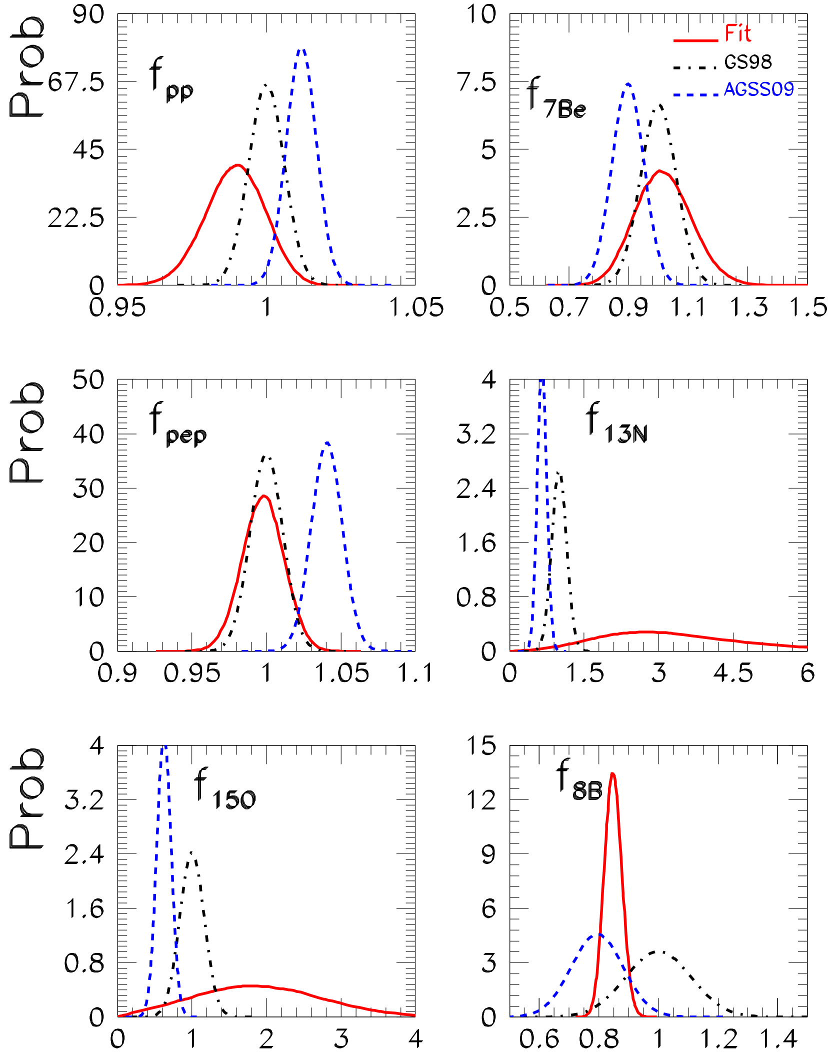
How the Sun Shines? Present Answer

Comparing with GS98 and AGSS09 Models



Both statistically equally probable

Plan of Lectures

Introduction: The New Minimal Standard Model

Effects of ν mass: Oscillations in Vaccum and Matter

Atmospheric Neutrinos

Solar Neutrinos

Accelerator and Reactor Neutrinos

Fitting all Together and Subleading effects

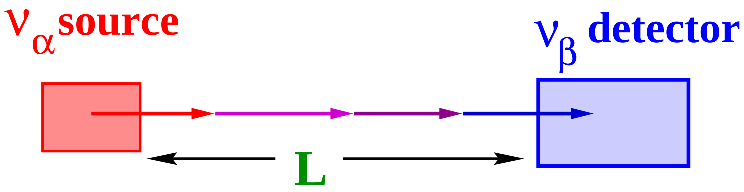
Summary

PS:The Near Future Experimental Program and Its Challenges

ν Oscillations: Accelerator and Reactor Searches

ν Oscillations: Accelerator and Reactor Searches

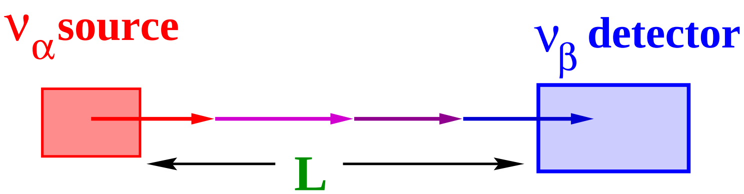
Appearance Experiment



Searches for
β diff α

ν Oscillations: Accelerator and Reactor Searches

Appearance Experiment



Searches for
β diff α

Experiment	$\langle \frac{E/\text{MeV}}{L/\text{m}} \rangle$		α	β
CCFR	100	FNAL	ν_μ, ν_e	ν_τ
E531	25	FNAL	ν_μ, ν_e	ν_τ
Nomad	13	CERN	ν_μ, ν_e	ν_τ
Chorus	13	CERN	ν_μ, ν_e	ν_τ
E776	2.5	BNL	ν_μ	ν_e
Karmen2	2.5	Rutherford	$\bar{\nu}_\mu$	$\bar{\nu}_e$
LSND	3	Los Alamos	$\bar{\nu}_\mu$	$\bar{\nu}_e$
Miniboone	3	Fermilab	ν_μ,$\bar{\nu}_\mu$	ν_e ,$\bar{\nu}_e$

ν Oscillations: Accelerator and Reactor Searches

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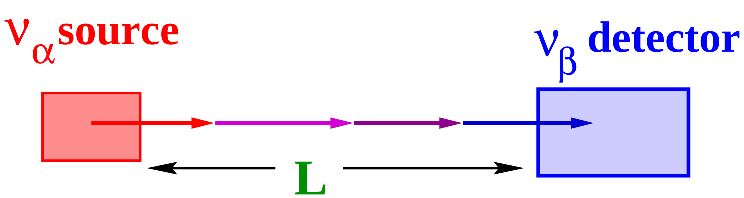
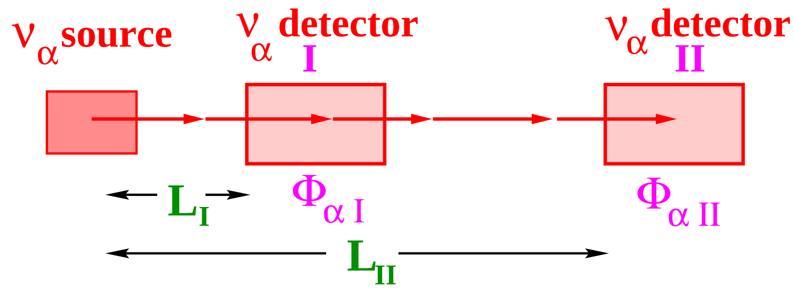


Searches for β diff α

Disappearance Experiment



Compares $\Phi_{\alpha\, I}$ and $\Phi_{\alpha\, II}$ to look for loss

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ν Oscillations: Accelerator and Reactor Searches

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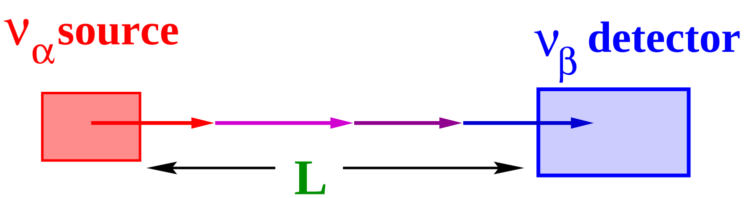

Searches for β diff α

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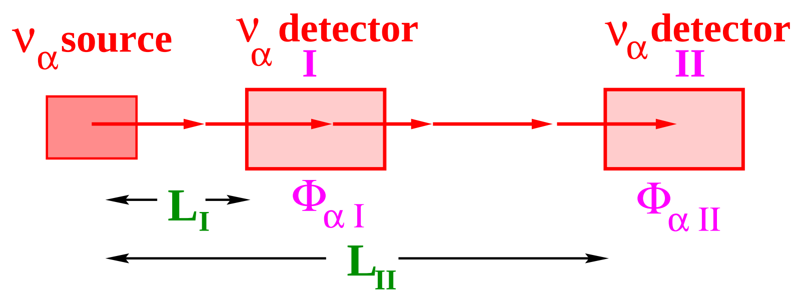

Compares $\Phi_{\alpha\,I}$ and $\Phi_{\alpha\,II}$ to look for loss

Experiment	$\langle \frac{E/MeV}{L/m} \rangle$		α
CDHSW	1.4	CERN	ν_μ
BugeyIII	0.05	Reactor	$\bar{\nu}_e$
Chooz	**0.005**	**Reactor**	$\bar{\nu}_e$

LSND

LSND

- The short distance signal for oscillation: $L = 30$ m with $\langle E_\nu \rangle \sim 30$ MeV
- Used the proton beam of Los Alamos $p + Target \rightarrow \pi^+ + X$

$$\pi^+ \rightarrow \nu_\mu \, \mu^+$$

$$\mu^+ \rightarrow e^+ \, \nu_e \, \overline{\nu_\mu}$$

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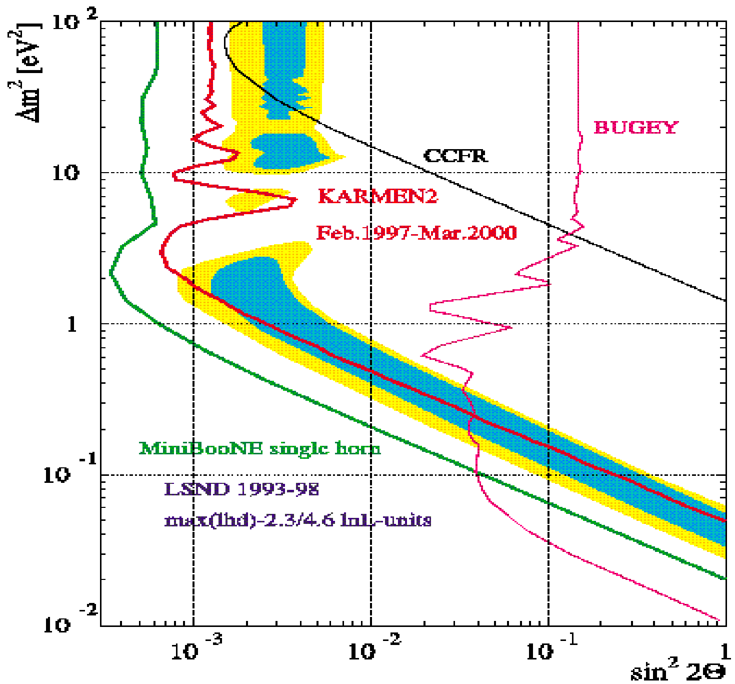
- observed $\overline{\nu_\mu} \to \overline{\nu_e}$ with probability $\langle P_{e\mu} \rangle = (0.26 \pm 0.07 \pm 0.05)\%$



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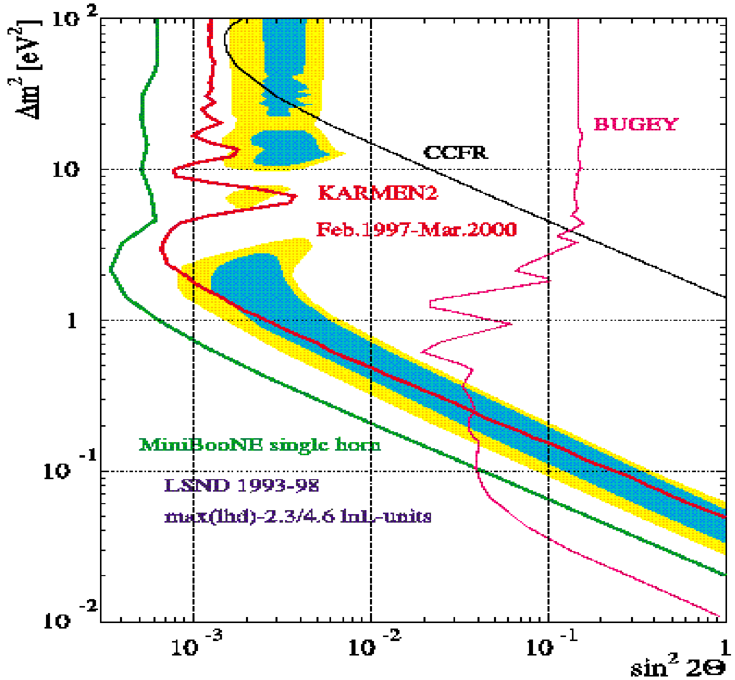

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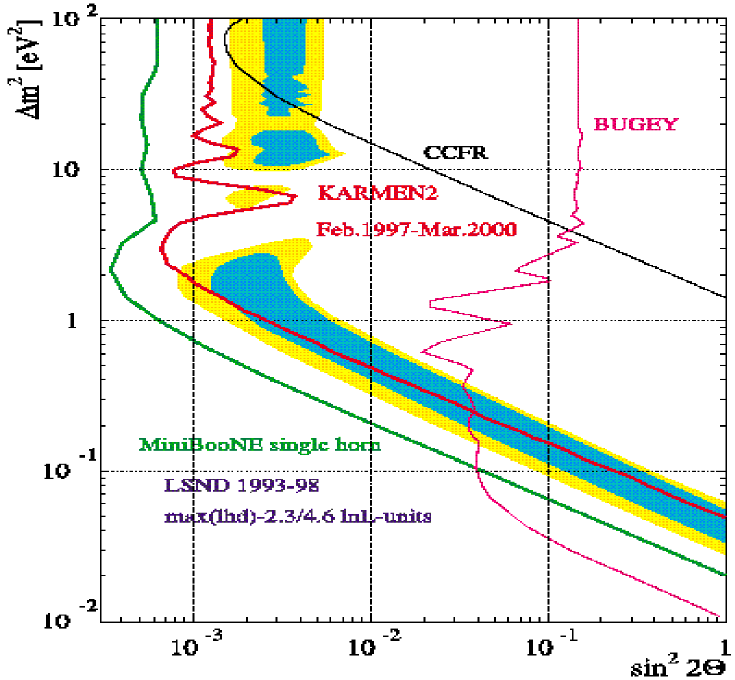

- $Karmen$ which searched for the same signal and did not observe oscillations.
- $MiniBoone$ in Fermilab has been running to solve this but....

MiniBooNE

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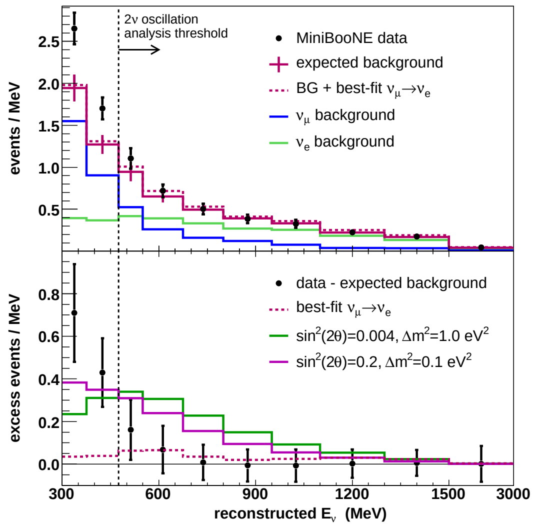
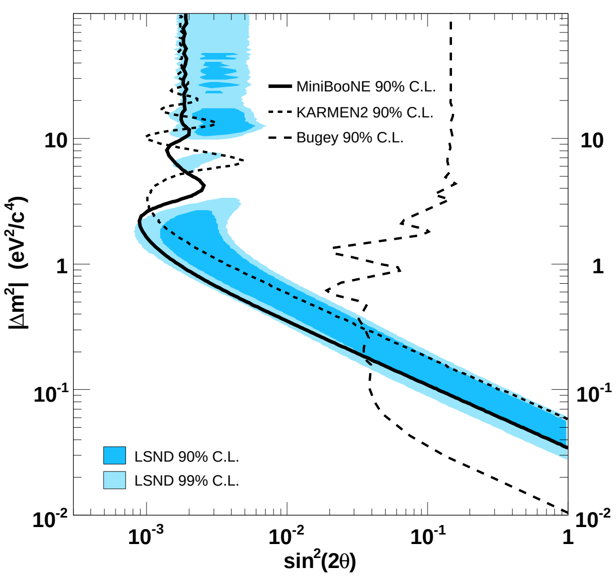
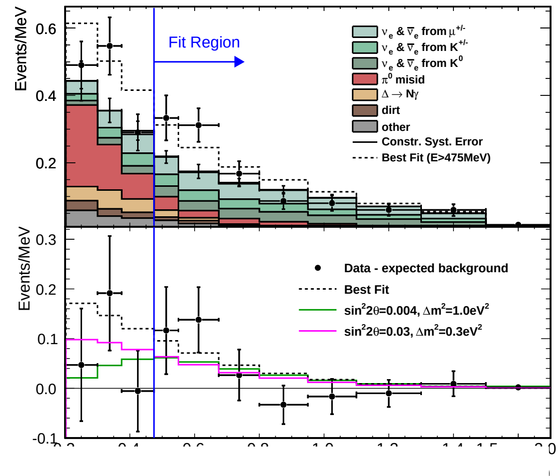
• Search for $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ with $E_\nu = 0.3 - 2$ GeV and $L = 540$ m

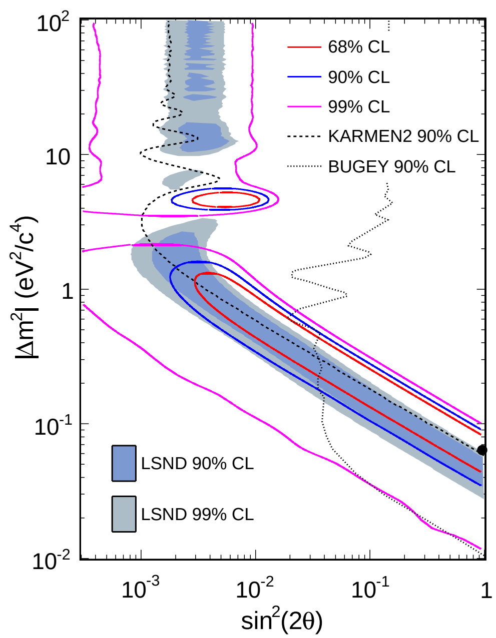
Neutrino Results (2007): No Signal, Bound





Antieutrino Results (2010): Signal





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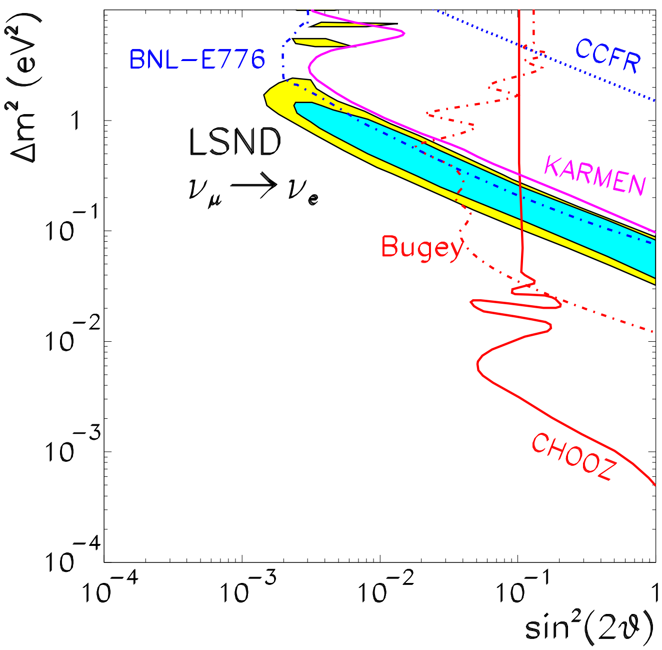
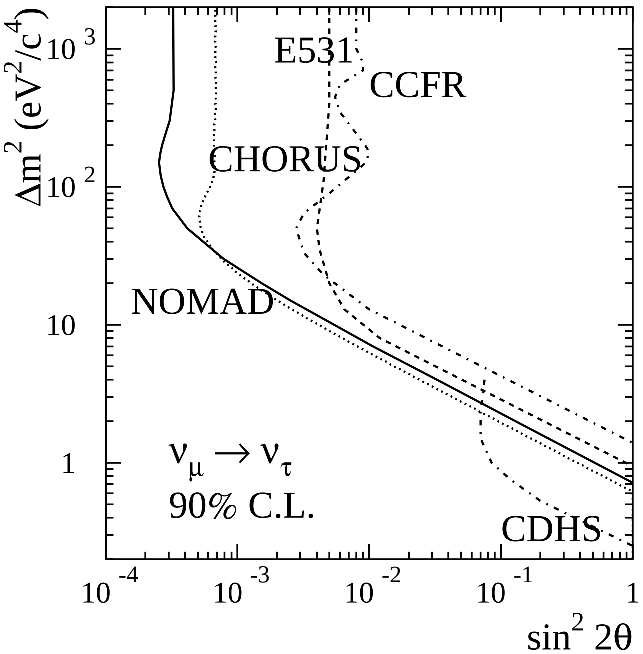
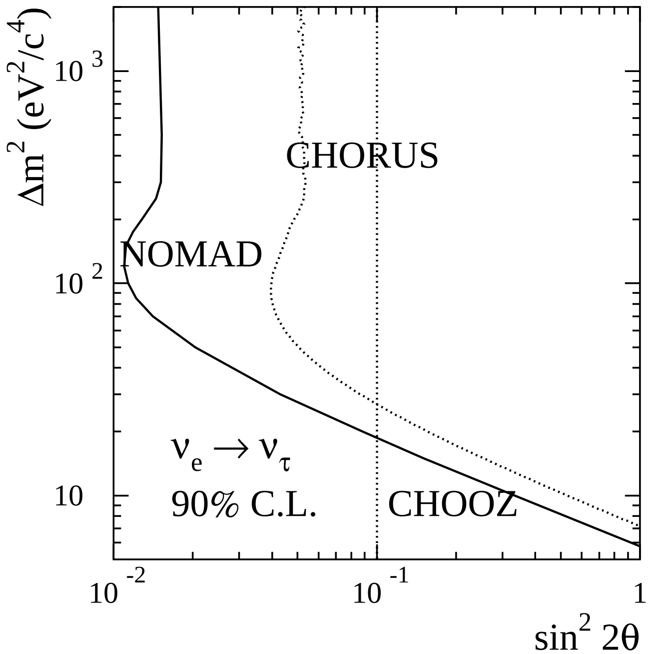
Summary of Searches at Short/Medium Baseline







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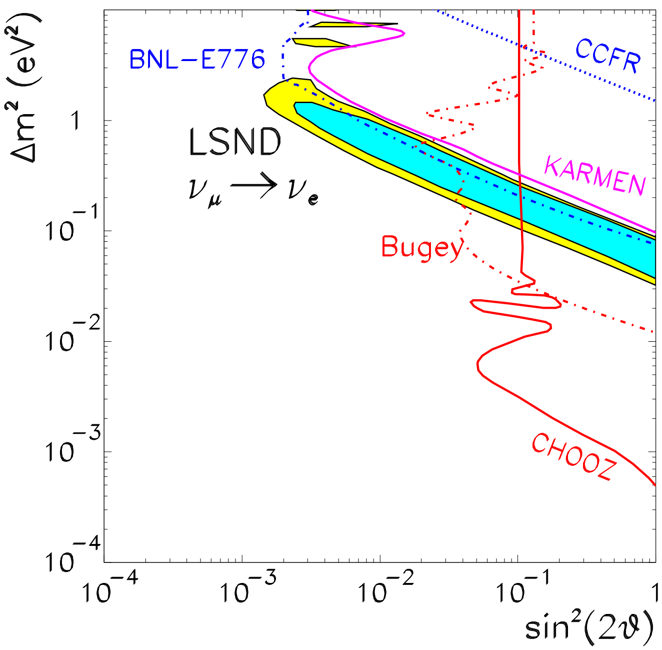
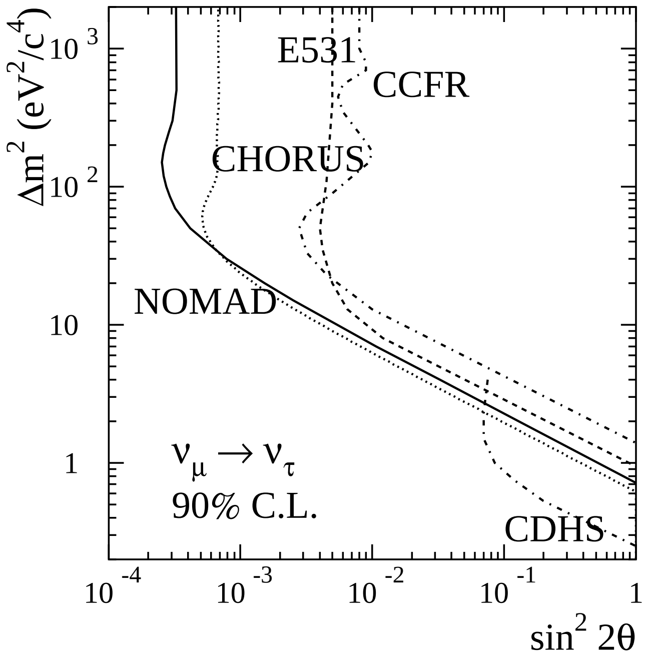
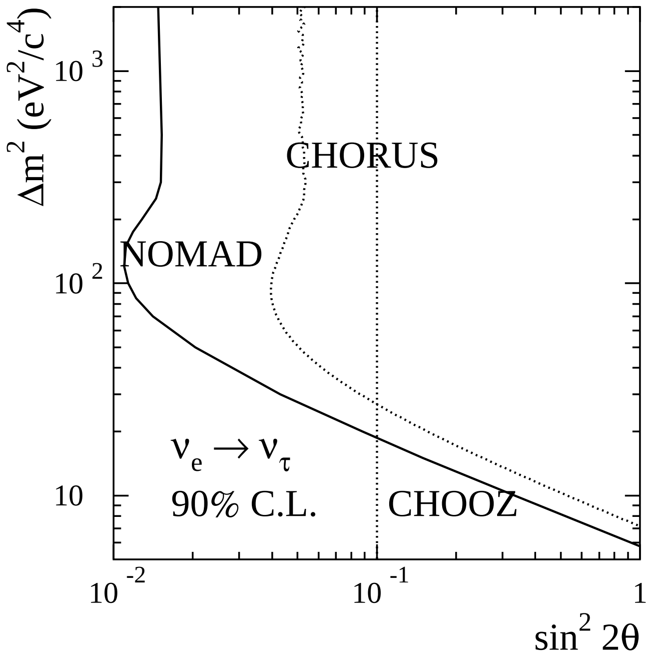
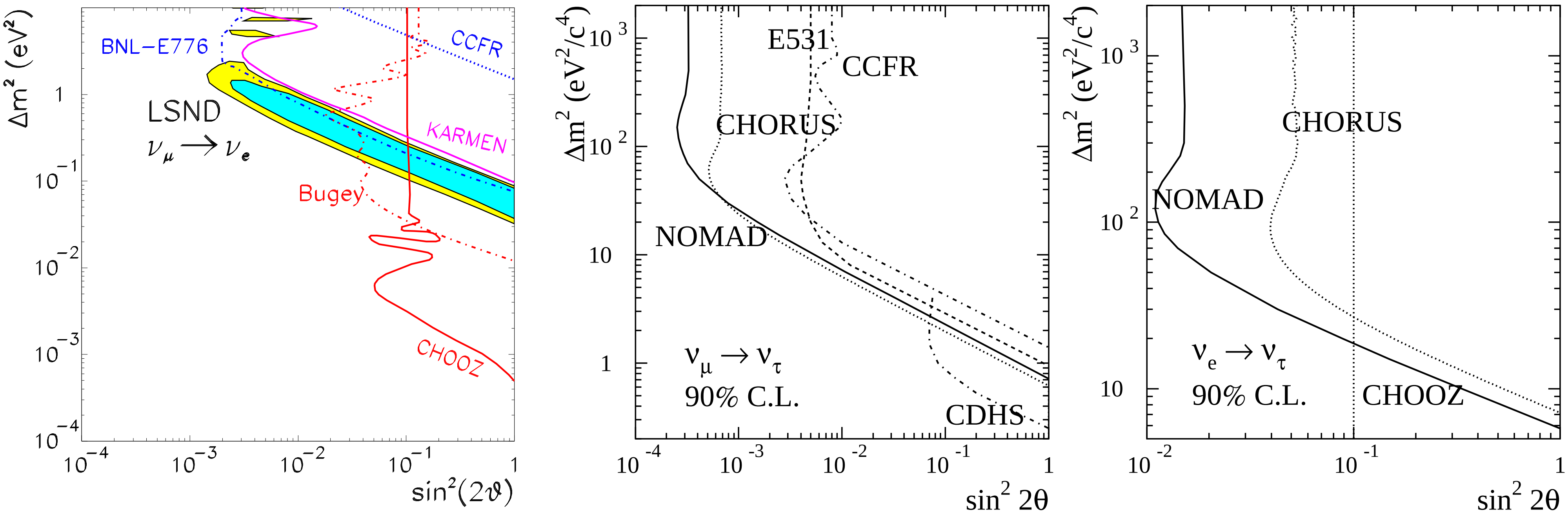






- CHOOZ Reactor disapp exper
  - ⇒ Lower E and longer L
  - ⇒ Sensitive $\Delta m^2_{\rm ATM}$
  - ⇒ Constraint ν_e effects in ATM

Summary of Searches at Short/Medium Baseline



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- To reach small $\Delta m^2 \sim 10^{-3}$ eV^2 with $\nu'_\mu s$
  - ⇒ very large L and intermediate E
  - ⇒ Long Baseline Exp at Accelerators

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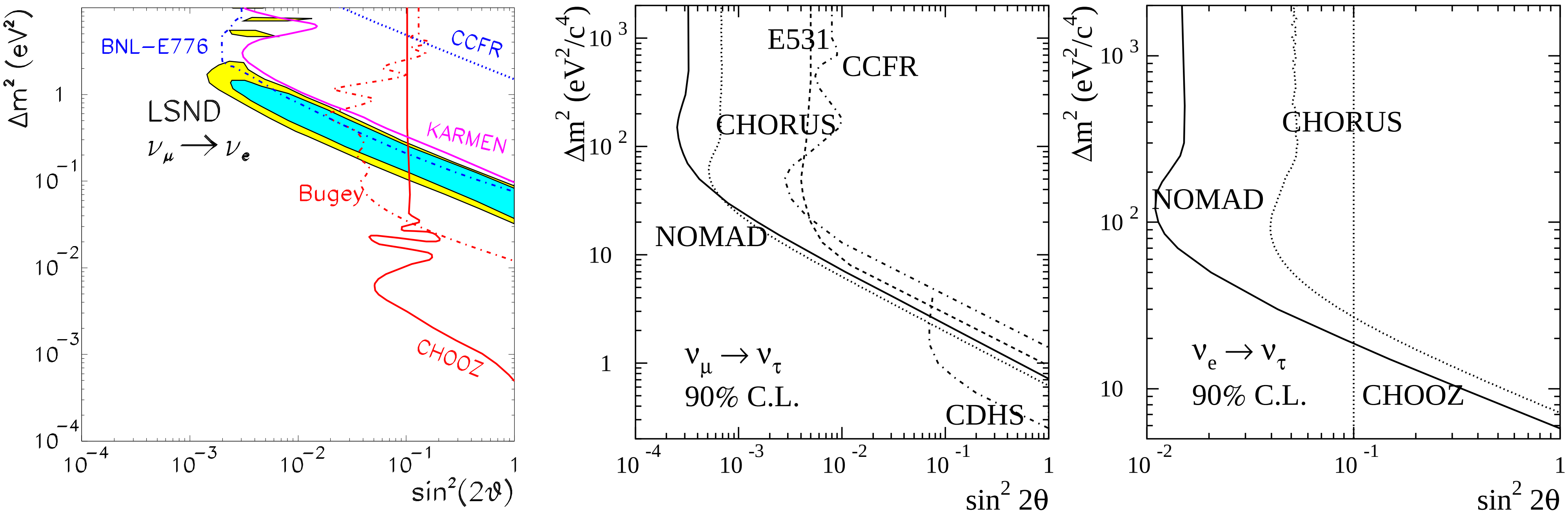
Summary of Searches at Short/Medium Baseline



- CHOOZ Reactor disapp exper
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- To reach small $\Delta m^2 \sim 10^{-3}$ eV^2 with $\nu_\mu' s$
  - $\Rightarrow$ very large L and intermediate E
  - $\Rightarrow$ Long Baseline Exp at Accelerators
- To lower $\Delta m^2 \sim 10^{-5}$ eV^2 with $\nu_e' s$
  - $\Rightarrow$ Long Baseline Exp at Reactors

ATM Test at Long Baseline Experiments

K2K	ν_μ at KEK	SK	L=250 km
MINOS	ν_μ at Fermilab	Soundan	L=735 km
Opera	ν_μ at CERN	Gran Sasso	L=740 km

ATM Test at Long Baseline Experiments

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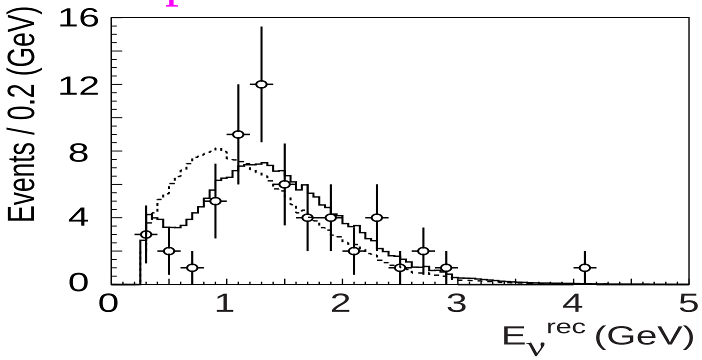
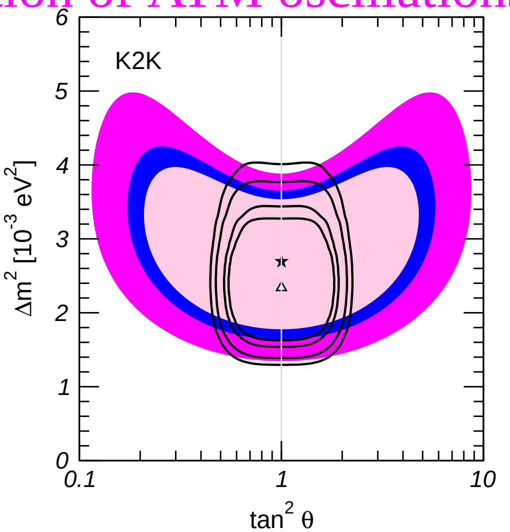
K2K 2004: spectral distortion



Confirmation of ATM oscillations



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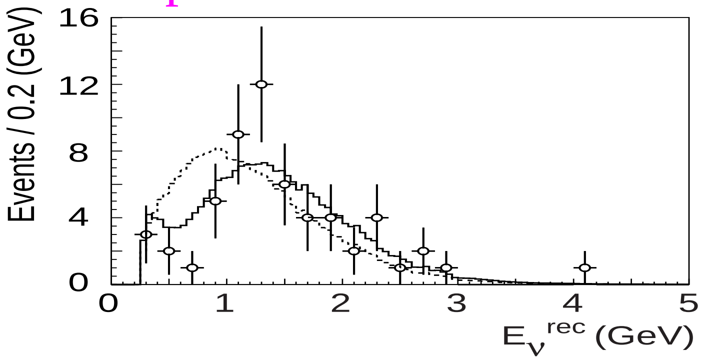
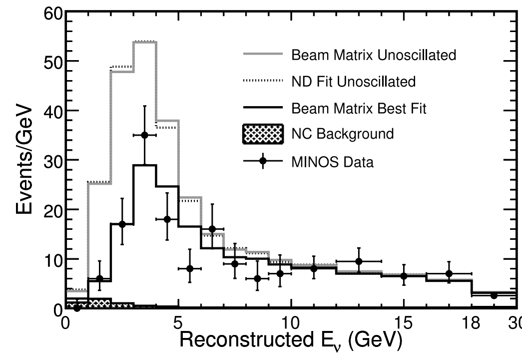


MINOS 2006-2010: spectral distortion



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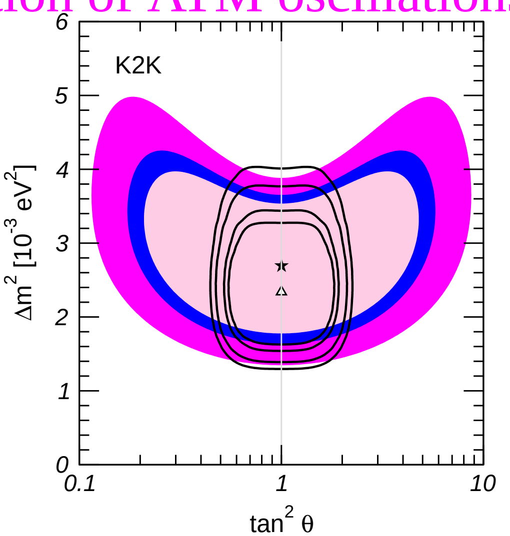
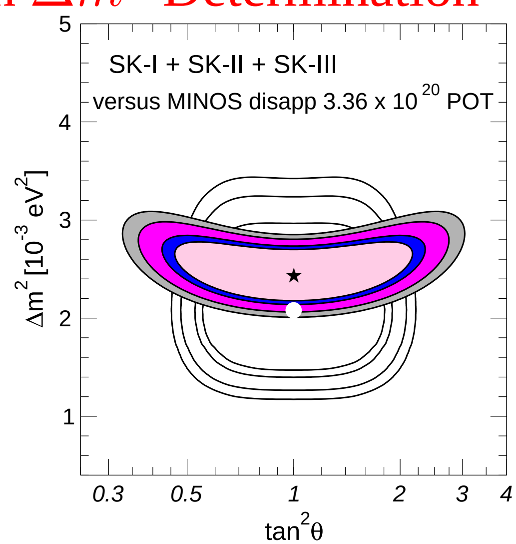


Impact on Δm^2 Determination

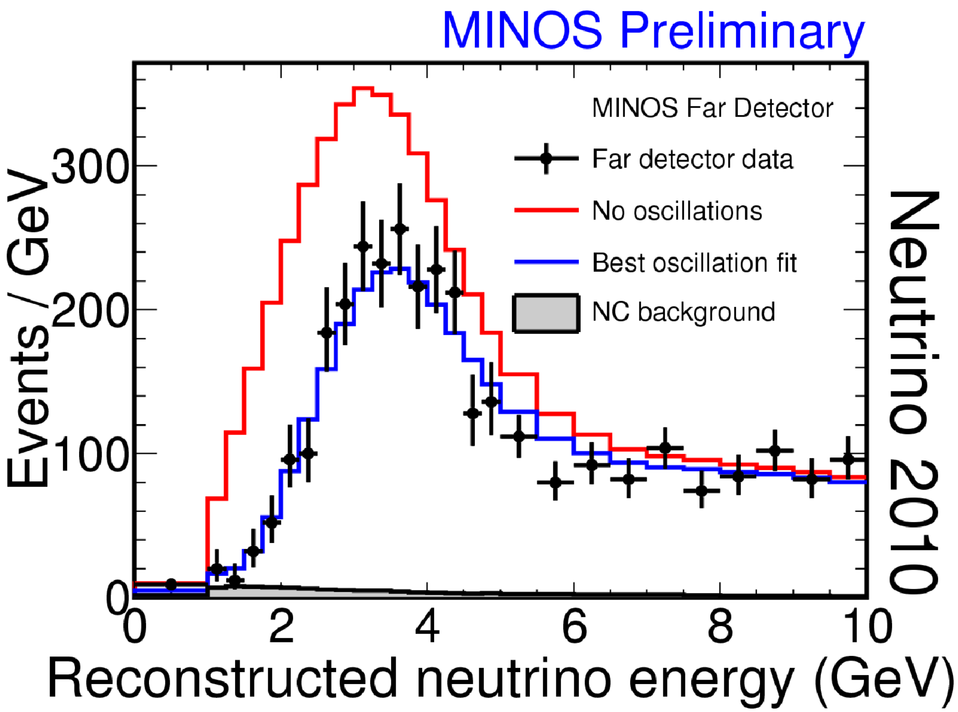


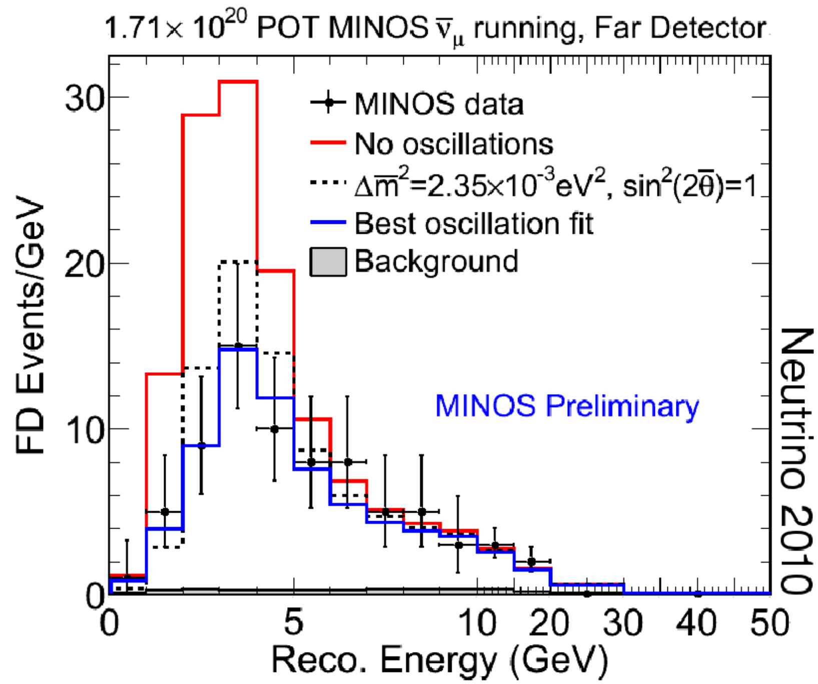
Little Puzzle with MINOS ν_μ versus $\bar{\nu}_\mu$ Disappearance

New Results Presented in ν2010:

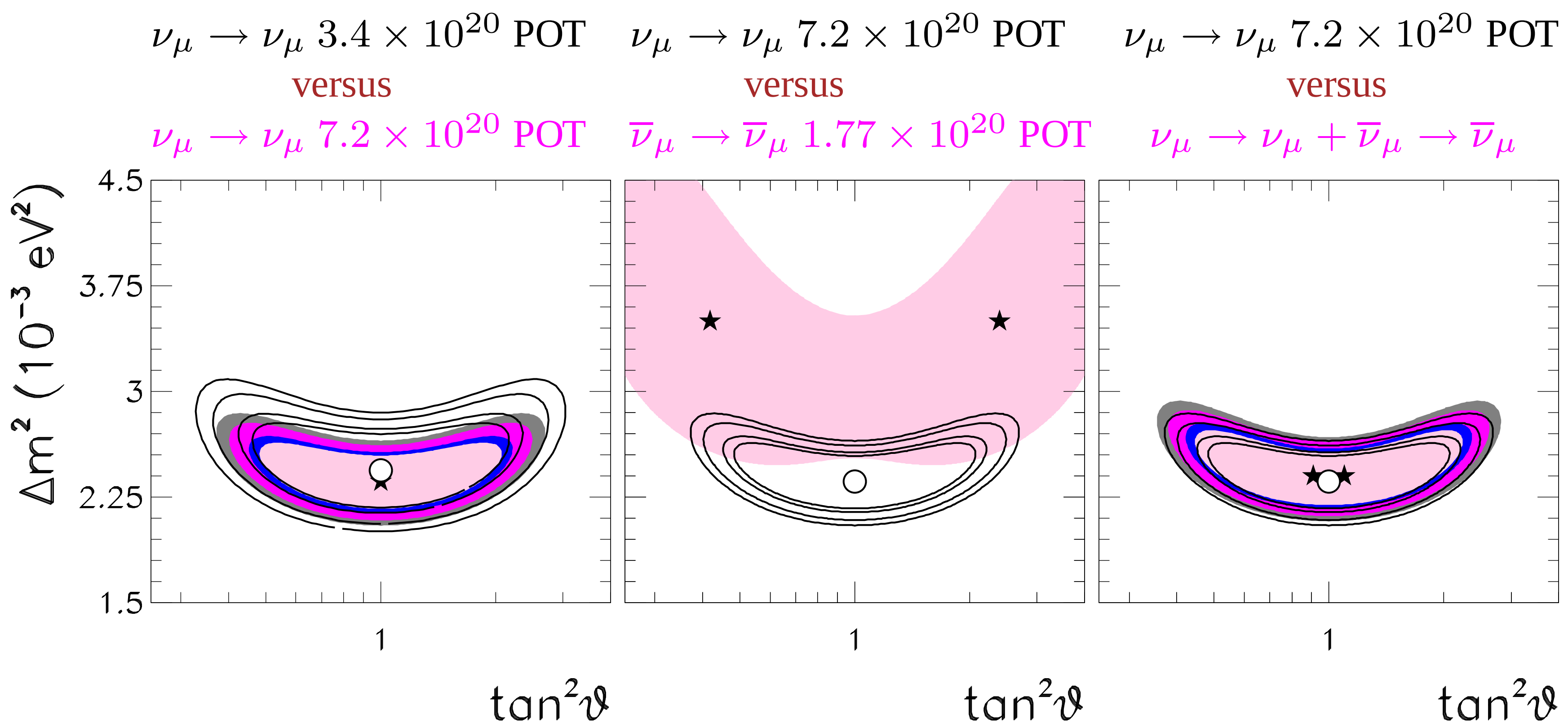
$\nu_\mu \to \nu_\mu$ 7.2×10^{20} POT

$\bar{\nu}_\mu \to \bar{\nu}_\mu$ 1.7×10^{20} POT





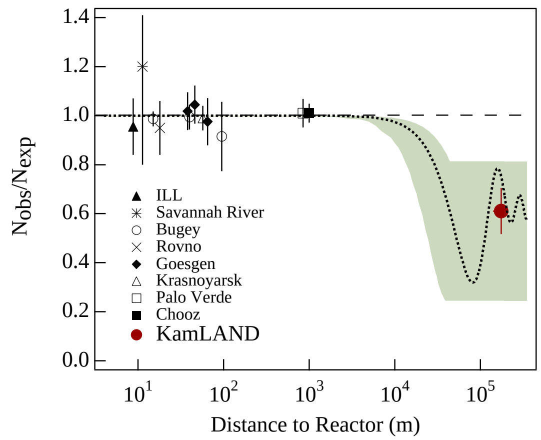
MINOS $\nu_\mu \to \nu_\mu$: Leading Oscillations



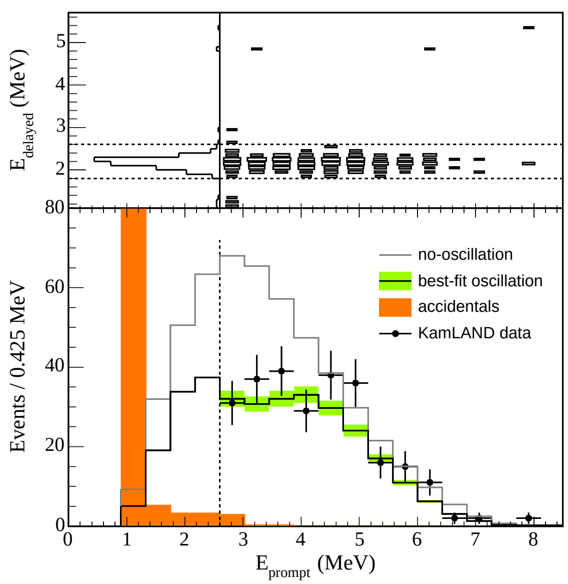
Terrestrial Test of LMA: KamLAND

- Search on $\overline{\nu_e}$ at L$\sim$ 180 km reactors, $E_{\overline{\nu}} \sim$ few MeV: $\bar{\nu}_e + p \to n + e^+$

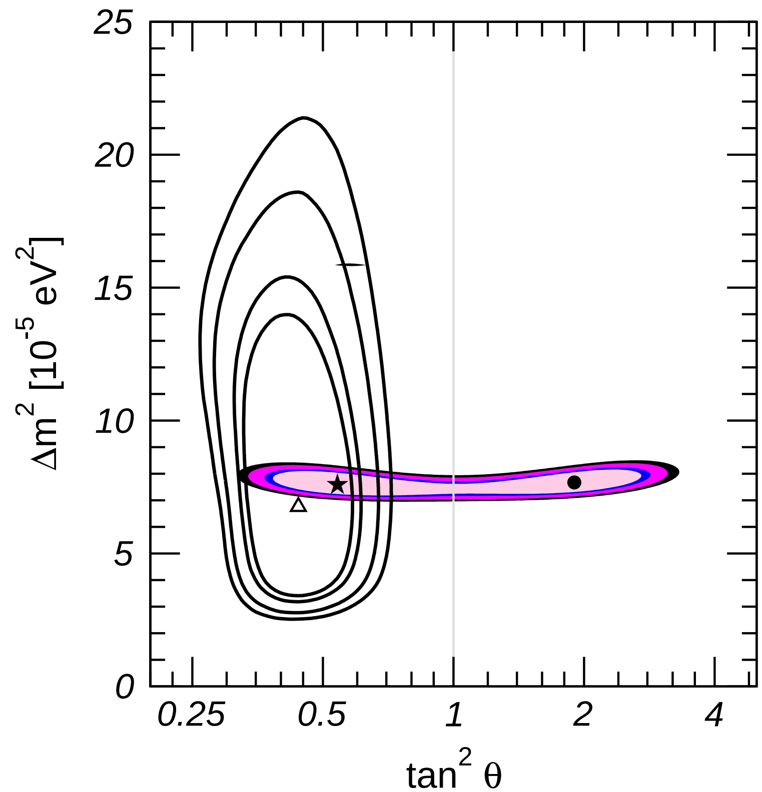
2002: Deficit $R_{\rm KLAND} = 0.611 \pm 0.094$



2004-: Significant Energy Distortion



Oscillation Analysis



Confirmation and Precision on Δm^2

Slight missmatch in best fit mixing angle

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Effects of ν mass: Oscillations in Vaccum and Matter

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- We have learned:

 $*$ Atmospheric ν_μ disappear ($>$ many σ) most likely to ν_τ

 $*$ K2K: accelerator ν_μ disappear at $L \sim 250$ Km with E-distortion (~ 2.5–4σ)

 $*$ MINOS: accelerator ν_μ disappear at $L \sim 735$ Km with E-distortion ($> 5\sigma$)

 $*$ Solar ν_e convert to ν_μ or ν_τ ($> 7\sigma$)

 $*$ KamLAND: reactor $\overline{\nu_e}$ disappear at $L \sim 200$ Km with E-distortion ($> 3\sigma$ CL)

 $*$ LSND and MINOS found some evidence for $\overline{\nu_\mu} \rightarrow \overline{\nu_e}$?

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  - $*$ LSND and MINOS found some evidence for $\overline{\nu_\mu} \to \overline{\nu_e}$?

All this implies that neutrinos are massive

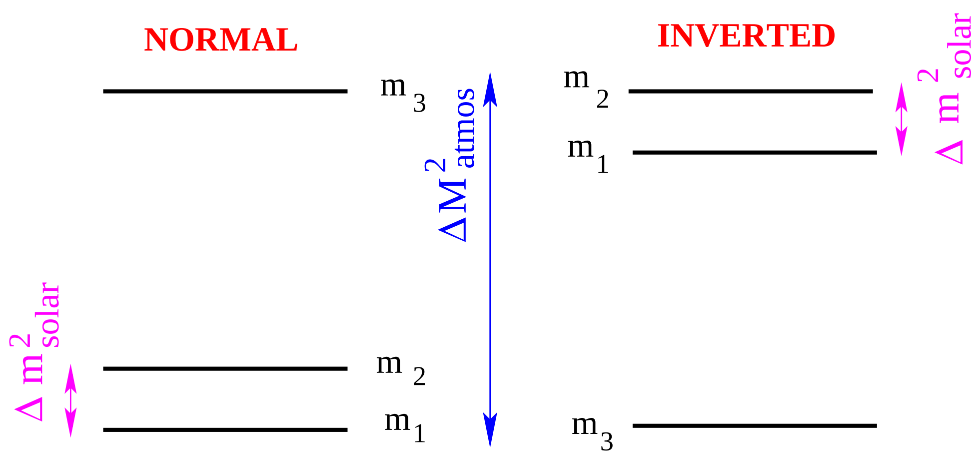
- We have important information (mostly constraints) from:
  - $*$ The line shape of the Z: $N_{\rm weak} = 3$
  - $*$ Limits from Short Distance Oscillation Searches at Reactor and Accelerators
  - $*$ Direct mass measurements: ${}^3H \to {}^3He + e^- + \bar{\nu}_e$ and ν-less $\beta\beta$ decay
  - $*$ From Astrophysics and Cosmology: BBN, CMBR, LSS ...

Solar+Atmospheric+Reactor+LBL 3ν Oscillations

U: 3 angles, 1 CP-phase
+ (2 Majorana phases)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Two mass schemes



2ν oscillation analysis $\Rightarrow \Delta m^2_{21} = \Delta m^2_{\odot} \ll \Delta M^2_{atm} \simeq \pm\Delta m^2_{32} \simeq \pm\Delta m^2_{31}$

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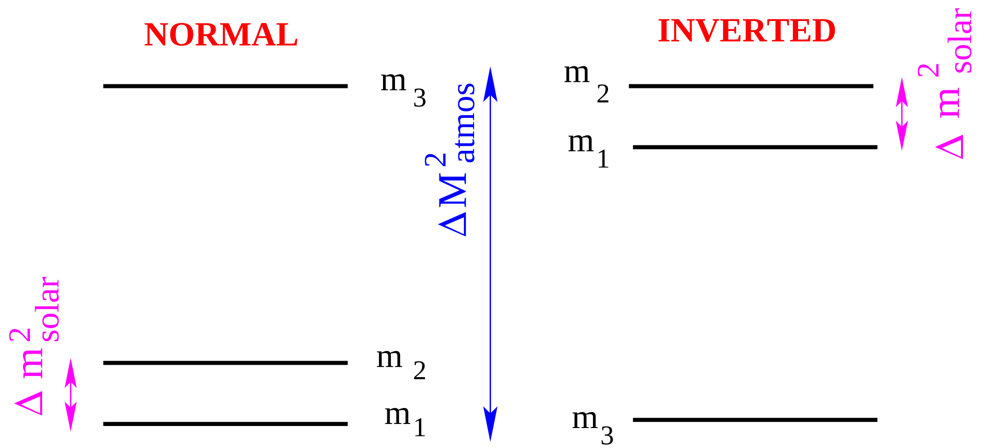

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Generic 3ν mixing effects:

– Effects due to θ_{13}

– Difference between Inverted and Normal

– Interference of two wavelength oscillations

– CP violation due to phase δ

3–ν Neutrino Oscillations

- In general one has to solve: $$i\frac{d\vec{\nu}}{dt} = H\,\vec{\nu}$$ $$H = U \cdot H_0^d \cdot U^\dagger + V$$

$$H_0^d = \frac{1}{2E_\nu}\text{diag}\left(-\Delta m^2_{21}, 0, \Delta m^2_{32}\right) \qquad V = \text{diag}\left(\pm\sqrt{2}G_F N_e, 0, 0\right)$$

$\Rightarrow$ Oscillation Probabilities Depend on the 6 parameters

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$\Rightarrow$ Oscillation Probabilities Depend on the 6 parameters

- Hierarchical approximation: $\Delta m^2_{21} \ll \Delta m^2_{31} \sim \Delta m^2_{32}$

$\Rightarrow$ Simplification of the relevant Oscillation Probabilities

3–ν Neutrino Oscillations: CHOOZ and KamLAND

- Hierarchical approximation: $\Delta m^2_{21} \ll \Delta m^2_{31} \sim \Delta m^2_{32}$

$$P^{3\nu}_{ee} = 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{31} L}{4E} - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \frac{\Delta m^2_{21} L}{4E}$$

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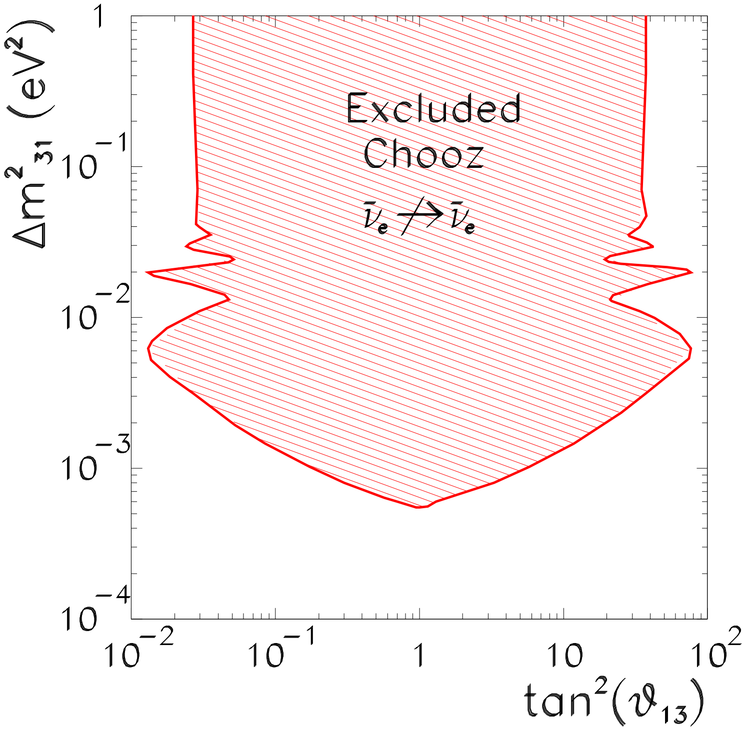
$*$ CHOOZ: L$\sim$ 1 km, E$\sim$ MeV $\Rightarrow \frac{\Delta m^2_{21} L}{4E} \simeq 0$

$$P^{CH}_{ee} = 1 - \sin^2 2\theta^2_{13} \sin^2 \left(\frac{\Delta m^2_{31} L}{4E}\right)$$



Limit on $\sin^2 \theta_{13} \lesssim 0.1$–$0.02$

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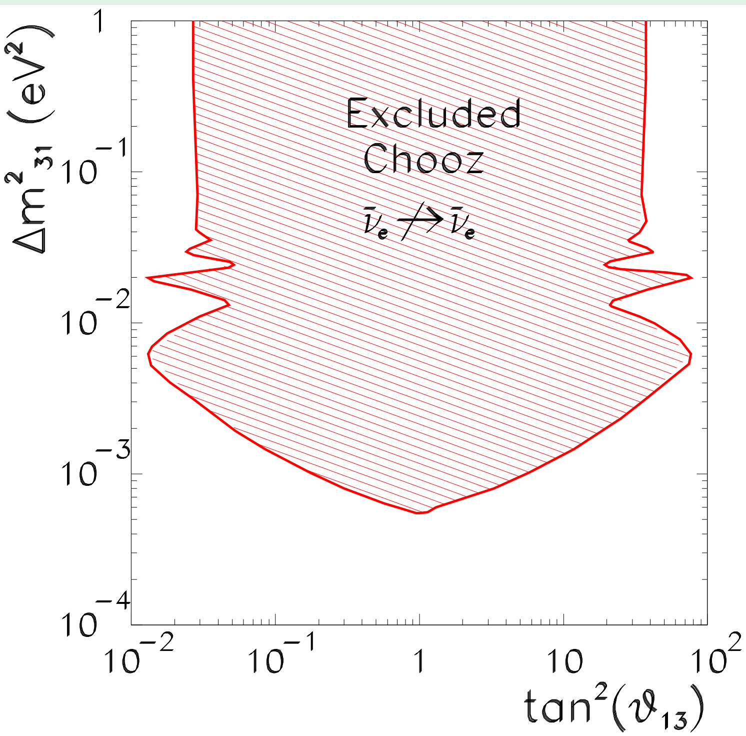


Limit on $\sin^2 \theta_{13} \lesssim 0.1$–$0.02$

$*$ KamLAND: L$\sim$ 180 km, E$\sim$ MeV

$\Rightarrow \frac{\Delta m^2_{31} L}{4E} \simeq \frac{1}{2}$

$$P^{KL,3\nu}_{ee} = c^4_{13} P^{2\nu}_{ee}(\Delta m^2_{12}, \theta_{12}) + s^4_{13}$$

3–ν Neutrino Oscillations: SOLAR

Convenient to use an intermediate basis

$$\nu_{e'} = c_{12}\nu_1 + s_{12}\nu_2 \quad \nu_{\mu'} = -s_{12}\nu_1 + c_{12}\nu_2 \quad \nu_{\tau'} = \nu_\tau$$

At lowest order in $EV \sin 2\theta_{13}/m_3$ and m_2^2/m_3^2

$$i\frac{d}{dt}\begin{pmatrix}\nu_{e'}\\ \nu_{\mu'}\\ \nu_{\tau'}\end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -m_2^2\cos 2\theta_{12} + 2EV\cos^2\theta_{13} & m_2^2 \sin 2\theta_{12} & \\ m_2^2 \cos 2\theta_{12} & m_2^2\cos 2\theta_{12} - 2EV\cos^2\theta_{13} & \\ & & 2m_3^2 \end{pmatrix}\begin{pmatrix}\nu_{e'}\\ \nu_{\mu'}\\ \nu_{\tau'}\end{pmatrix}$$

Solving one gets:

$$P_{ee}^{\odot,3\nu} = c_{13}^4 P_{ee}^{\odot,2\nu}(\Delta m_{12}^2, \theta_{12}, V' = V\cos^2\theta_{13}) + s_{13}^4$$

3ν Solar Neutrino Oscillations: θ_{13}

For solar neutrinos

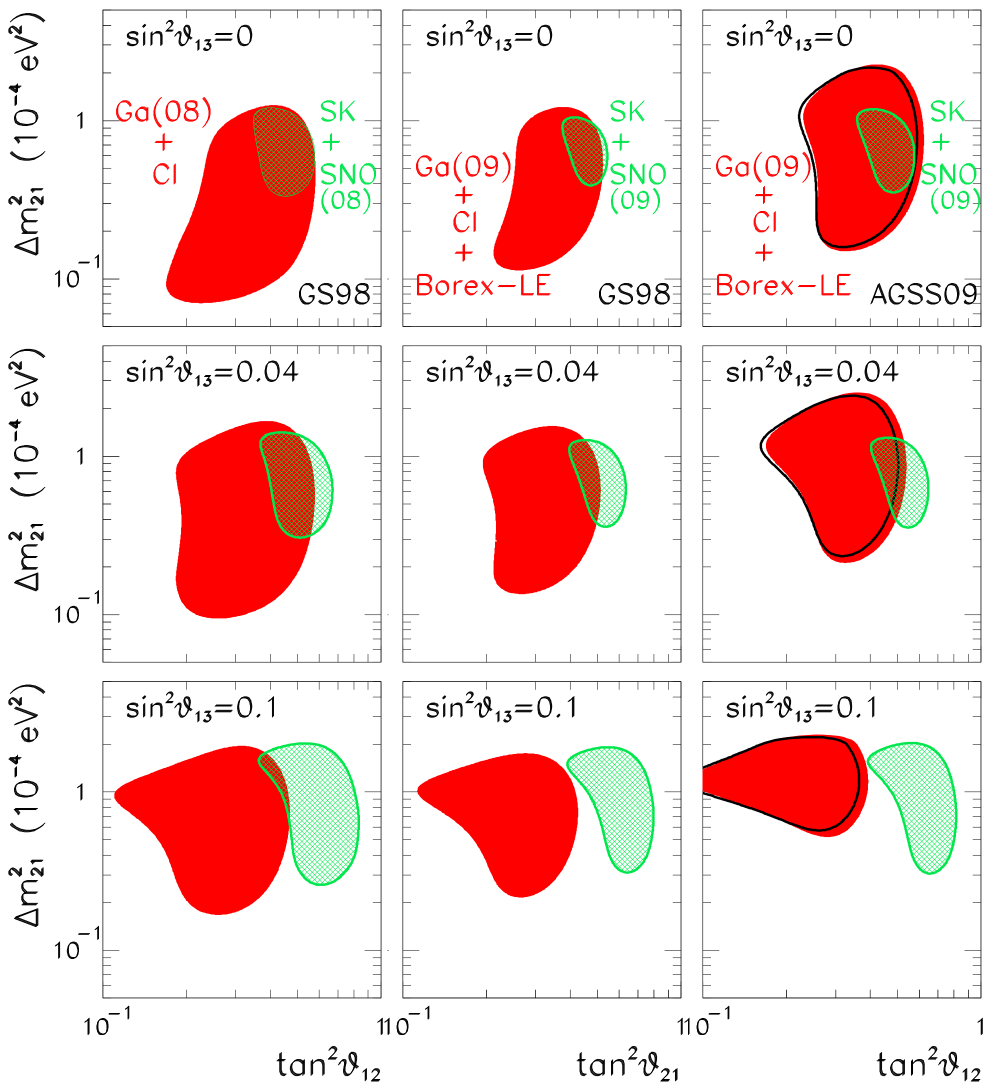
$$P_{ee}^{3\nu} = \sin^4\theta_{13} + \cos^4\theta_{13} P_{ee}^{2\nu}(\Delta m^2_{21}, \theta_{12})$$

For $E_\nu \lesssim$ few $\times$ 100 KeV (Cl,Ga)

$$P_{ee}^{2\nu}(\Delta m^2_{21}, \theta_{12}) \simeq 1 - \frac{1}{2}\sin^2(2\theta_{12})$$

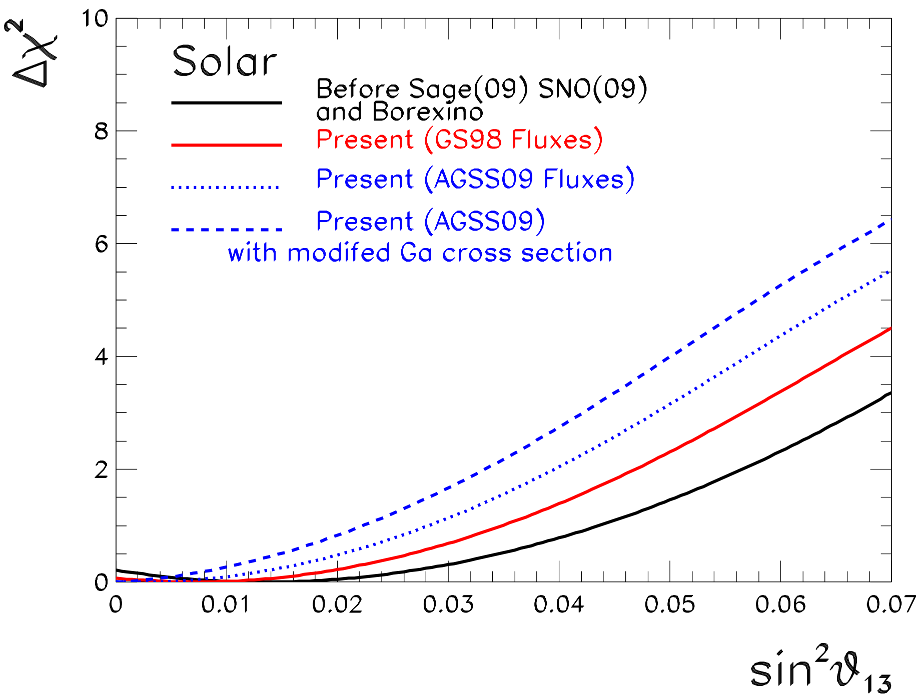
For $E_\nu \gtrsim$ few $\times$ 1 MeV (SNO,SK)

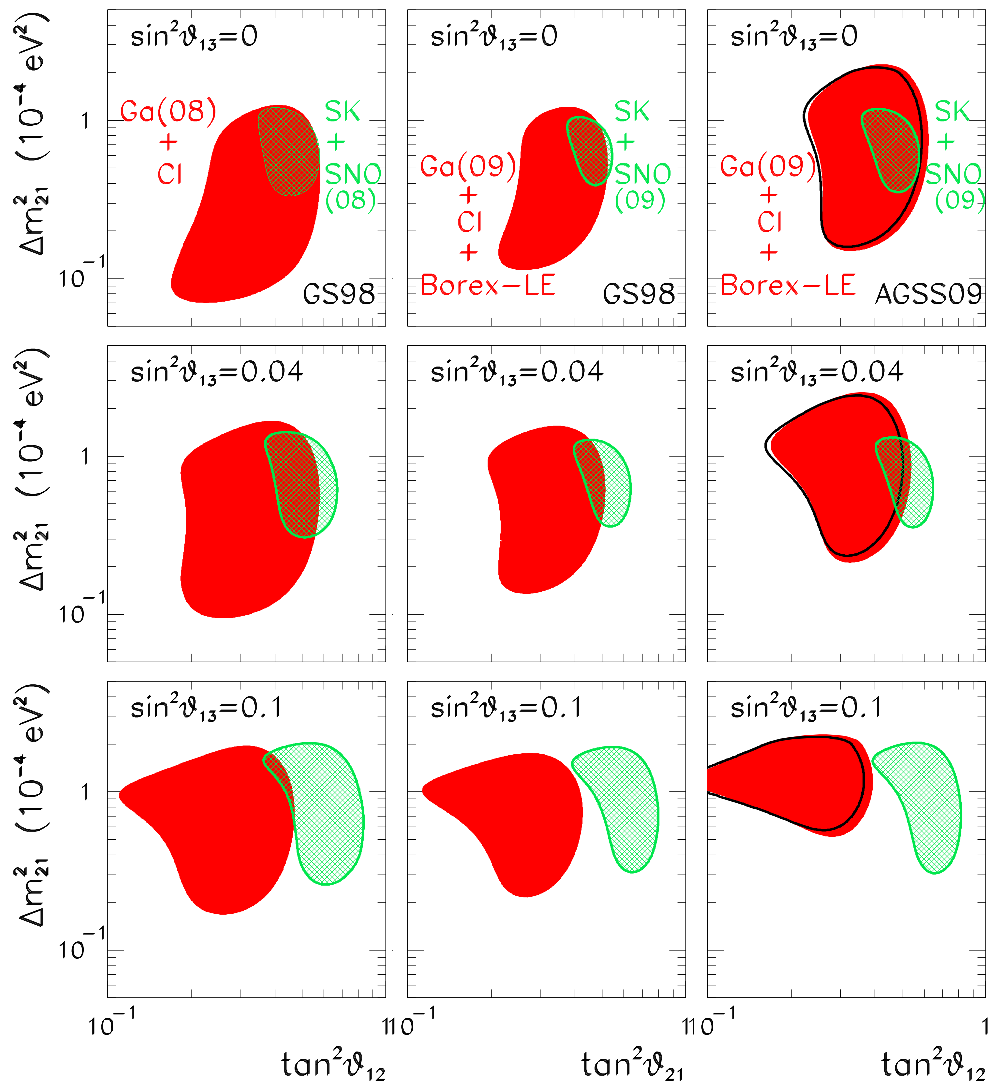
$$P_{ee}^{2\nu}(\Delta m^2_{21}, \theta_{12}) \simeq \sin^2(\theta_{12})$$



Solar Neutrino Oscillations: θ_{13}

$\theta_{13} = 0$ Best fit



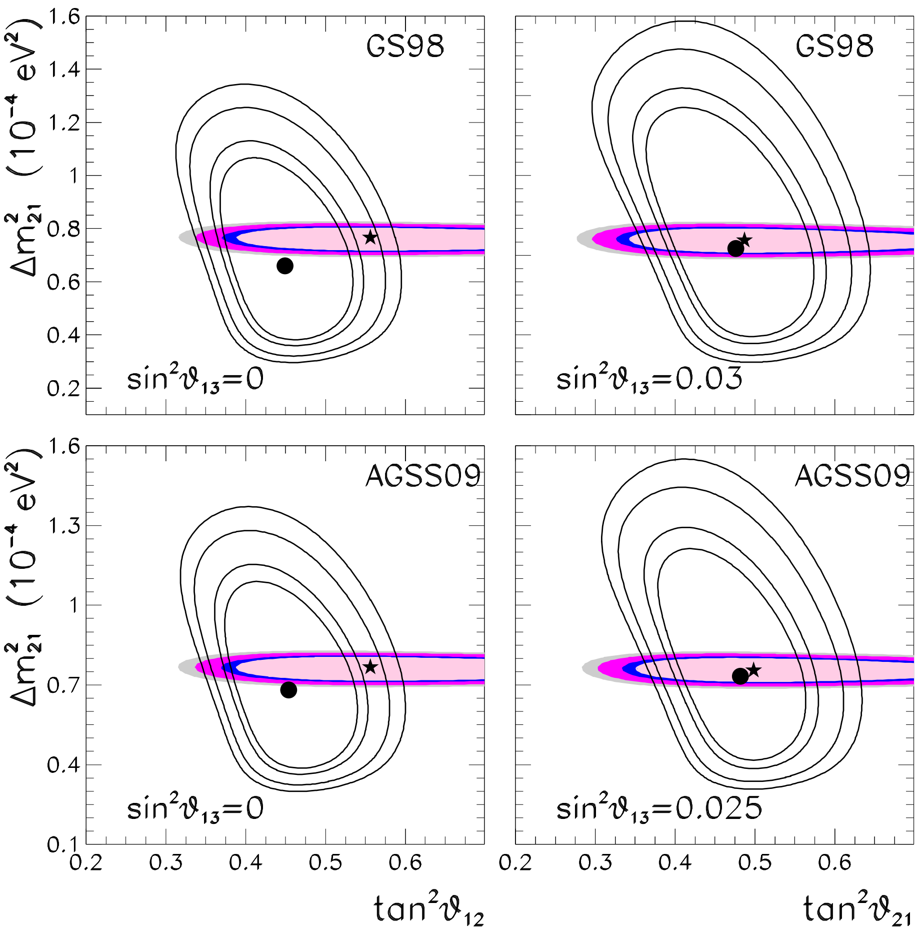


Solar + KamLAND and θ_{13}

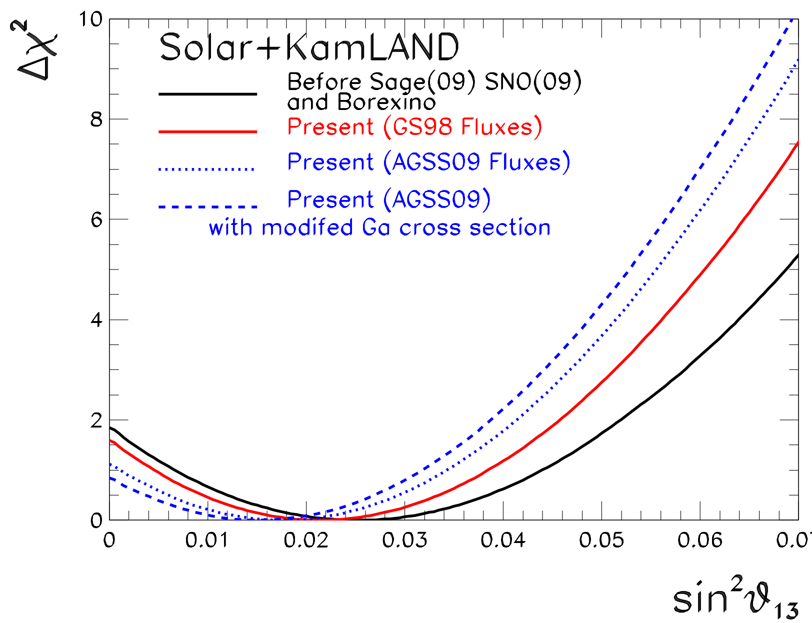
For KamLAND: $P_{ee}^{3\nu} = \sin^4\theta_{13} + \cos^4\theta_{13} P_{ee}^{2\nu}(\Delta m^2_{21}, \theta_{12})$

With $P_{ee}^{2\nu,\mathrm{kam}} = 1 - \frac{1}{2}\sin^2(2\theta_{12})\sin^2\frac{\Delta m^2_{21} L}{2E}$

Better Agreement with $\theta_{13} \neq 0$



But Not very significant



$$\sin^2\theta_{13} = \begin{array}{ll} 0.021 \pm 0.017 & \mathrm{for GS98} \\ 0.017 \pm 0.017 & \mathrm{for AGSS09} \end{array}$$

3–ν Atmospheric Neutrino Oscillation: Effect of θ_{13}

- In general one has to solve: $i\frac{d\vec{\nu}}{dt} = H\,\vec{\nu}$ $\qquad H = U \cdot H_0^d \cdot U^\dagger + V$

$$H_0^d = \frac{1}{2E_\nu}\text{diag}\left(-\Delta m_{21}^2, 0, \Delta m_{32}^2\right) \qquad V = \text{diag}\left(\pm\sqrt{2}G_F N_e, 0, 0\right)$$

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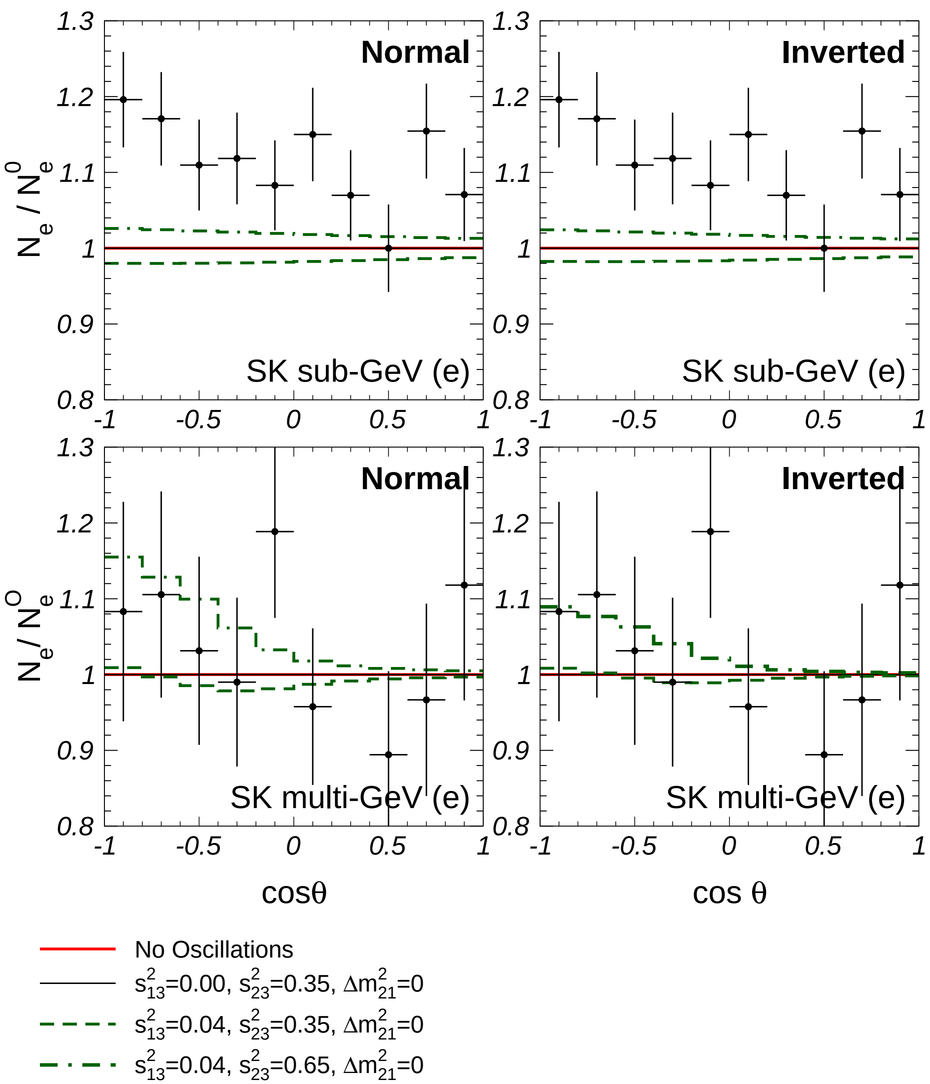
- Hierarchical approximation: $\Delta m^2_{21} \ll \Delta m^2_{31} \sim \Delta m^2_{32}$ $\Rightarrow$ neglect Δm^2_{21} in ATM

$$\begin{aligned}
P_{ee} &= 1 - 4 s^2_{13,m} c^2_{13,m}\, S_{31} \\
P_{\mu\mu} &= 1 - 4 s^2_{13,m} c^2_{13,m} s^4_{23}\, S_{31} - 4 s^2_{13,m} s^2_{23} c^2_{23}\, S_{21} - 4 c^2_{13,m} s^2_{23} c^2_{23}\, S_{32} \\
P_{e\mu} &= 4 s^2_{13,m} c^2_{13,m} s^2_{23}\, S_{31} \\
S_{ij} &= \sin^2\left(\frac{\Delta\mu^2_{ij}}{4E_\nu} L\right) \\
\Delta\mu^2_{21} &= \frac{\Delta m^2_{32}}{2}\left(\frac{\sin 2\theta_{13}}{\sin 2\theta_{13,m}} - 1\right) - E_\nu V_e \\
\Delta\mu^2_{32} &= \frac{\Delta m^2_{32}}{2}\left(\frac{\sin 2\theta_{13}}{\sin 2\theta_{13,m}} + 1\right) + E_\nu V_e \\
\Delta\mu^2_{31} &= \Delta m^2_{32}\frac{\sin 2\theta_{13}}{\sin 2\theta_{13,m}} \\
\sin 2\theta_{13,m} &= \frac{\sin 2\theta_{13}}{\sqrt{(\cos 2\theta_{13} \mp \frac{2E_\nu V_e}{\Delta m^2_{31}})^2 + \sin^2 2\theta_{13}}}
\end{aligned}$$

3–ν Atmospheric Neutrino Oscillation: Effect of θ_{13}



$$\frac{N_e}{N_{e0}} - 1 = \overline{P_{e\mu}}\,\bar{r}\left(s^2_{23} - \frac{1}{\bar{r}}\right)$$

$$\bar{r} = \frac{N_{\mu 0}}{N_{e0}}$$

$$P_{e\mu} = 4 s^2_{13,m} c^2_{13,m} \sin^2\left(\frac{\Delta m^2_{31}\, L}{4E_\nu}\,\frac{\sin 2\theta_{13}}{\sin 2\theta_{13,m}}\right)$$

$$\sin 2\theta_{13,m} = \frac{\sin 2\theta_{13}}{\sqrt{\left(\cos 2\theta_{13} \mp \frac{2E_\nu V_e}{\Delta m^2_{31}}\right)^2 + \sin^2 2\theta_{13}}}$$

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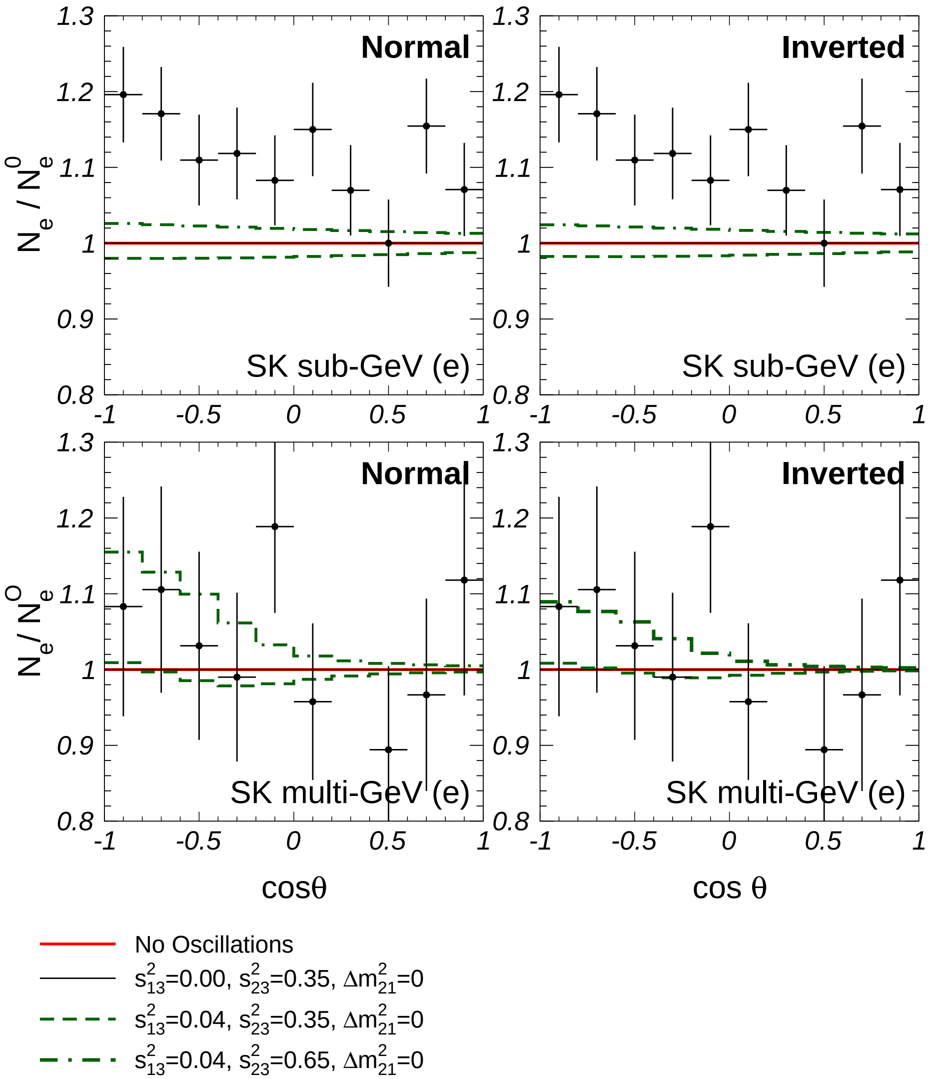
3–ν Atmospheric Neutrino Oscillation: Effect of θ_{13}



$$\frac{N_e}{N_{e0}} - 1 = \overline{P_{e\mu}}\bar{r}\left(s^2_{23} - \frac{1}{\bar{r}}\right)$$

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- Multi-GeV : Enhancement due to Matter

  Larger Effect in Normal

  Possible Sensitivity to Mass Ordering

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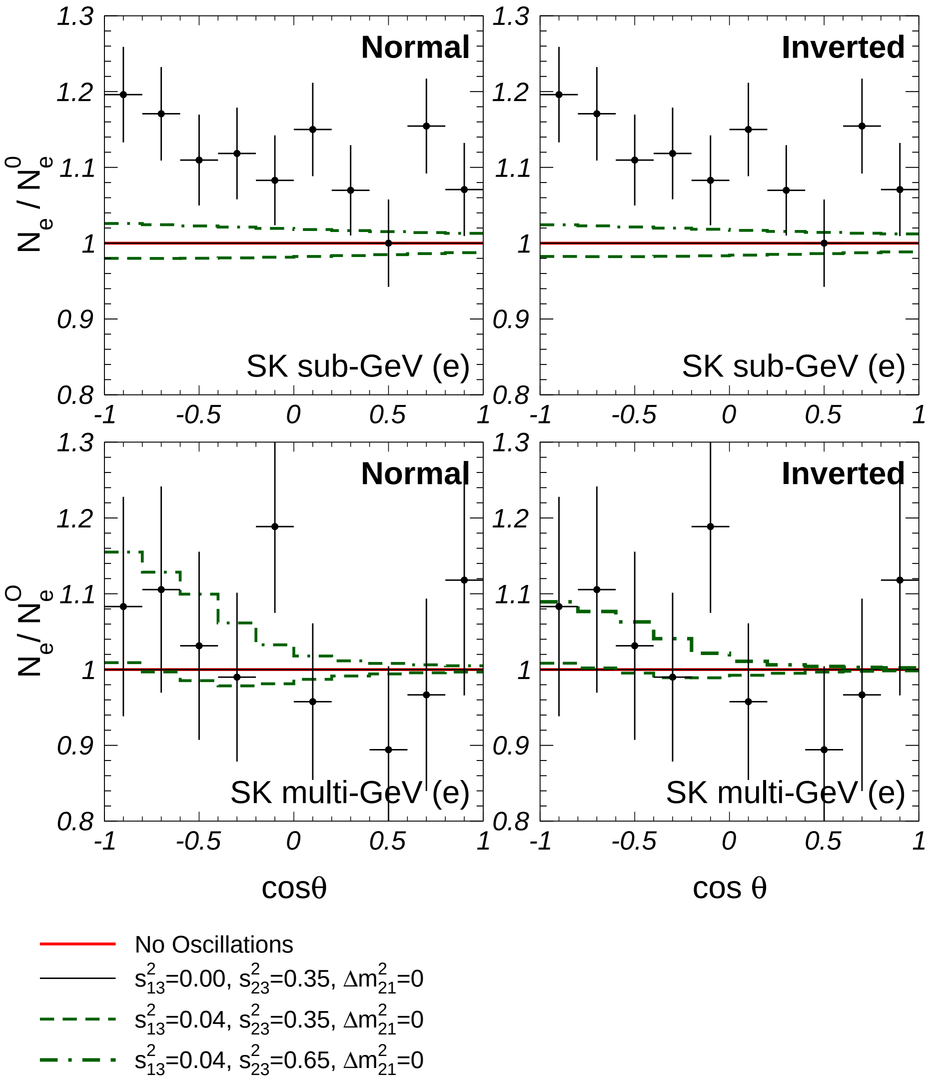
3–ν Atmospheric Neutrino Oscillation: Effect of θ_{13}



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- Multi-GeV : Enhancement due to Matter

  Larger Effect in Normal

  Possible Sensitivity to Mass Ordering

- Sub-GeV: Vacuum Osc: Smaller Effect

$$r \simeq 2 \Rightarrow \begin{array}{l} \theta_{23} < \frac{\pi}{4} \Rightarrow s^2_{23} < \frac{1}{2} \Rightarrow N_e(\theta_{13}) < N_{e0} \\ \theta_{23} > \frac{\pi}{4} \Rightarrow s^2_{23} > \frac{1}{2} \Rightarrow N_e(\theta_{13}) > N_{e0} \end{array}$$

Δm^2_{21} effects in ATM Data

- In general one has to solve: $i\frac{d\vec{\nu}}{dt} = H\,\vec{\nu}$ $\qquad H = U \cdot H_0^d \cdot U^\dagger + V$

$$H_0^d = \frac{1}{2E_\nu}\mathrm{diag}\left(-\Delta m^2_{21}, 0, \Delta m^2_{32}\right) \qquad V = \mathrm{diag}\left(\pm\sqrt{2}G_F N_e, 0, 0\right)$$

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- Neglecting θ_{13}:

$$\begin{aligned}
P_{ee} &= 1 - P_{e2} \\
P_{e\mu} &= c^2_{23} P_{e2} \\
P_{\mu\mu} &= 1 - c^4_{23} P_{e2} - 2 s^2_{23} c^2_{23}\left[1 - \sqrt{1 - P_{e2}}\cos\phi\right] \\
P_{e2} &= \sin^2 2\theta_{12,m}\sin^2\left(\frac{\Delta m^2_{21}\,L}{4E_\nu}\,\frac{\sin 2\theta_{12}}{\sin 2\theta_{12,m}}\right) \\
\sin 2\theta_{12,m} &= \frac{\sin 2\theta_{12}}{\sqrt{(\cos 2\theta_{12} \mp \frac{2E_\nu V_e}{\Delta m^2_{21}})^2 + \sin^2 2\theta_{12}}} \\
\phi &\approx (\Delta m^2_{31} + s^2_{12}\,\Delta m^2_{21})\frac{L}{2E_\nu}
\end{aligned}$$

Δm^2_{21} effects in ATM Data

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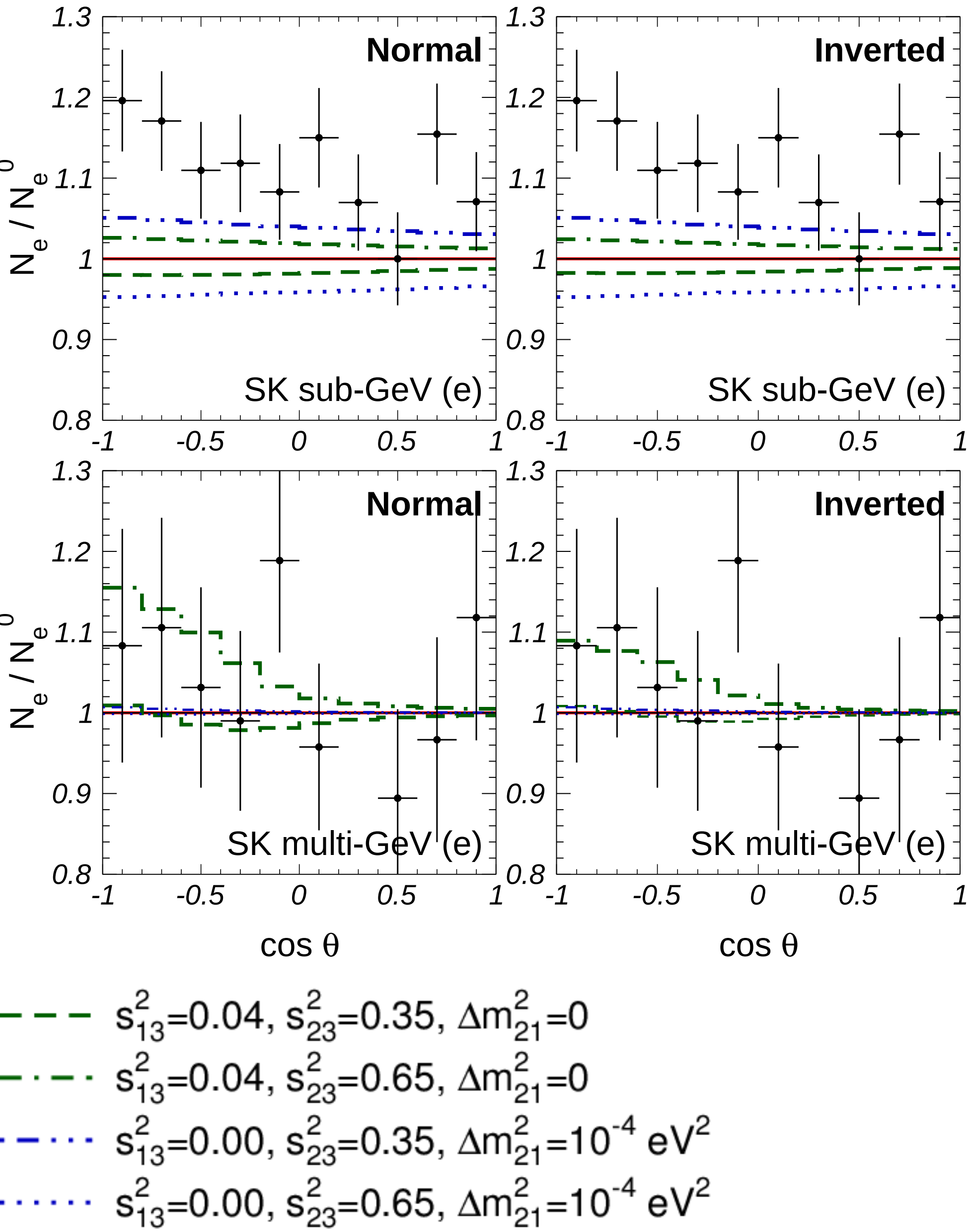

$$\frac{N_e}{N_{e0}} - 1 = \overline{P_{e2}}\,\bar{r}\,(c^2_{23} - \frac{1}{\bar{r}})$$

$$P_{e2} = \sin^2 2\theta_{12,m} \sin^2\left(\frac{\Delta m^2_{21}\, L}{4E_\nu}\,\frac{\sin 2\theta_{12}}{\sin 2\theta_{12,m}}\right)$$

$$\sin 2\theta_{12,m} = \frac{\sin^2 2\theta_{12}}{\sqrt{(\cos 2\theta_{12} \mp \frac{2EV_e}{\Delta m^2_{21}})^2 + \sin^2 2\theta_{12}}}$$

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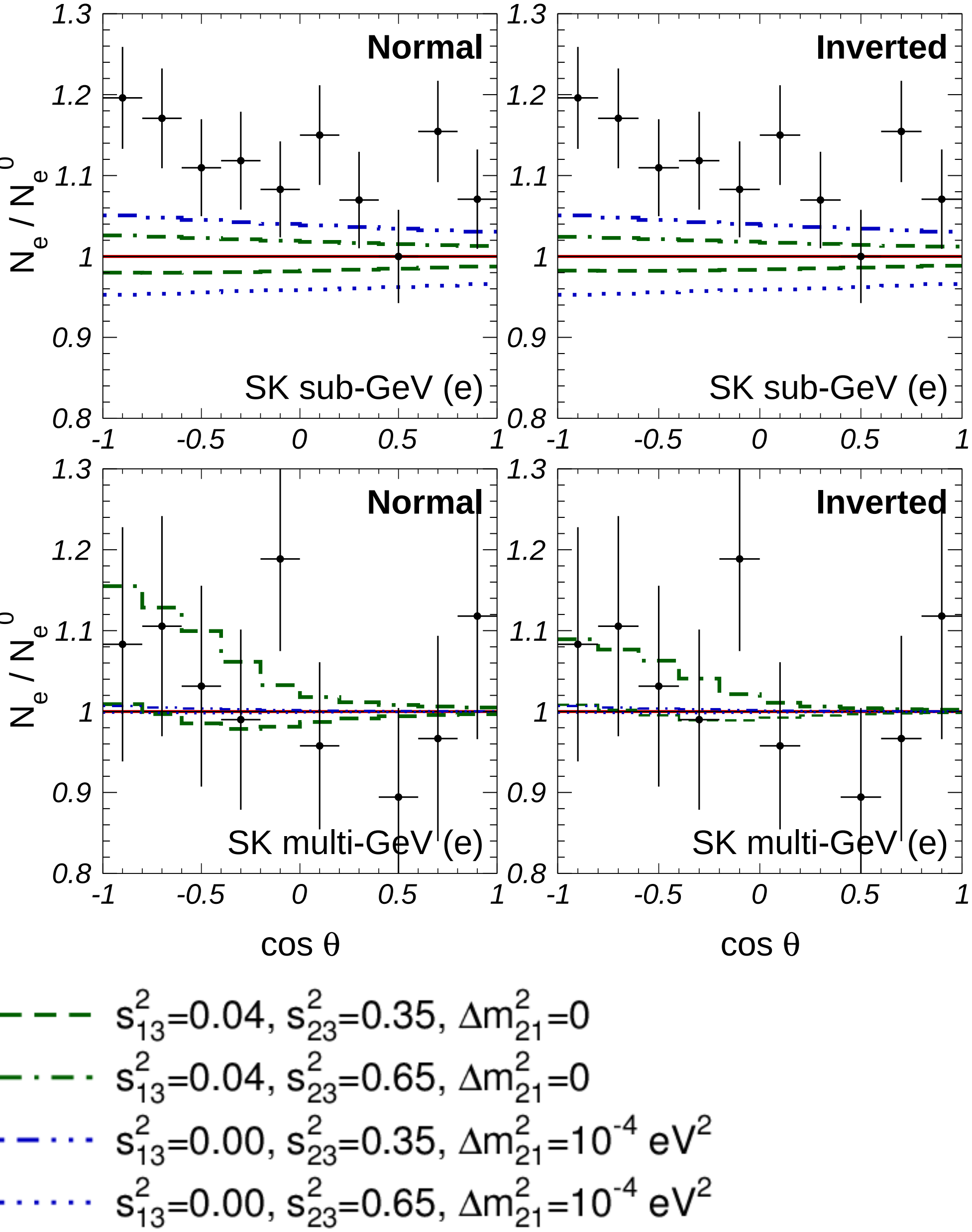
Δm^2_{21} effects in ATM Data



$$\frac{N_e}{N_{e0}} - 1 = \overline{P_{e2}}\,\bar{r}\left(c^2_{23} - \frac{1}{\bar{r}}\right)$$

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$$\sin 2\theta_{12,m} = \frac{\sin^2 2\theta_{12}}{\sqrt{(\cos 2\theta_{12} \mp \frac{2EV_e}{\Delta m^2_{21}})^2 + \sin^2 2\theta_{12}}}$$

For Sub-GeV:

$$P_{e2} = \frac{(\Delta m^2_{21})^2}{(2EV_e)^2}\sin^2 2\theta_{12}\sin^2\frac{V_e L}{2}$$

$$\theta_{23} < \frac{\pi}{4} \Rightarrow c^2_{23} > \frac{1}{2} \Rightarrow N_e(\theta_{13}) > N_{e0}$$

$$\theta_{23} > \frac{\pi}{4} \Rightarrow c^2_{23} < \frac{1}{2} \Rightarrow N_e(\theta_{13}) < N_{e0}$$

$\Rightarrow$ Sensitiv to Deviations from Maximal θ_{23}

$\Rightarrow$ Sensitivity to Octant of θ_{23}

(even for vanishing θ_{13})

$\Rightarrow$ Effect proportional to $(\Delta m^2_{21})^2$

Beyond Hierarchical: Effect $\theta_{13} \times \Delta m^2_{21}$ in ATM

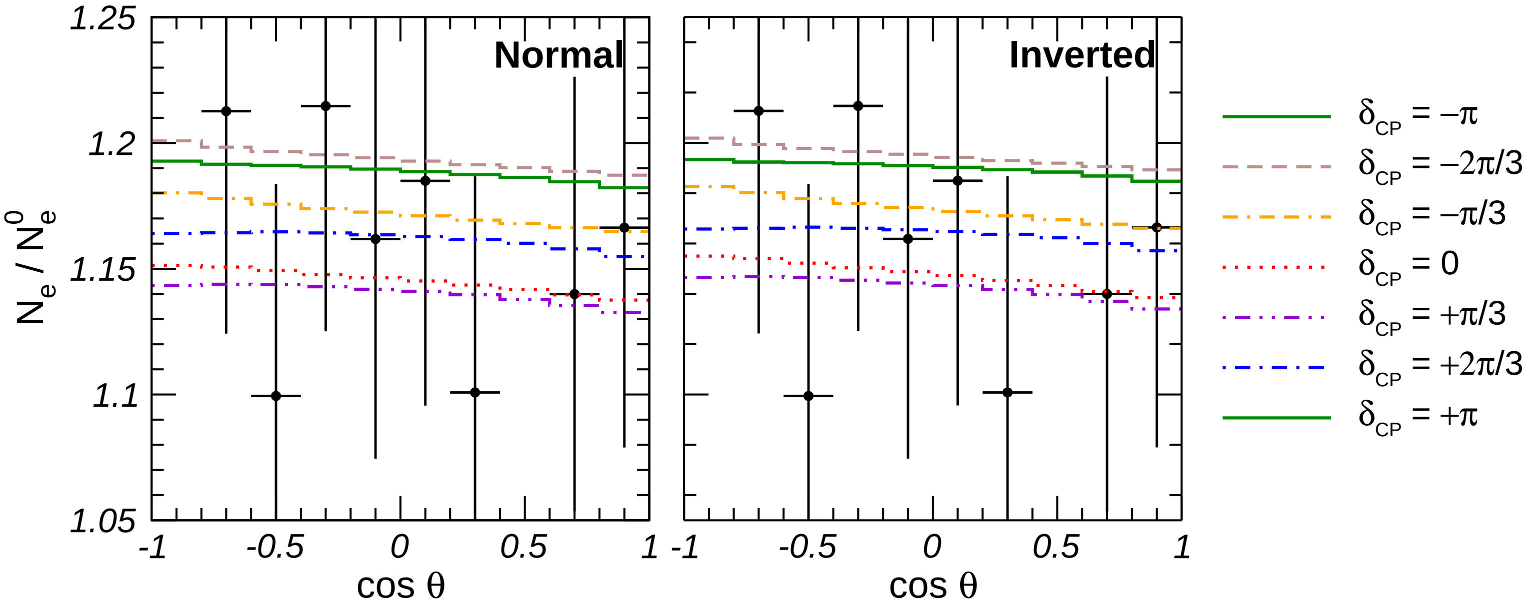
For sub-GeV energies

$$\frac{N_e}{N_e^0} - 1 \simeq \overline{P_{e2}}\,\overline{r}(c_{23}^2 - \frac{1}{\overline{r}}) + 2\tilde{s}_{13}^2\overline{r}(s_{23}^2 - \frac{1}{\overline{r}}) - \overline{r}\tilde{s}_{13}\tilde{c}_{13}^2 \sin 2\theta_{23}(\cos\delta_{CP}\,\overline{R_2} - \sin\delta_{CP}\overline{I_2})$$

$$P_{e2} = \sin^2 2\theta_{12,m}\,\sin^2\frac{\phi_m}{2} \qquad \sin 2\theta_{12,m} = \frac{\sin 2\theta_{12}}{\sqrt{(\cos 2\theta_{12} \mp \frac{2E_\nu V_e}{\Delta m^2_{21}})^2 + \sin^2 2\theta_{12}}}$$

$$R_2 = -\sin 2\theta_{12,m}\cos 2\theta_{12,m}\,\sin^2\frac{\phi_m}{2} \qquad I_2 = -\frac{1}{2}\sin 2\theta_{12,m}\,\sin\phi_m$$

$$\tilde{\theta}_{13} \approx \theta_{13}\left(1 + \frac{2E_\nu V_e}{\Delta m^2_{31}}\right) \qquad \phi \approx (\Delta m^2_{31} + s^2_{12}\,\Delta m^2_{21})\frac{L}{2E_\nu}$$



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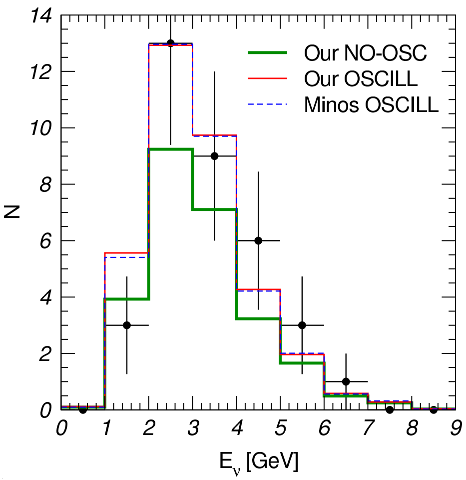
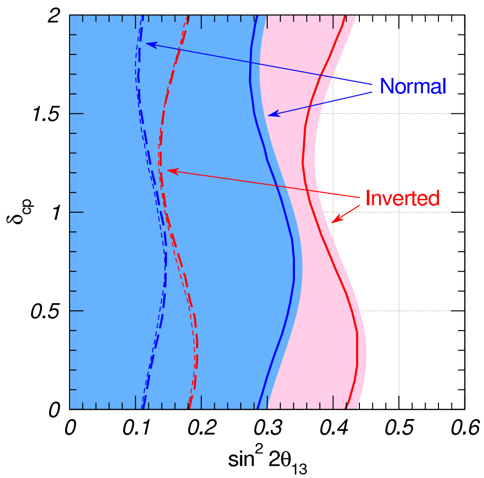
MINOS $\nu_\mu \to \nu_e$: θ_{13}

$$P_{\mu e}^{\rm MINOS} \simeq \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{31} L}{4E} + \mathcal{O}(\delta, \Delta m^2_{21})$$

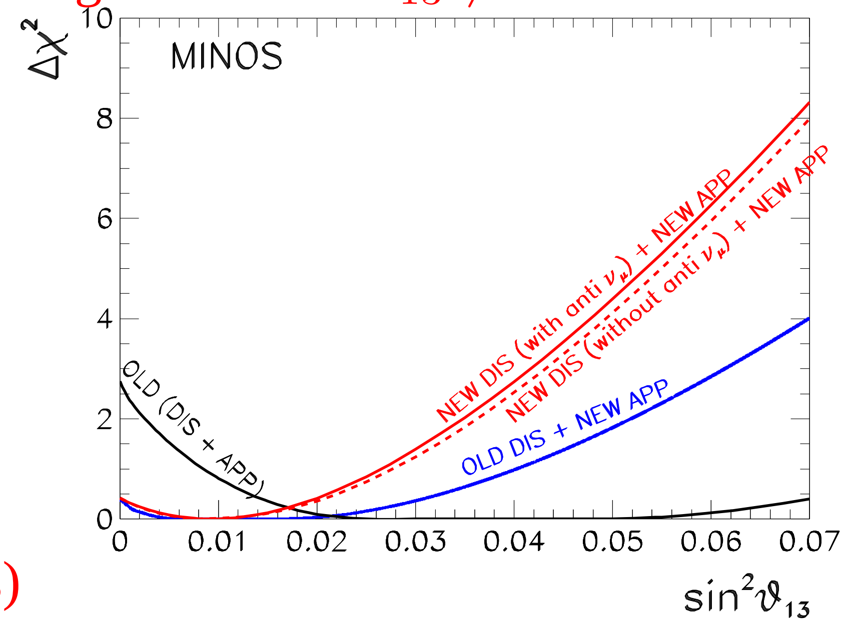
With 3.15×10^{20} POT: arXiv:0909.4996

35 events for $27 \pm 5 \pm 2$ bckg (1.5 σ excess)



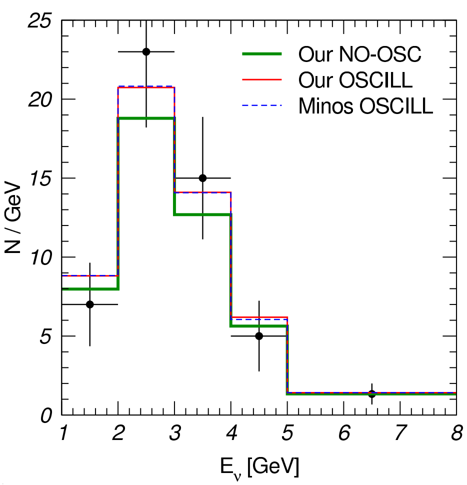


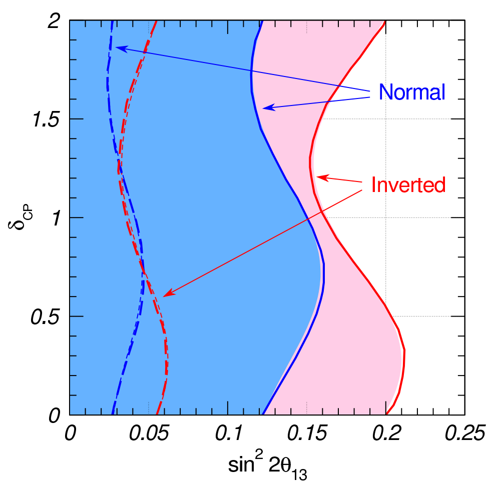
Significance of $\theta_{13} \neq 0$ decreased



With 7×10^{20} POT: April 2010

54 events for $49.1 \pm 7 \pm 2.7$ bckg (0.7 σ excess)





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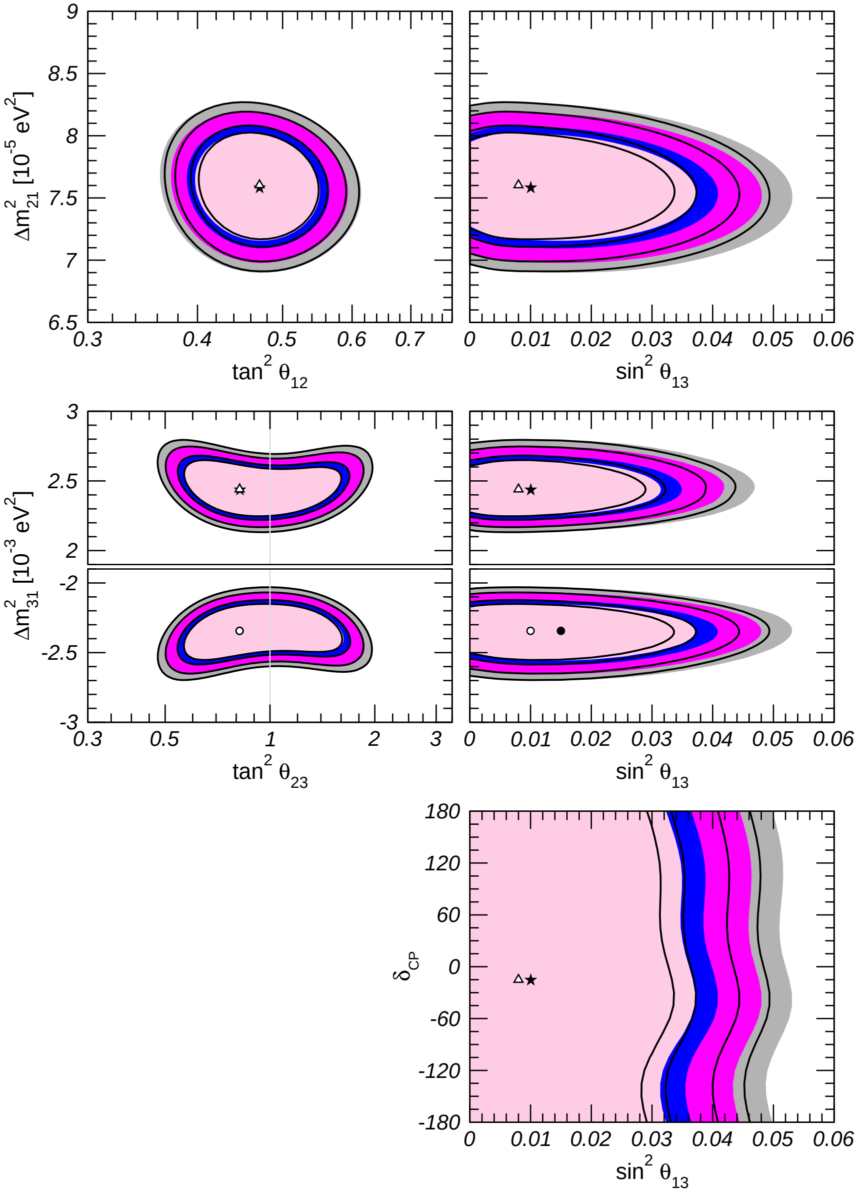
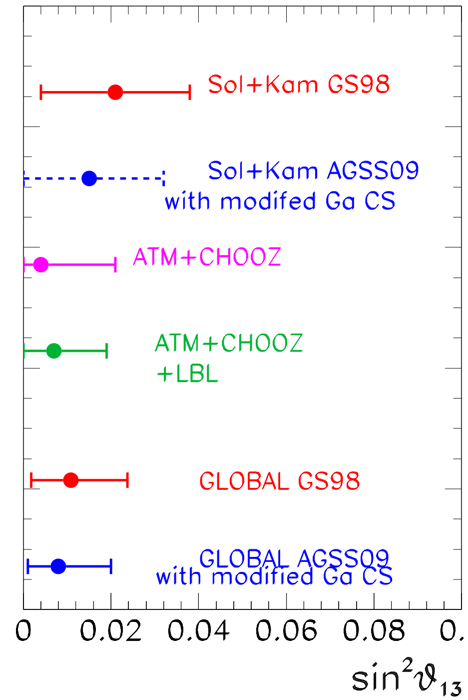
GLOBAL ANALYSIS

GS98 (full) or AGSS09' (lines)





GS98

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$$\frac{\Delta m^2_{31}}{10^{-3}\ \mathrm{eV}^2} = \begin{cases} -2.41 \pm 0.09 \left({+0.30 \atop -0.27}\right) \\ +2.44 \pm 0.09 \left({+0.30 \atop -0.27}\right) \end{cases}$$

$$\theta_{12} = 34.4 \pm 1.0 \left({+3.2 \atop -2.9}\right)^\circ$$

$$\theta_{23} = 42.3^{+5.2}_{-2.8} \left({+11.3 \atop -7.0}\right)^\circ$$

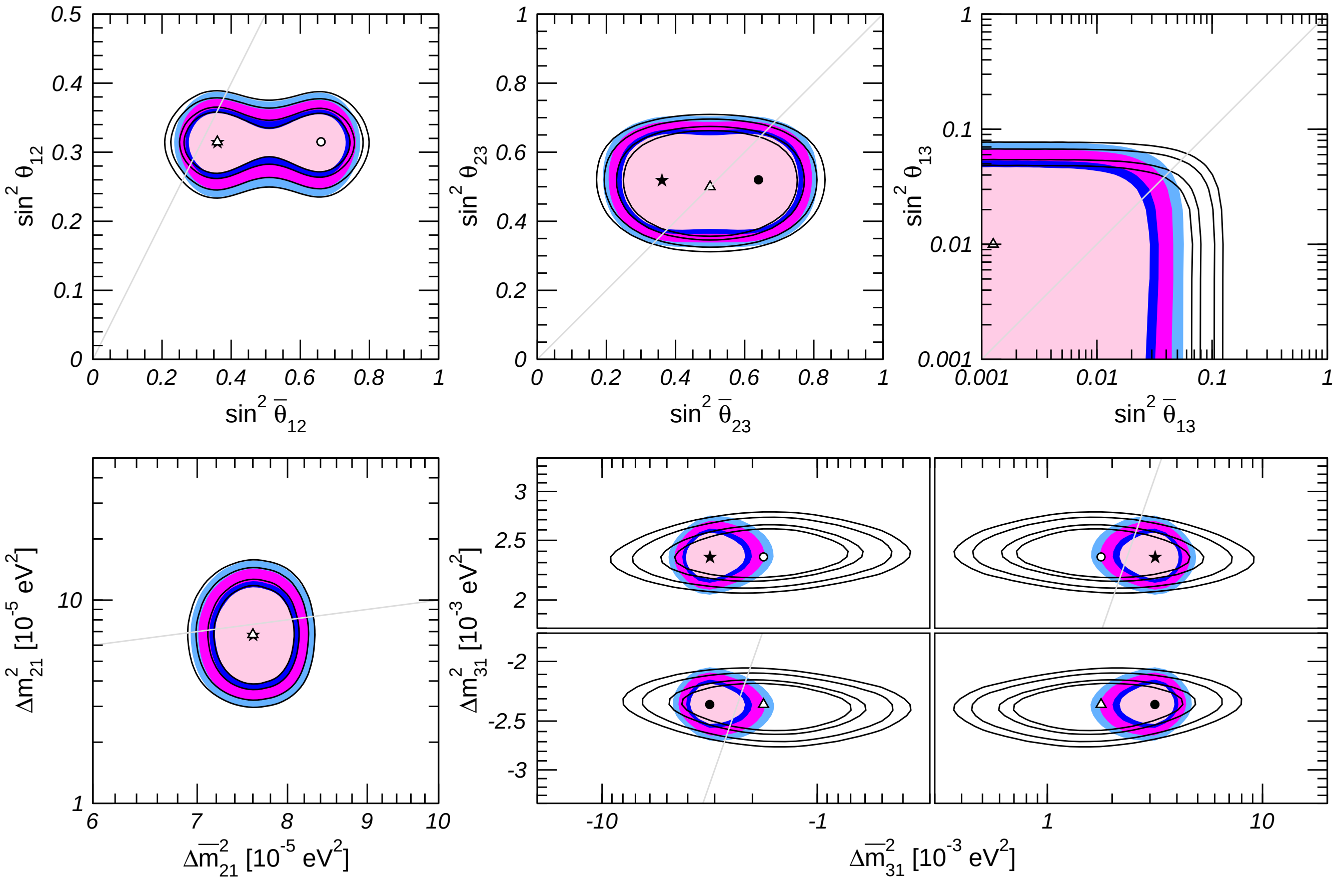
$$\theta_{13} = 5.9^{+2.9}_{-2.7} \ (\leq 12.6)^\circ$$

$$\left[\sin^2\theta_{13} = 0.0108^{+0.013}_{-0.009} \ (\leq 0.048)\right]$$

$$\delta_{CP} \in [0,\ 360]$$

LSND/MiniBOONE/MINOS:CPT Violation?

From Global Fit to Solar+ATM+Reactor+MINOS with 3ν and $3\overline{\nu}$ and C$\not{P}$T



$\chi^{2,CPT}_{min} - \chi^{2,C\not{P}T}_{min} = 4.5$

Driven Mostly By MINOS

But $\Delta\overline{m}^2_{31} \leq 3 \times 10^{-3}$

Not possible to explain

MiniBooNE only with CPT

More (sterile) states

+ CPT or new CP violation?

Akhmedov,Schwetz 1007.4171

Barger,Marfatia (2003)...

Summary and Some Open Questions

Present Determination of NMSM: $\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2} = 7.59 \pm 0.20 \ \binom{+0.61}{-0.69}$,

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$$|U_{LEP}|_{3\sigma} = \begin{pmatrix} 0.79 \to 0.86 & 0.50 \to 0.61 & 0.00 \to 0.20 \\ 0.25 \to 0.53 & 0.47 \to 0.73 & 0.56 \to 0.79 \\ 0.21 \to 0.51 & 0.42 \to 0.69 & 0.61 \to 0.83 \end{pmatrix}$$

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with structure

$$|U_{\mathrm{LEP}}| \simeq \begin{pmatrix} \frac{1}{\sqrt{2}}(1+\mathcal{O}(\lambda)) & \frac{1}{\sqrt{2}}(1-\mathcal{O}(\lambda)) & \epsilon \\ -\frac{1}{2}(1-\mathcal{O}(\lambda)+\epsilon) & \frac{1}{2}(1+\mathcal{O}(\lambda)-\epsilon) & \frac{1}{\sqrt{2}} \\ \frac{1}{2}(1-\mathcal{O}(\lambda)-\epsilon) & -\frac{1}{2}(1+\mathcal{O}(\lambda)-\epsilon) & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \begin{matrix} \lambda \sim 0.2 \\ \epsilon \lesssim 0.2 \end{matrix}$$

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very different from quark's $|U_{\mathrm{CKM}}| \simeq \begin{pmatrix} 1 & \mathcal{O}(\lambda) & \mathcal{O}(\lambda^3) \\ \mathcal{O}(\lambda) & 1 & \mathcal{O}(\lambda^2) \\ \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) & 1 \end{pmatrix}$ $\quad \lambda \sim 0.2$

Summary and Some Open Questions

We still ignore:

(1) Is $\theta_{13} \neq 0$? How small?

(2) Is $\theta_{23} = \frac{\pi}{4}$? If not, is it $>$ or $<$?

(3) Is there CP violation in the leptons (is $\delta \neq 0, \pi$)?

(4) What is the ordering of the neutrino states?

(5) Mass Scale: are neutrino masses

hierarchical: $m_i - m_j \sim m_i + m_j$?

degenerated: $m_i - m_j \ll m_i + m_j$?

(6) Dirac or Majorana?

We have unsolved puzzles:

–We lack a sensible explanation of LSND and MiniBooNE

–We have 2σ difference between ν_μ and $\bar{\nu}_\mu$ in MINOS

Also interesting information on:

–Natural (atmospheric, solar, earth) neutrino fluxes

–More exotic forms of new physics

Plan of Lectures

Introduction: The New Minimal Standard Model

Effects of ν mass: Oscillations in Vaccum and Matter

Atmospheric Neutrinos

Solar Neutrinos

Accelerator and Reactor Neutrinos

Fitting all Together and Subleading effects

Summary

PS:The Near Future Experimental Program and Its Challenges

Neutrino Parameters: Future Strategies

- Oscillation Probabilities in Earth: $\Delta_{ij} = \frac{\Delta m^2_{ij}}{2E_\nu}$ $B_\pm = \Delta_{31} \pm V_E$ $(V_E \sim 10^{-13}\text{ eV})$

$\tilde{J} = c_{13} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 2\theta_{12}$

$$\begin{aligned}
P_{\nu_e\nu_\mu(\bar\nu_e\bar\nu_\mu)} &\simeq s^2_{23} \sin^2 2\theta_{13} \left(\frac{\Delta_{31}}{B_\mp}\right)^2 \sin^2\left(\frac{B_\mp L}{2}\right) \\
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– θ_{13}: Very intense ν_μ or ν_e beam with low background

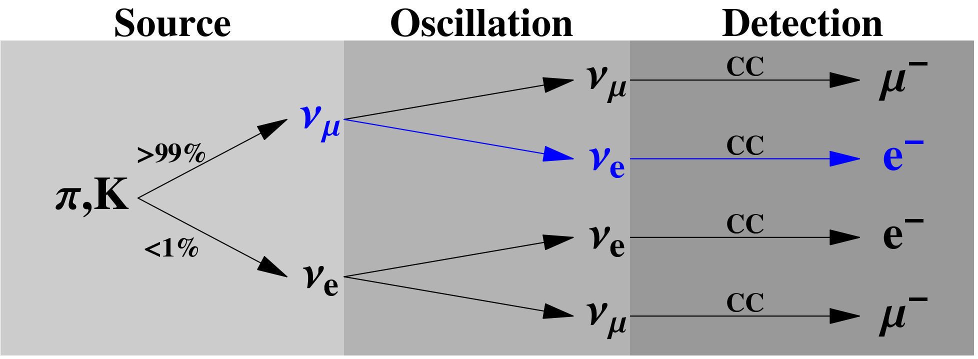
– $\not{C}$P: All angles and Δm^2 non vanishing $\Rightarrow$ Δm^2_{21} in LMA and θ_{13} not too small

Intense beams with exchangeable initial state ($\nu/\bar\nu$)

Experimental Set-ups

Experimental Set-ups

- Conventional (=from π decay) Superbeams



	Exp	L	$\langle E \rangle$
Off-Axis	T2K (Japan)	295 km	0.76 GeV
	Nova (Fermilab)	812 km	2.22 GeV

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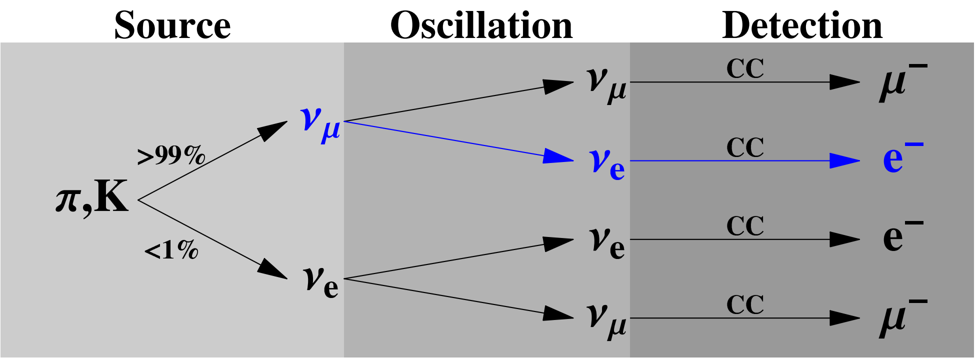

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CHOOZII, Daya-Bay, Reno

$\langle E \rangle \sim 4$ MeV $\quad L \sim 1$–2 km

Concha Gonzalez-Garcia

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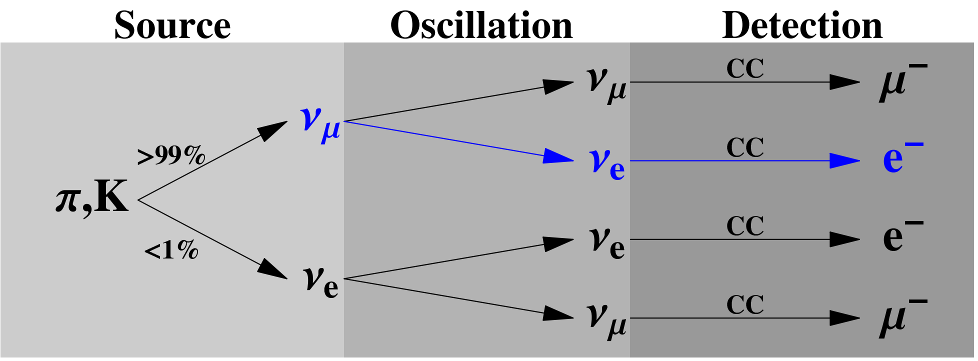

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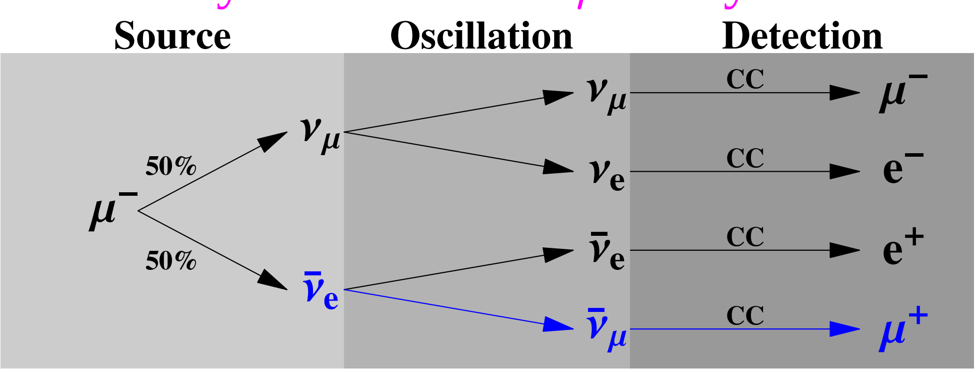

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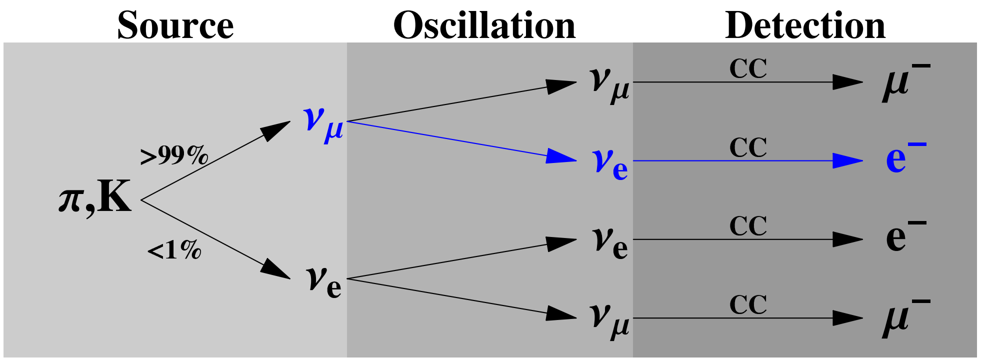

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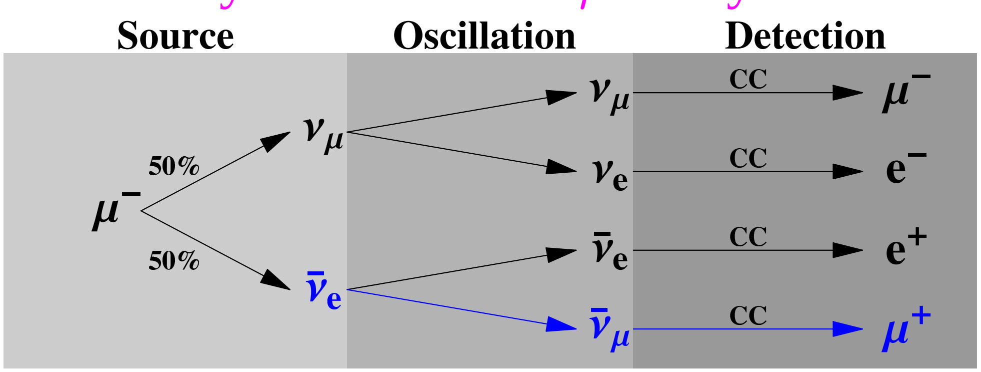

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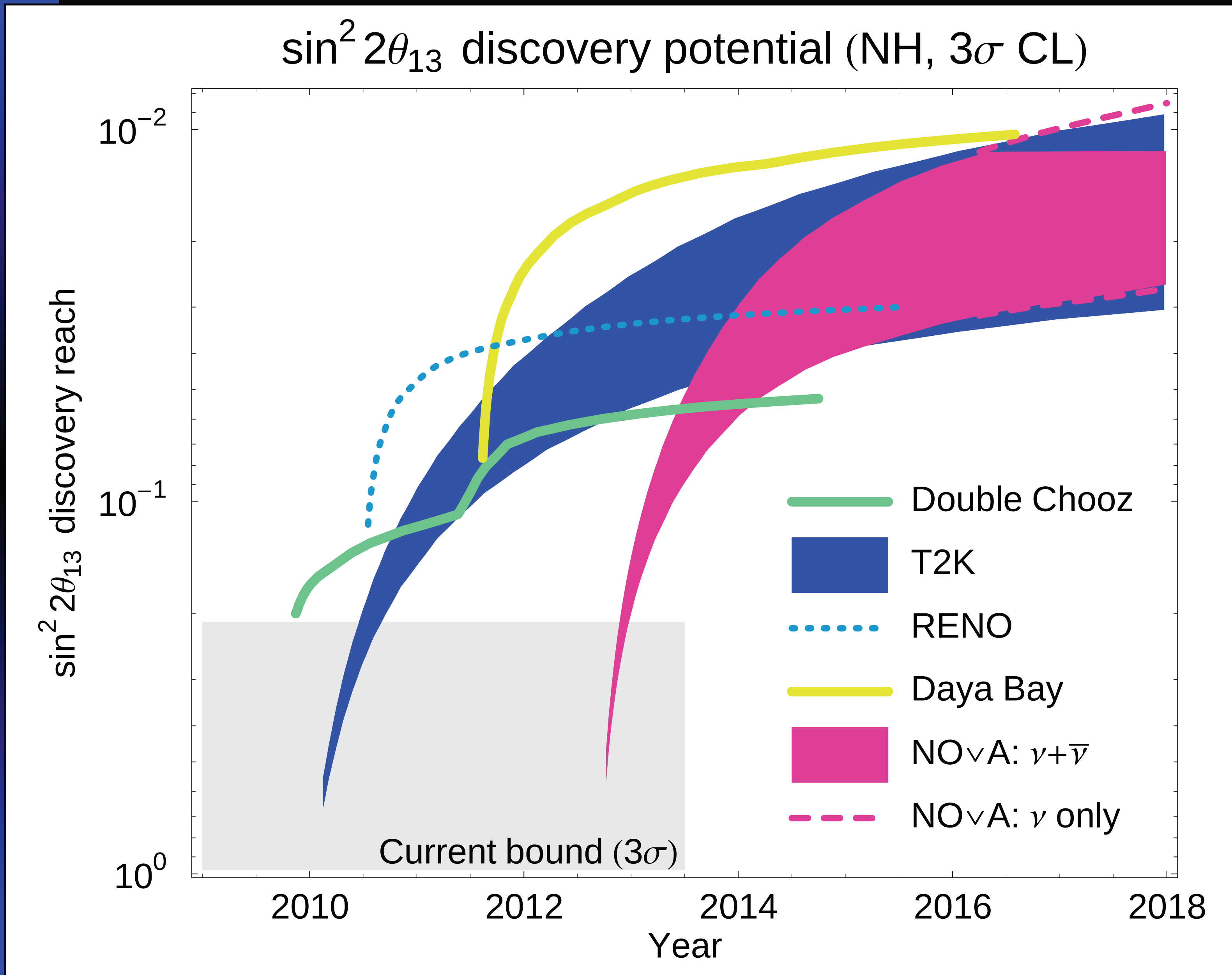
- β-beam : Beam of pure ν_e or $\overline{\nu}_e$ from heavy-ion decay:

$${}^6_2He^{++} \to {}^6_3 Li^{++}\, e^-\, \nu_e$$

Possible Time Scale in Sensitivity to θ_{13}



PH, M. Lindner, T. Schwetz, W. Winter, arXiv:0907.1896.

Challenge at Future LBL: Parameter Degeneracies

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$$P_{\nu_e\nu_\mu(\bar\nu_e\bar\nu_\mu)}(\theta_{13},\delta) = P_{\nu_e\nu_\mu(\bar\nu_e\bar\nu_\mu)}(\theta'_{13},\delta')$$

(c) ($\text{sgn}\ \Delta m_{31}^2$, δ) ambiguity:

$$P_{\nu_e\nu_\mu(\bar\nu_e\bar\nu_\mu)}(\text{sgn}\Delta m_{31}^2,\delta) = P_{\nu_e\nu_\mu(\bar\nu_e\bar\nu_\mu)}(-\text{sgn}\Delta m_{31}^2,\delta')$$

Concha Gonzalez-Garcia

Challenge at Future LBL: Parameter Degeneracies

$$P_{\nu_e\nu_\mu(\bar\nu_e\bar\nu_\mu)} \simeq s_{23}^2 \sin^2 2\theta_{13} \left(\frac{\Delta_{31}}{B_\mp}\right)^2 \sin^2\left(\frac{B_\mp L}{2}\right)$$
$$+ \tilde J \frac{\Delta_{12}}{V_E}\frac{\Delta_{31}}{B_\mp} \sin\left(\frac{V_E L}{2}\right) \sin\left(\frac{B_\mp L}{2}\right) \cos\delta \cos\left(\frac{\Delta_{31} L}{2}\right)$$
$$\pm \tilde J \frac{\Delta_{12}}{V_E}\frac{\Delta_{31}}{B_\mp} \sin\left(\frac{V_E L}{2}\right) \sin\left(\frac{B_\mp L}{2}\right) \sin\delta \sin\left(\frac{\Delta_{31} L}{2}\right) + \dots$$
$$P(\nu_\mu \to \nu_\mu) \simeq 1 - c_{13}^2 \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta_{31} L}{2}\right) + \mathcal{O}(\Delta_{12}, s_{13}^2)$$

$\Delta_{ij} = \frac{\Delta m_{ij}^2}{2E_\nu}$ $\quad B_\pm = \Delta_{31} \pm V_E$

$\tilde J = c_{13} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 2\theta_{12}$

(a) $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$ ambiguity:

$$P_{\nu_\mu\nu_\mu} \propto \sin^2 2\theta_{23} \text{ and } P_{\nu_e\nu_\mu(\bar\nu_e\bar\nu_\mu)}(\theta_{23},\theta_{13},\delta) = P_{\nu_e\nu_\mu(\bar\nu_e\bar\nu_\mu)}(\tfrac{\pi}{2}-\theta_{23},\theta'_{13},\delta')$$

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$$\Downarrow$$

If only total number of ν_e, ν_μ, $\overline\nu_e$ and $\overline\nu_\mu$ at given L are measured $\Rightarrow$ 8-fold degeneracy

Future LBL: Cures of Degeneracies

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- Go to *magic* baselines:

At $L = \frac{2\pi}{V_e} \sim 7500$ km $\Rightarrow$ Only first term survives

Unambiguous determination of θ_{13} but no sensitivity to CP violation (also a bit too long...)

Future LBL: Cures of Degeneracies

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(θ_{13}, δ) ambiguity broken but others remain

Future LBL: Cures of Degeneracies

$$
\begin{aligned}
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- Combine measurements at different L and different E

Need several detectors and/or several beams

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  Need detector with good energy resolution

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- Use "silver" oscillation channels $\nu_e \to \nu_\tau$ and $\nu_\mu \to \nu_\tau$

  Need higher beam energy and high resolution detector