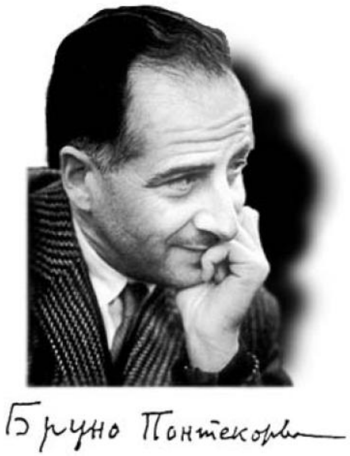


Pontecorvo summer school of neutrino physics
Alushta, September 26 – October 6, 2010

Neutrinoless double beta decay and related topics

Fedor Šimkovic

JINR Dubna, Russia

Comenius University, Bratislava, Slovakia

Study of the $0\nu\beta\beta$-decay is one of the highest priority issues in particle and nuclear physics

OUTLINE

- *Introduction (neutrino properties)*
- *Kurie functions for β-decay of ^3H and ^{187}Re*
- *Detection of relic neutrinos on ^3H, ^{187}Re and ^3He*
- *$0\nu\beta\beta$-decay is a particle physics problem (LNV mechanisms)*
- *$0\nu\beta\beta$-decay is a nuclear physics problem (Current status of NMEs)*
- *On the relation between $0\nu\beta\beta$-decay and $2\nu\beta\beta$-decay NMEs*
- *Co-existence of few mechanisms of the $0\nu\beta\beta$-decay*
- *Neutrinoless double electron capture (resonant transition)*
- *(Partly) bosonic ν and $2\nu\beta\beta$-decay*
- *Analogues of the $0\nu\beta\beta$-decay*
- *ν magnetic moment*
- *Conclusion*

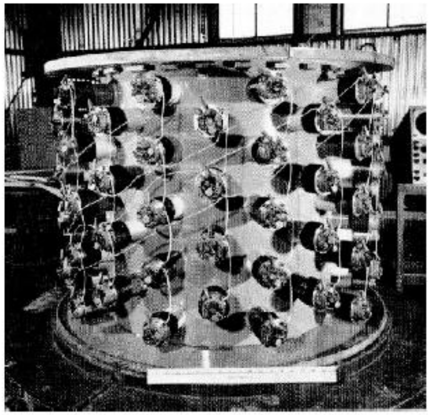
Pauli proposes existence of "neutron" (with spin ½ and mass not more than 0.01 mass of proton) in nucleus. β-decay is then a three body decay with continues distribution of energy among constituents.

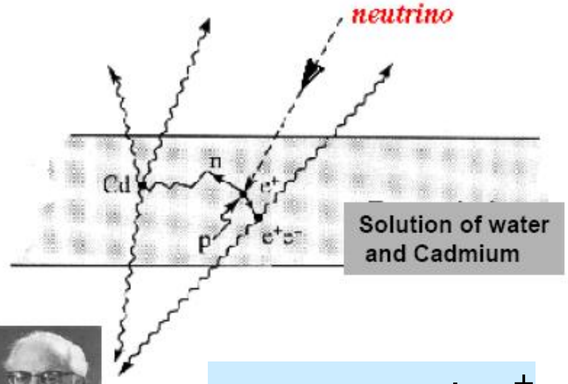
I have done a terrible thing I invented a particle that cannot be detected

W. Pauli

4 December 1930
A letter to Tuebingen

Detector at Savannah River
Nuclear reactor (1956)





$$\bar{\nu} + p \rightarrow n + e^+$$

Reines: 1995 Nobel Prize

3 events per hour

We are happy to inform you (Pauli) that we have definitely detected ν

Reines & Cowan

Signals due to:

i) e^+ annihilation,

ii) n-capture

$$\sigma = (1.1\pm0.3)\ 10^{-43}\ \mathrm{cm}^2$$

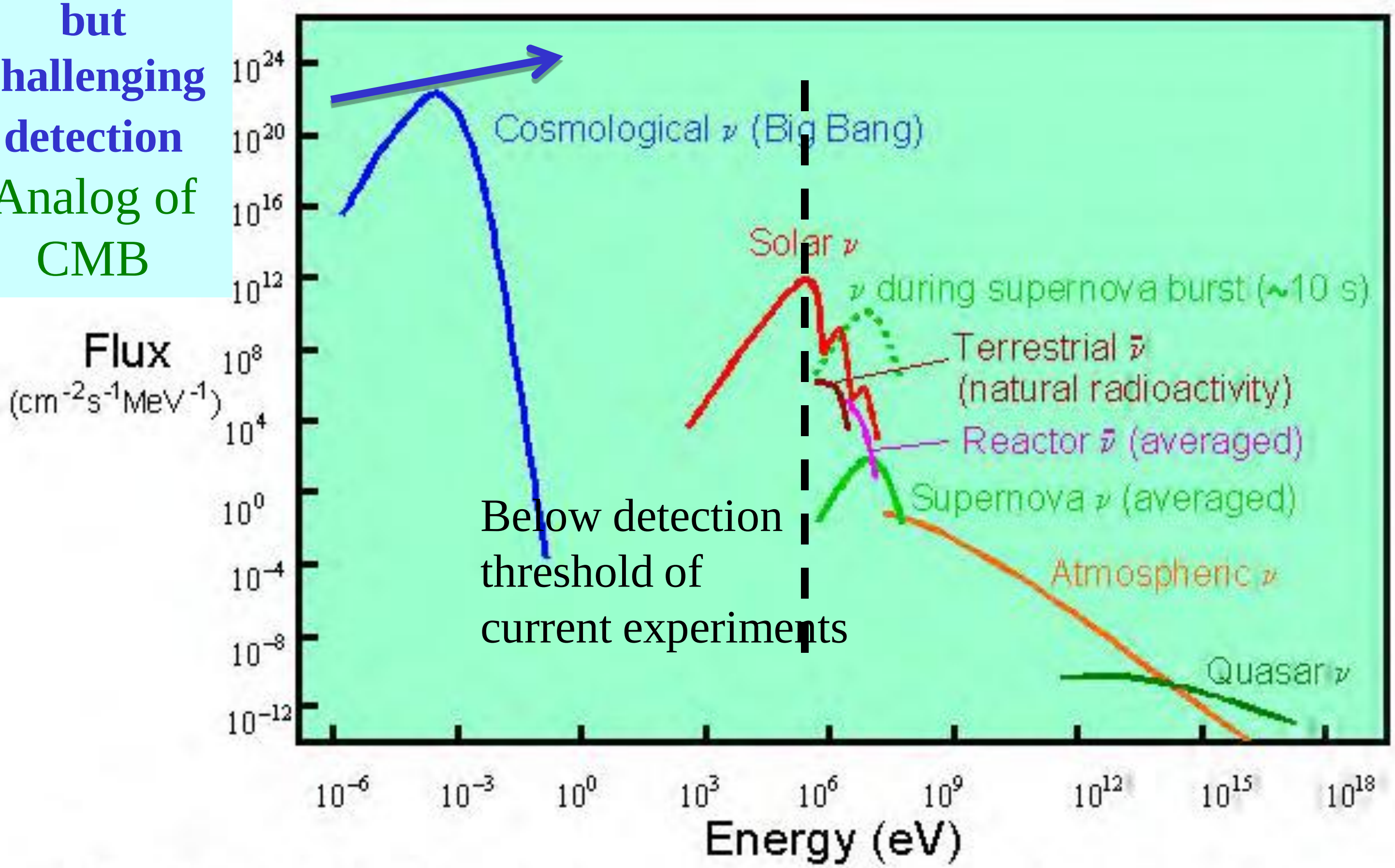
in agreement with Fermi theory of β-decay

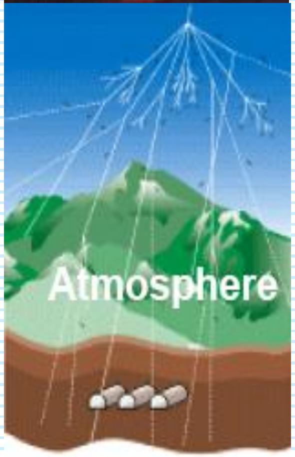
Sources of neutrinos

The Sun is the most intense detected source with a flux on Earth of 6 10^{10} $\nu/cm^2 s$

Abundant but challenging detection

Analog of CMB





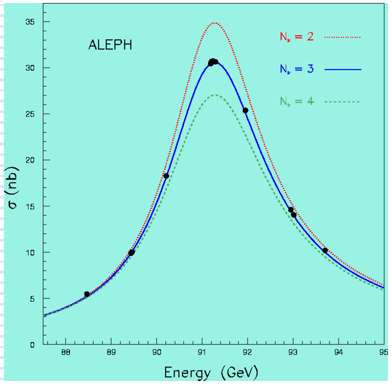
Flux on Earth of neutrinos from different sources as a function of energy

D. Vignaud and M. Spiro, Nucl. Phys.A 654 (1999) 350

Fundamental properties of neutrinos

Like most people, physicists enjoy a good mystery. When you start investigating a mystery you rarely know where it is going

After 54 years we know



- 3 families of light (V-A) neutrinos: ν_e, ν_μ, ν_τ
- ν are massive: we know mass squared differences
- relation between flavor states and mass states (neutrino mixing) only partially known

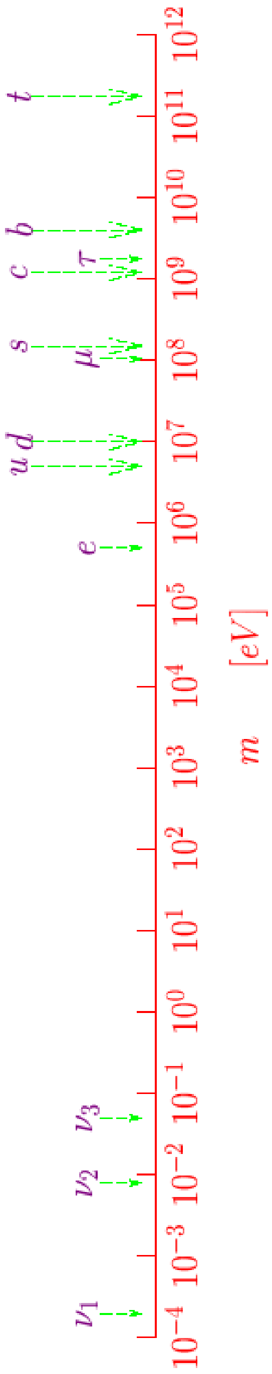
Claim for evidence of the $0\nu\beta\beta$-decay

H.V. Klapdor-Kleingrothaus et al.,NIM A 522, 371 (2004); PLB 586, 198 (2004)

- Absolute ν mass scale from the $0\nu\beta\beta$-decay. (cosmology, ^{3}H, ^{187}Rh ?)
- ν's are their own antiparticles – Majorana.

No answer yet

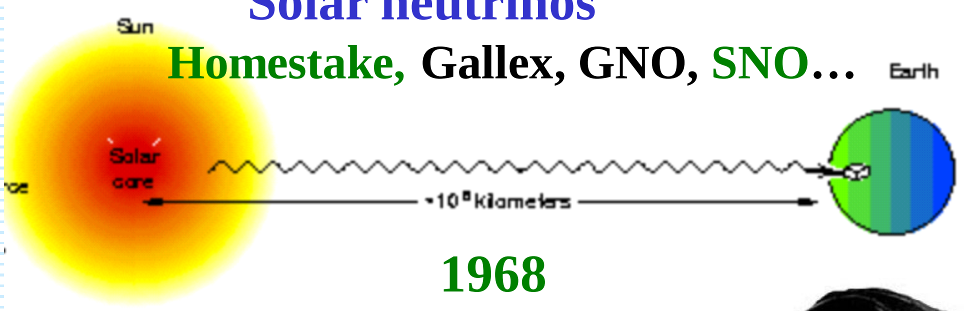
- Is there a CP violation in ν sector? (leptogenesis)
- Are neutrinos stable?
- What is the magnetic moment of ν?
- Sterile neutrinos?
- Statistical properties of ν? Fermionic or partly bosonic?



Neutrino oscillations ⇒ Massive neutrinos

Solar neutrinos

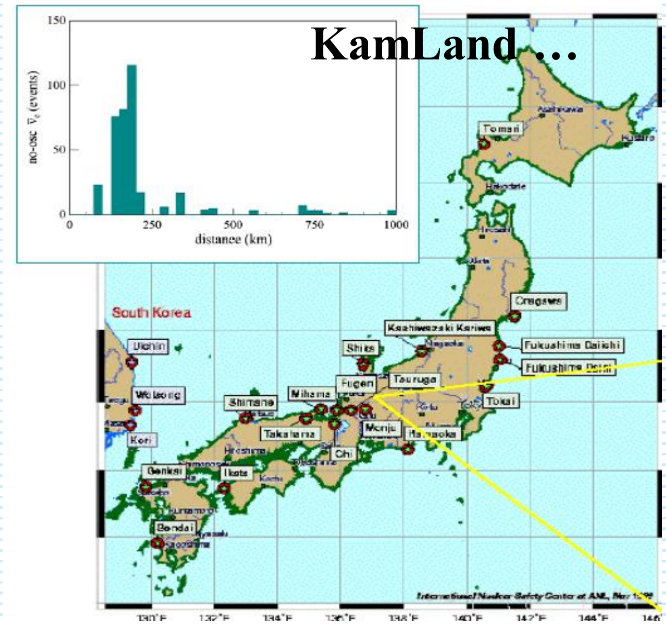
Homestake, Gallex, GNO, SNO…



1968

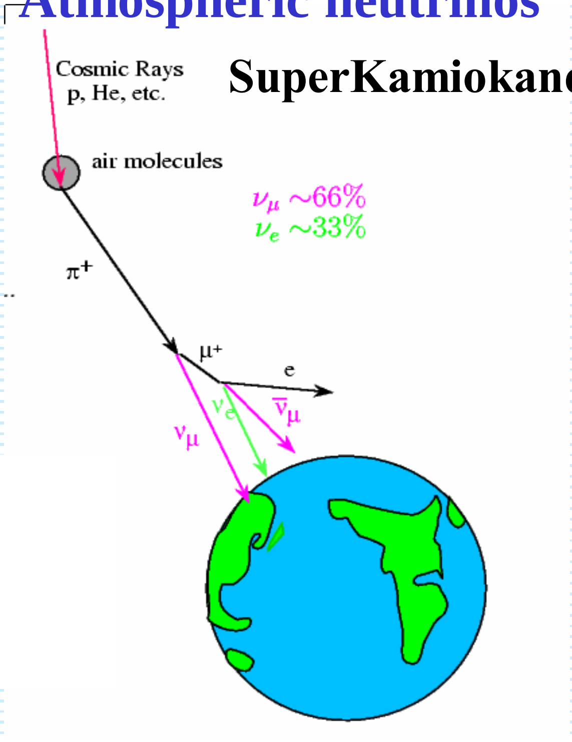
Reactor neutrinos

KamLand …



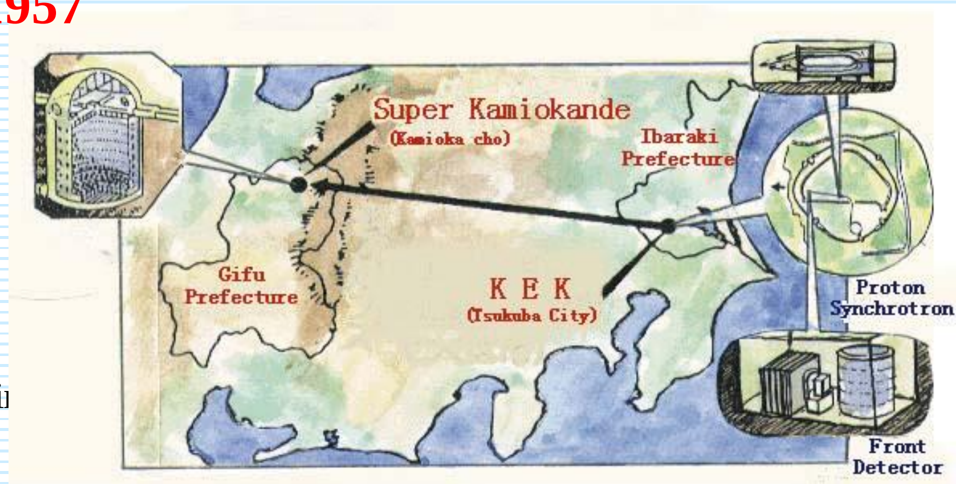
Atmospheric neutrinos

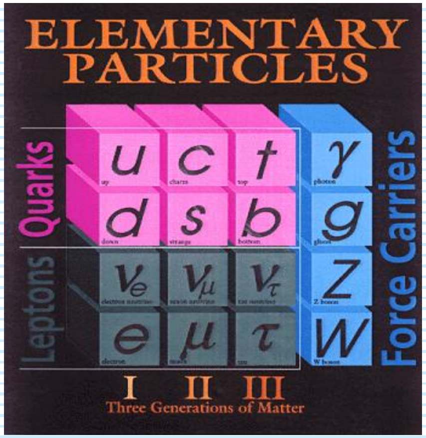
SuperKamiokande…



Dubna, 1957

Accelerator neutrinos





Standard Model Lepton Universality

Particle	*Symbol*	*Anti − p.*	*mass* [MeV]	L_e	L_μ	L_τ	*life − time* [s]
electron	e^-	e^+	0.511	1	0	0	*stable*
el.neutrino	ν_e	$\overline{\nu}_e$	$< 2.2\ 10^{-6}$	1	0	0	*stable*
muon	μ^-	μ^+	105.6	0	1	0	$2.2\ 10^{-6}$
muon neutr.	ν_μ	$\overline{\nu}_\mu$	< 0.19	0	1	0	*stable*
tau	τ^-	τ^+	1777.	0	0	1	$2.9\ 10^{-13}$
tau neutrino	ν_τ	$\overline{\nu}_\tau$	< 18.2	0	0	1	*stable*

Lepton Family Number Violation

NEW PHYSICS
massive neutrinos, SUSY...

Total Lepton Number Violation

$\nu_{e,\mu\tau} \leftrightarrow \nu_{e,\mu\tau}$, $\overline{\nu}_{e,\mu\tau} \leftrightarrow \overline{\nu}_{e,\mu\tau}$	*observed*
$\mu^+ \to e^+ + \gamma$	$R \le 1.2 \times 10^{-11}$
$\mu^+ \to e^+ + e^- + e^+$	$R \le 1.0 \times 10^{-12}$
$K^+ \to \pi^+ + e^- + \mu^+$	$R \le 4.7 \times 10^{-12}$
$\tau^- \to e^- + \mu^+ + \mu^-$	$R \le 1.8 \times 10^{-6}$
$Z^0 \to e^\pm + \mu^\mp$	$R \le 1.7 \times 10^{-6}$
$\mu_b^- + (A,Z) \to (A,Z) + e^-$	$R \le 1.2 \times 10^{-11}$

$\nu_{e,\mu\tau} \leftrightarrow \overline{\nu}_{e,\mu\tau}$	*not observed*
$K^+ \to \pi^- + e^+ + \mu^+$	$R \le 5 \times 10^{-10}$
$\tau^- \to \pi^- + \pi^+ + e^+$	$R \le 1.9 \times 10^{-6}$
$W^- + W^- \to e^- + e^-$	
$(A,Z) \to (A,Z+2) + e^- + e^-$	$T^{0\nu} \ge 1.9 \times 10^{-25}$
$\mu_b^- + (A,Z) \to (A,Z-2) + e^+$	$R \le 3.6 \times 10^{-11}$
$e^- + e^- \to \pi^- + \pi^-$	?

1937 Beginning of Majorana neutrino physics

Ettore Majorana discoveres the possiility of existence of truly neutral fermions

Charged fermion (electron) + electromagnetic field

$$\left(i\gamma^{\mu}\partial_{\mu} - e\gamma^{\mu}A_{\mu} - m\right)\Psi = 0$$

$$\left(i\gamma^{\mu}\partial_{\mu} + e\gamma^{\mu}A_{\mu} - m\right)\Psi^{c} = 0$$

$\Psi^{c} = \Psi$ **forbidden**

Neutral fermion (neutrino) + electromagnetic field

$$\left(i\gamma^{\mu}\partial_{\mu} - m\right)\nu = 0$$

$$\left(i\gamma^{\mu}\partial_{\mu} - m\right)\nu^{c} = 0$$

$\nu^{c} = \nu$ **allowed**

Majorana condition

Symmetric Theory of Electron and Positron
Nuovo Cim. 14 (1937) 171

Here is the beginning of Nonstandard Neutrino Properties

Study of the 0νββ-decay is one of the highest priority issues in particle and nuclear physics

ν

0νββ-decay:
⇐ No | Yes ⇒

ν

US

***APS Joint Study on the Future of Neutrino Physics* (physics/0411216)**
We recommend, as a high priority, a phased program of sensitive searches for neutrinoless double beta decay (first in the list of recommendations)

Europe

ASPERA road map:

- ***Requirement for construction and operation of two double-beta decay experiments with a European lead role or shared equally with non-European partners (GERDA, COBRA, CUORE, SuperNEMO)***
- ***We finally reiterate the importance of assessing and reducing the uncertainty in our knowledge of the corresponding nuclear matrix elements, experimentally and theoretically.***

The answer to the question whether neutrinos are their own antiparticles is of central importance, not only to our understanding of neutrinos, but also to our understanding of the origin of mass.

$$\frac{1}{T_{1/2}} = G^{0\nu}(E_0, Z)|M'^{0\nu}|^2|\langle m_{\beta\beta}\rangle|^2 \ ,$$

$$m_{\beta\beta} = \sum_{i=1}^{3} U_{ei}^2 \ m_i$$

Absolute ν mass scale

Normal or inverted Hierarchy of ν masses

CP-violating phases

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

An accurate knowledge of the nuclear matrix elements, which is not available at present, is however a pre-requisite for exploring neutrino properties.

Mass matrix of light neutrinos

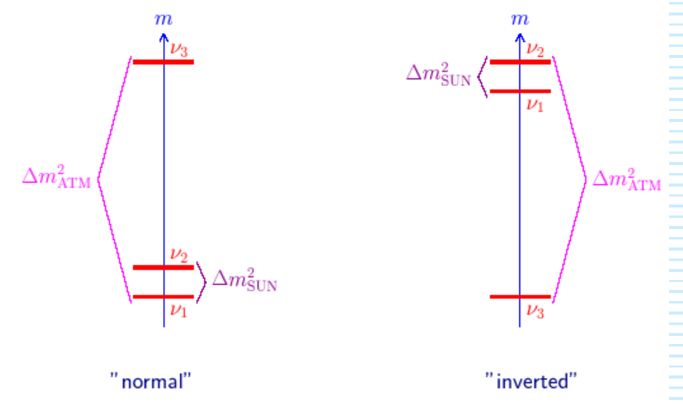
Flavor eigenstates | **Large off diagonal elements !** | **Mass eigenstates**

$$m_{\beta\beta} = \sum_k \left(U^L_{ek}\right)^2 \xi_k m_k$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{\mu e} & M_{\mu\mu} & M_{\mu\tau} \\ M_{\tau e} & M_{\tau\mu} & M_{\tau\tau} \end{pmatrix}$$

$$U_{PMNS} = \begin{pmatrix} 0.83 & 0.55 & 0.05 \\ 0.34-0.45 & 0.56-0.62 & 0.70 \\ 0.34-0.45 & 0.55-0.62 & 0.70 \end{pmatrix}$$



0νββ-decay
Majorana condition

$$C\, \overline{\chi_k}^T(x) = \xi_k\, \chi_k(x)$$

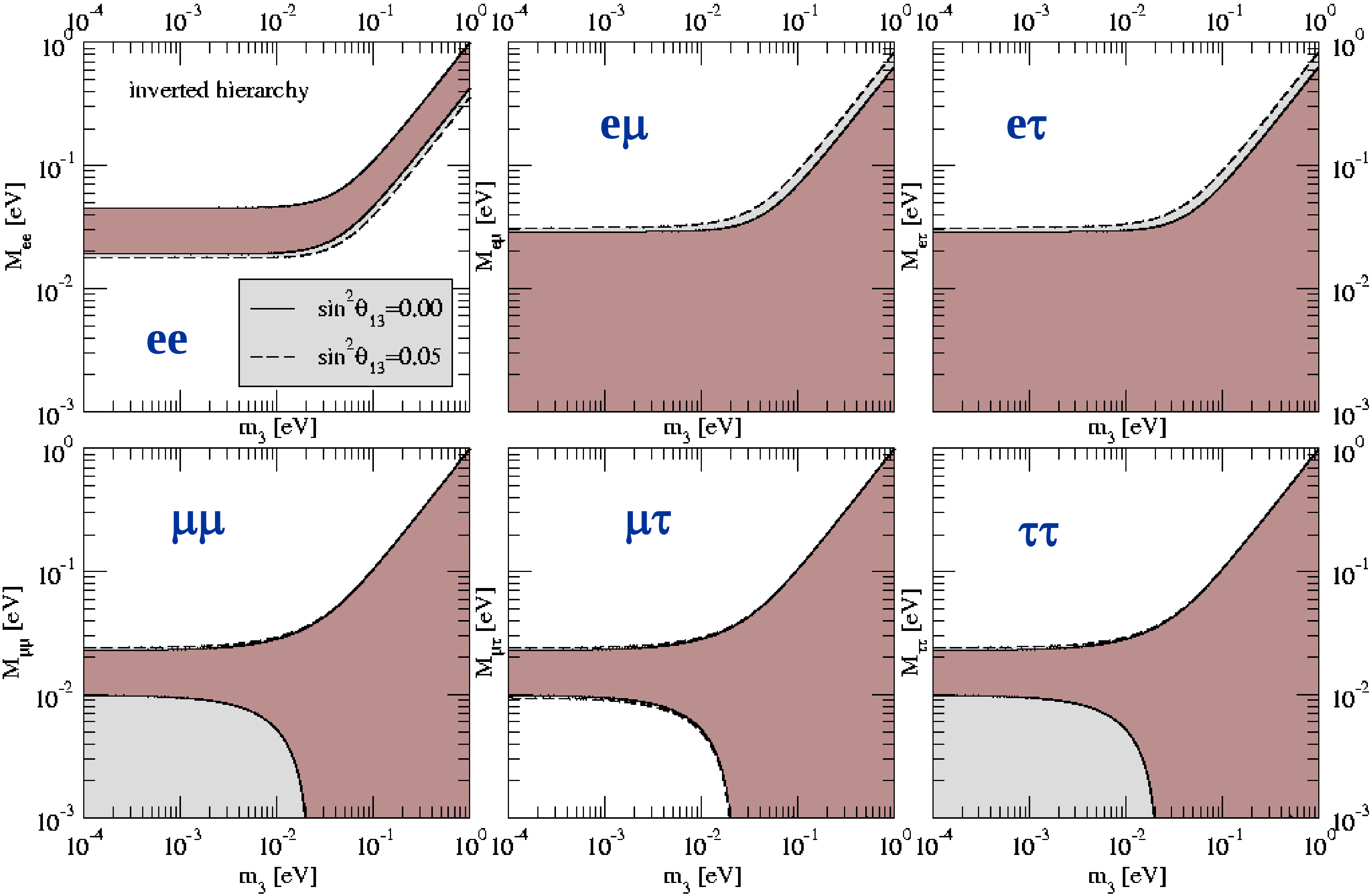
$$< \nu_{eL}(x_1) \nu_{eL}{}^T(x_2) > \; = \; -\sum_k \left(U^L_{ek}\right)^2 \xi_k \frac{1+\gamma_5}{2} S_k(x_1 - x_2) \frac{1+\gamma_5}{2} C$$

$$= \frac{i}{(2\pi)^4} \sum_k \left(U^L_{ek}\right)^2 \xi_k m_k \int \frac{e^{iq(x_1-x_2)} dq}{q^2 + m_k^2} \frac{1+\gamma_5}{2} C$$

Effective mass of Majorana neutrinos

ν-masses in flavor basis: Inverted hierarchy

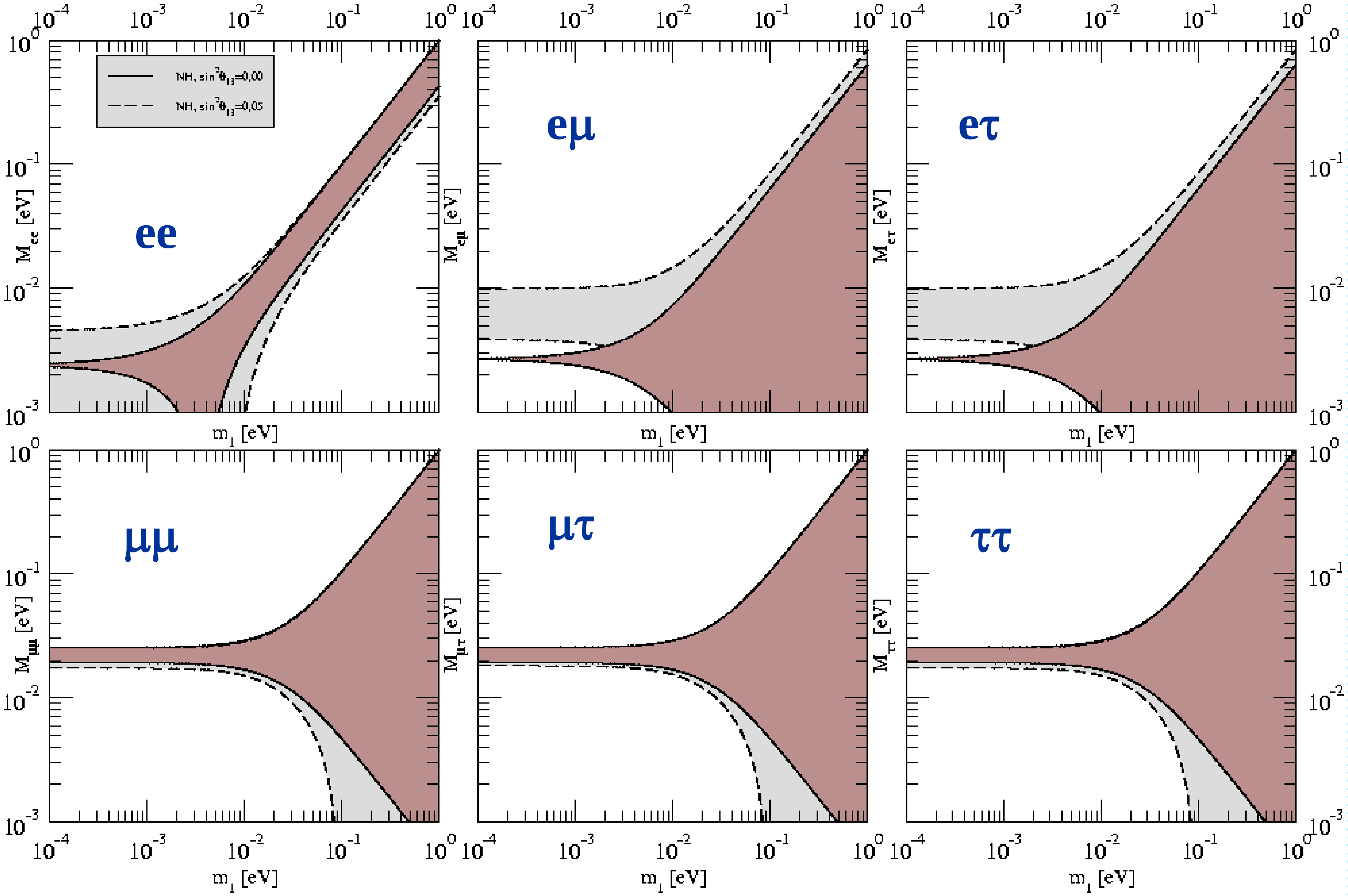
$$\begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{\mu e} & M_{\mu\mu} & M_{\mu\tau} \\ M_{\tau e} & M_{\tau\mu} & M_{\tau\tau} \end{pmatrix}$$



Gozdz,Kaminski, F.Š, Faessler, Acta Phys. Pol. 37 (2006) 2203

ν-masses in flavor basis: Normal hierarchy

$$\begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{\mu e} & M_{\mu\mu} & M_{\mu\tau} \\ M_{\tau e} & M_{\tau\mu} & M_{\tau\tau} \end{pmatrix}$$



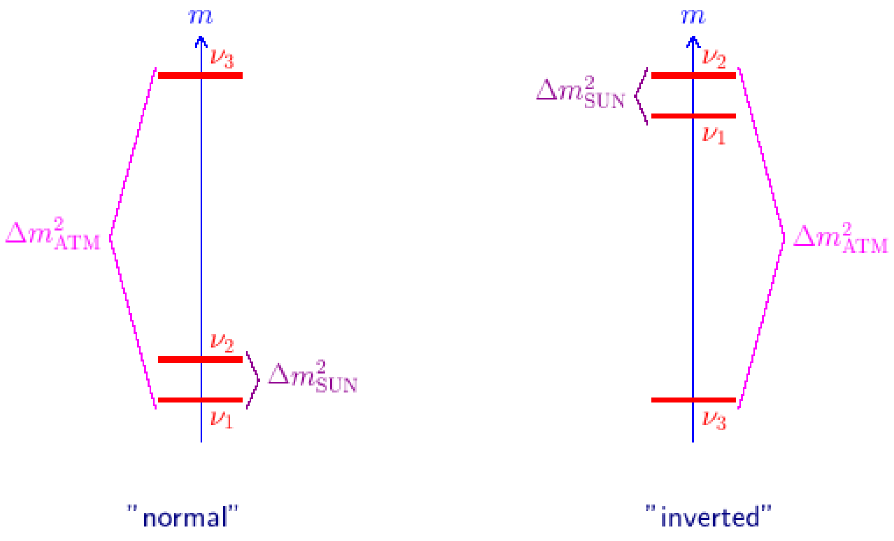
Gozdz,Kaminski, F.Š, Faessler, Acta Phys. Pol. 37 (2006) 2203

Neutrinos mass spectrum

We need 3mass-eigenstates
to explain 2 different Δm^2:

$|m_2^2 - m_1^2| = \Delta m^2_{sol} \sim 3\ 10^{-5}\ eV^2$

$|m_3^2 - m_2^2| = \Delta m^2_{atm} \sim 2\ 10^{-3}\ eV^2$

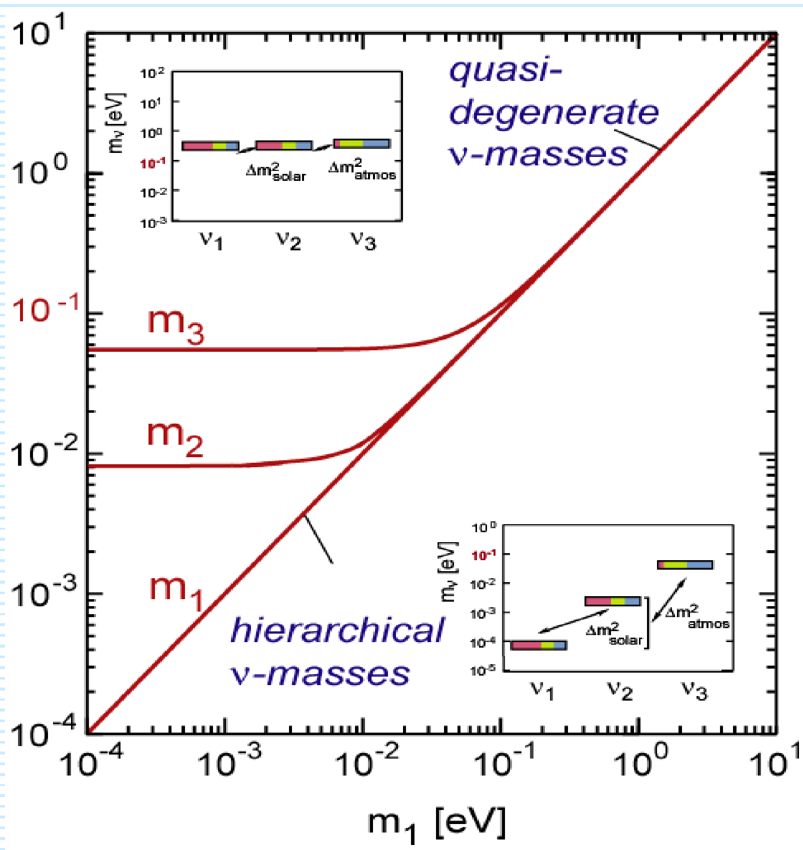


Absolute mass scale of neutrinos

0νββ-decay $m_{\beta\beta} = \sum_{i=1}^{3} U_{ei}^2\ m_i$

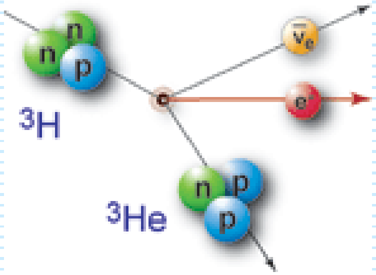
Tritium decay $m_\beta = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2\ m_i^2}$

Cosmology $\sum_{i=1}^{3} m_i$



The single β-decay, absolute ν mass scale and relic ν

Tritium beta decay: $^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$

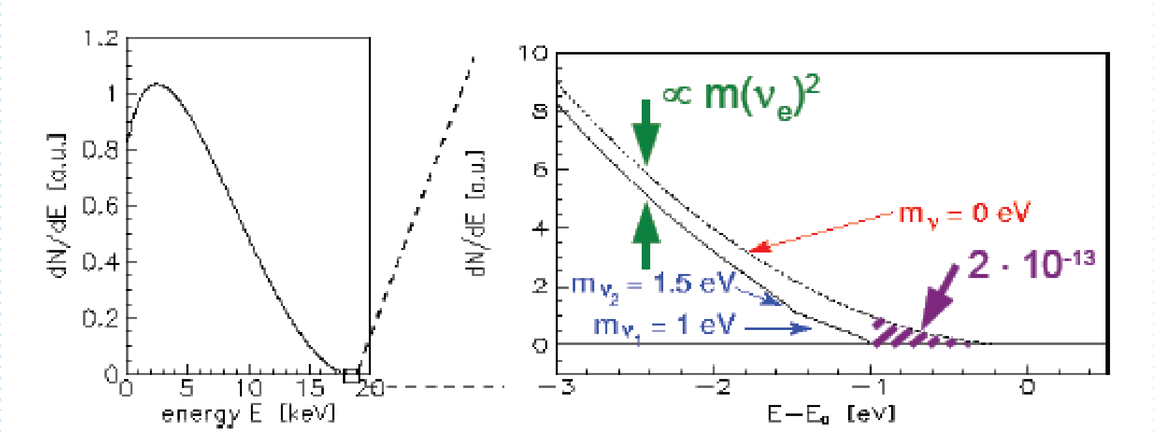


$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

1934 – Fermi pointed out that shape of electron spectrum in β–decay near the endpoint might be sensitive to neutrino mass

First measured by Hanna and Pontecorvo with estimation $m_\nu \sim 1$ keV [Phys. Rev. 75, 983 (1940)]

$Q = M_H - M_{He} - m_e = 1858$ keV



Troitsk

$m_\nu^2 = -2.3 \pm 2.5 \pm 2.0$ eV^2

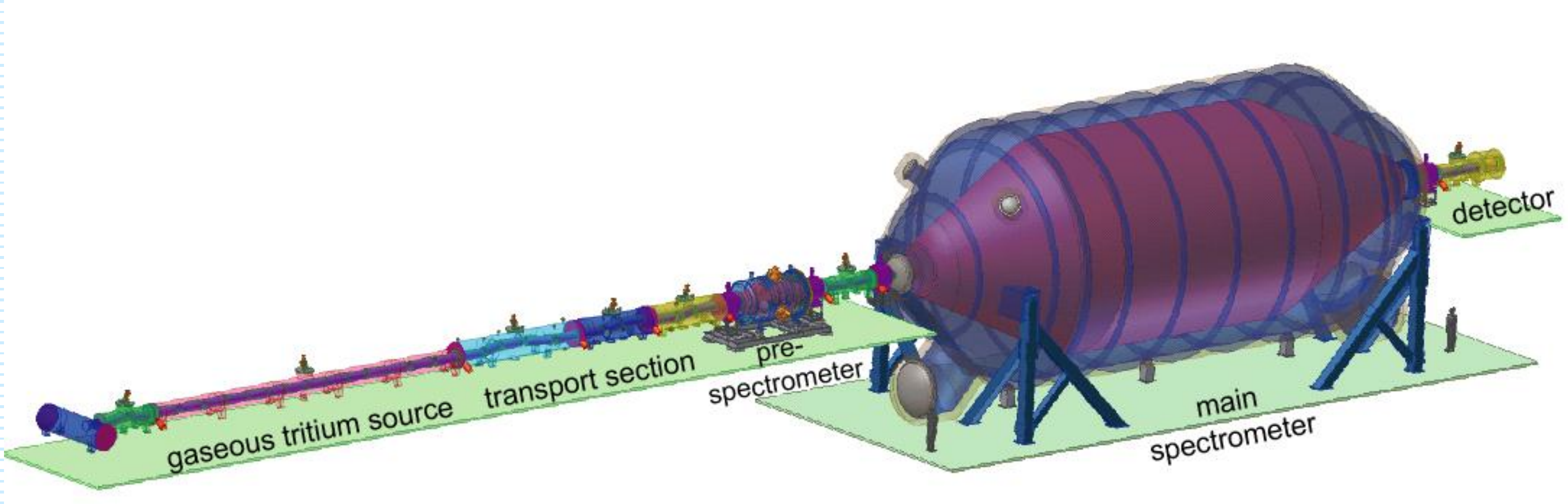
$m_\nu \leq$ 2.2 eV (95% CL.)

Mainz

$m_\nu^2 = -1.2 \pm 2.2 \pm 2.1$ eV^2

$m_\nu \leq$ 2.2 eV (95% CL.)

Karlsruhe TRItium Neutrino experiment (KATRIN)



Evidence for neutrino mass signal
KATRIN discovery potential:

No neutrino mass signal
KATRIN sensitivity

$m_\beta = 0.35$ eV (5σ)
$m_\beta = 0.30$ eV (3σ)

$$m_\beta = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2 m_i^2} < 0.2\ eV \qquad m_\beta \approx m_1$$

Relativistic approach to ^{3}H decay

Standard approach

- non-relativistic nuclear w.f.
- nuclear recoil neglected
- phase space analysis

$$E_e^{\max} = M_i - M_f - m_\nu$$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

Relativistic EPT approach

- Analogy with n-decay
  (^{3}H,^{3}He) $\leftrightarrow$ (n,p)
- nuclear recoil of 3.4 eV by E_e^{max}
- relevant only phase space

$$E_e^{\max} = \frac{1}{2M_f}\left[M_i^2 + m_e^2 - \left(M_f^2 - m_\nu^2\right)\right]$$

$$\begin{aligned}
\frac{d\Gamma}{dE_e} &= \frac{1}{(\pi)^3}(G_F\cos\theta_c)^2 F(Z,E_e) p_e \\
&\times \frac{M_i^2}{(m_{12})^2}\sqrt{y\left(y + 2m_\nu \frac{M_f}{M_i}\right)} \\
&\times \Bigg[(g_V+g_A)^2 y\left(y + m_\nu\frac{M_f}{M_i}\right)\frac{M_i^2(E_e^2 - m_e^2)}{3(m_{12})^4} \\
&\quad (g_V+g_A)^2\left(y + m_\nu\frac{M_f + m_\nu}{M_i}\right)\frac{(M_iE_e - m_e^2)}{m_{12}^2} \\
&\quad \times \left(y + M_f\frac{M_f + m_\nu}{M_i}\right)\frac{(M_i^2 - M_iE_e)}{m_{12}^2} \\
&\quad - (g_V^2 - g_A^2)M_f\left(y + m_\nu\frac{(M_f + M_\nu)}{M_i}\right) \\
&\quad \times \frac{(M_iE_e - m_e^2)}{(m_{12})^2} \\
&\quad + (g_V - g_A)^2 E_e\left(y + m_\nu\frac{M_f}{M_i}\right)\Bigg]
\end{aligned}$$

$$\begin{aligned}
y &= E_e^{max} - E_e \\
(m_{12})^2 &= M_i^2 - 2M_iE_e + m_e^2
\end{aligned}$$

Numerics:
Practically the same dependence of Kurie function on m_ν for $E_e \approx E_e^{max}$

lor Simkovic

F.Š., R. Dvornický, A. Faessler, PRC 77 (2008) 055502

Rhenium β-decay

$$^{187}Re \rightarrow ^{187}Os + e^- + \tilde{\nu}_e$$

- Beta emitter of g.s.→g.s. transition with lowest known Q value **(2.47 keV)**
- Relative high half-live **($T_{1/2}$=4.35 x 10^{10} y)** ~ age of the Universe
- Natural abundance **63%**

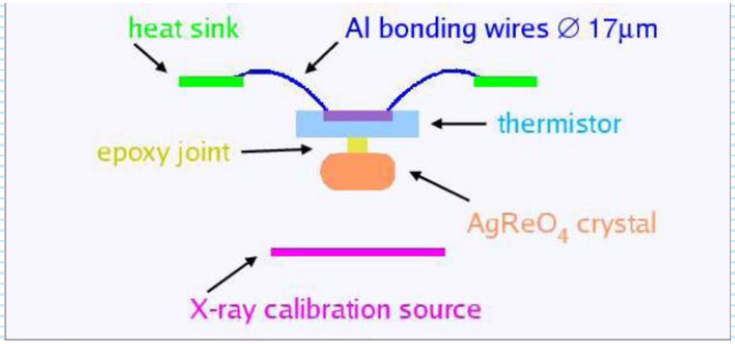
first unique forbidden β-decay ⇒ $5/2^+ \rightarrow 1/2^- \Rightarrow \Delta J^\pi = 2^-$

MIBETA ($AgReO_4$, 10*(250-350) mg Milano/Como)
MANU (Re metalic crystals, 1.5 mg, Genova)

m_ν^2 = -141±211±90 eV^2
m_ν = 15.6 eV (90% c.l.)

The entire energy is measured in the detector except the neutrino including the molecular & atomic excitations

M.Sisti et al., NIMA 520 (2004) 125



Microcalorimeter Arrays for a Rhenium Experiment (MARE)

MARE II: 5000 – 50 000 detectors (MIBETA 10)
Expected sensitivity m_ν = 0.2 eV

Spectrum of emitted electrons in rhenium β-decay

$$\frac{d\Gamma}{dE} = \frac{G_F^2 V_{ud}^2}{2\pi^3} |M|^2 pE(E_0 - E)\sqrt{(E_0 - E)^2 - m_\nu^2}\ \frac{1}{3} R^2 \left(p^2 F_1(Z,E) + k^2 F_0(Z,E) \right)$$

$$k = \sqrt{(E_0 - E)^2 - m_\nu^2}$$

Electron $p_{3/2}$ decay channel clearly dominates

$$\Gamma_S / \Gamma_P = 1.011 \times 10^{-4}$$

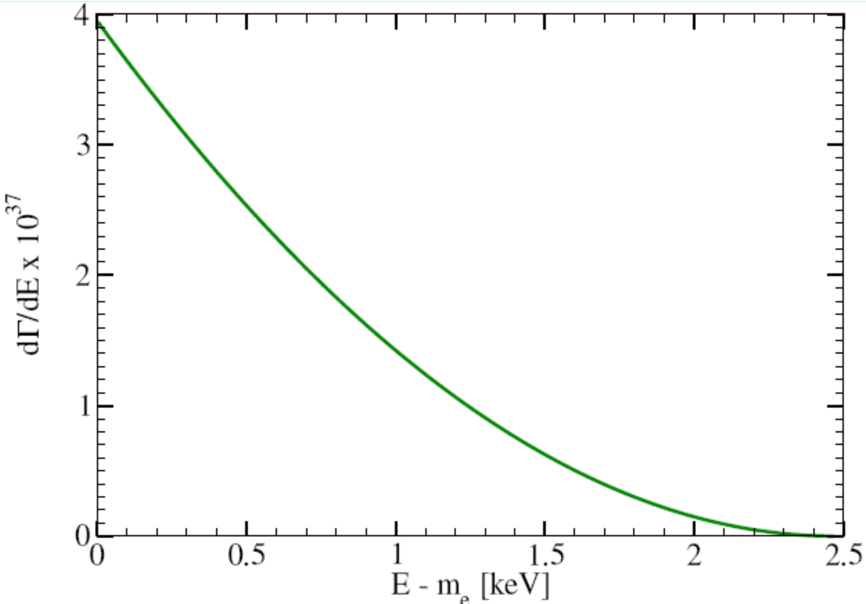
In agreement with
Arnaboldi et al.: PRL 96, 042503 (2006)

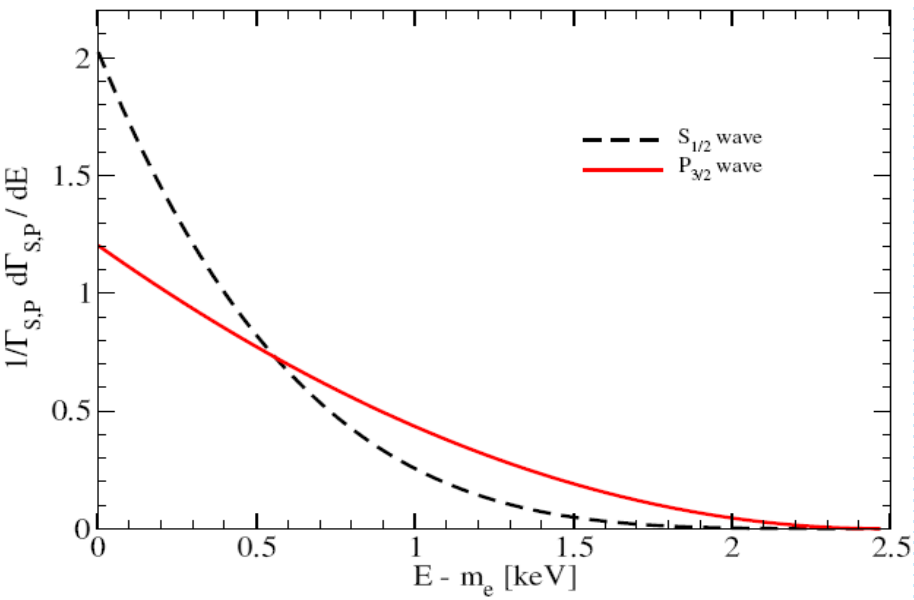
Electron in the $p_{3/2}$ state

$$p^{max} \cong 50 keV$$

Electron in the $s_{1/2}$ state

$$k^{max} = 2.47 keV$$





Kurie plots for rhenium (MARE) and tritium (KATRIN) β-decay

Rhenium

$$B_{\rm Re} = \frac{G_F V_{ud}}{\sqrt{2\pi^3}} \frac{g_A}{\sqrt{2J_i+1}} \left| \langle {}^{187}Os \| \sqrt{\frac{4\pi}{3}} \sum_n \tau_n^+ \frac{r_n}{R} \{\sigma_1 \otimes Y_1\}_2 \| {}^{187}Re \rangle \right|$$

$$\times \sqrt{\frac{1}{3} R^2 p^2 \frac{F_1(Z,E)}{F_0(Z,E)}}$$

$$K(E_e)/B_{\rm Re} \cong (E_0 - E_e) \sqrt[4]{1 - \frac{m_\nu^2}{(E_0 - E_e)^2}}$$

Properly normalized Kurie functions are practically the same by the endpoint !

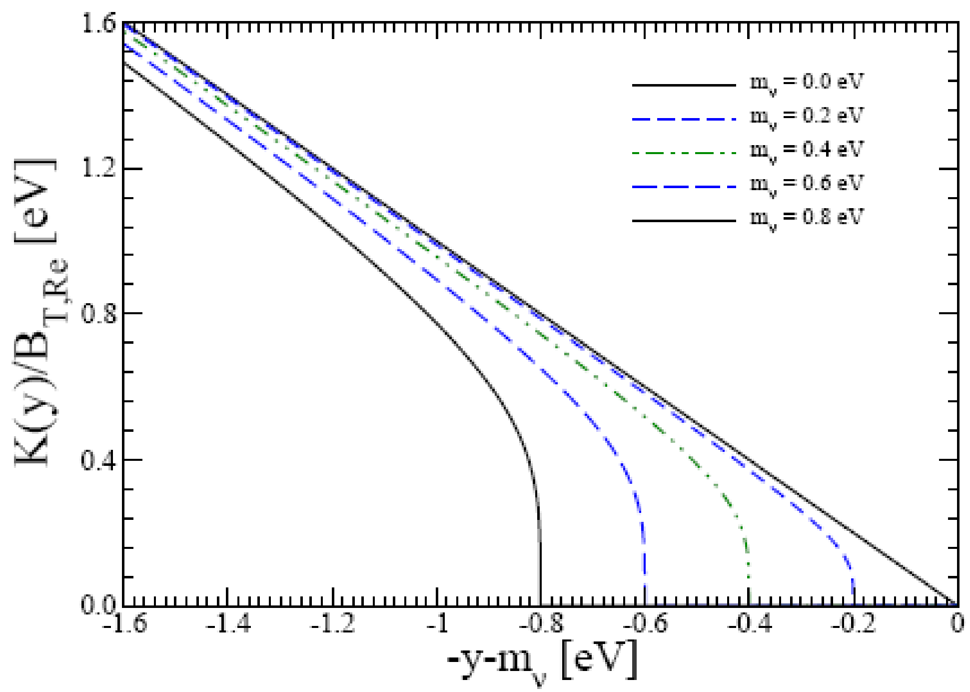
$$\mathbf{K(E)/B_{Re} \cong K(y)/B_T}$$

Tritium

$$B_T = \frac{G_F V_{ud}}{\sqrt{2\pi^3}} \sqrt{g_V^2 + 3g_A^2}$$

$$K(y)/B_T = \left(\sqrt{y(y+2m_\nu)}(y+m_\nu)\right)^{1/2}$$

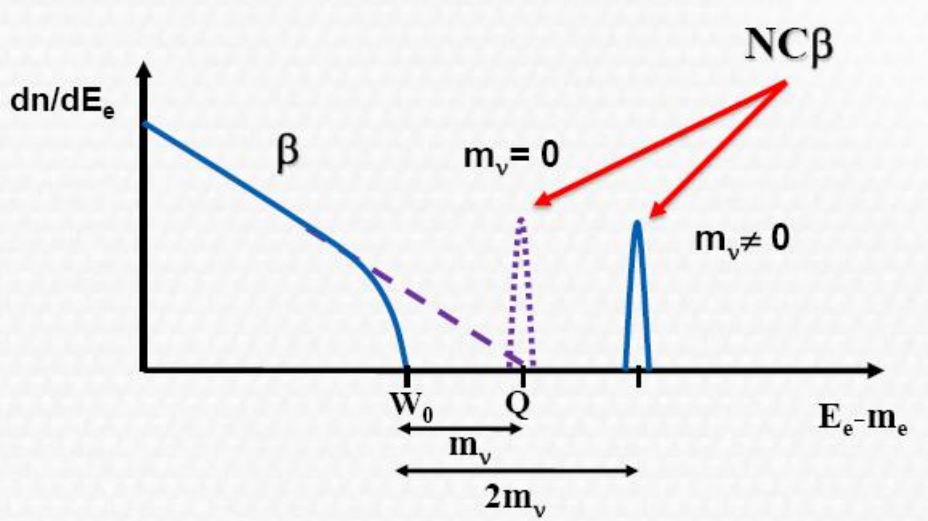
$$y = E_e^{max} - E_e$$



Relic neutrinos

The neutrino capture via the β–decaying nucleus might be a tool to detect cosmological neutrinos

$$\nu + (A,Z) \rightarrow (A,Z+1) + e^-$$



The density of relic ν: $\langle\eta\rangle = 56\ \mathrm{cm}^{-3}$

Temperature

$$T_\nu^{\,0} = \left(\frac{4}{11}\right)^{1/3} T_\gamma^{\,0} \approx (1.945 \pm 0.001)K \rightarrow \mathrm{k_B} T_\nu \approx (1.676 \pm 0.001) \times 10^{-4}\, eV$$

$$\rightarrow T_\gamma^{\,0} = (2.725 \pm 0.001)K = (2.348 \pm 0.001) \times 10^{-4}\, eV$$

Mean momentum

$$\langle p_\nu^0 \rangle = \frac{7}{2}\frac{\zeta(4)}{\zeta(3)} T_\nu^0 \approx 3.151 T_\nu^0 \approx 5.314 \times 10^{-4}\, eV$$

Gravitational clustering of relic neutrinos

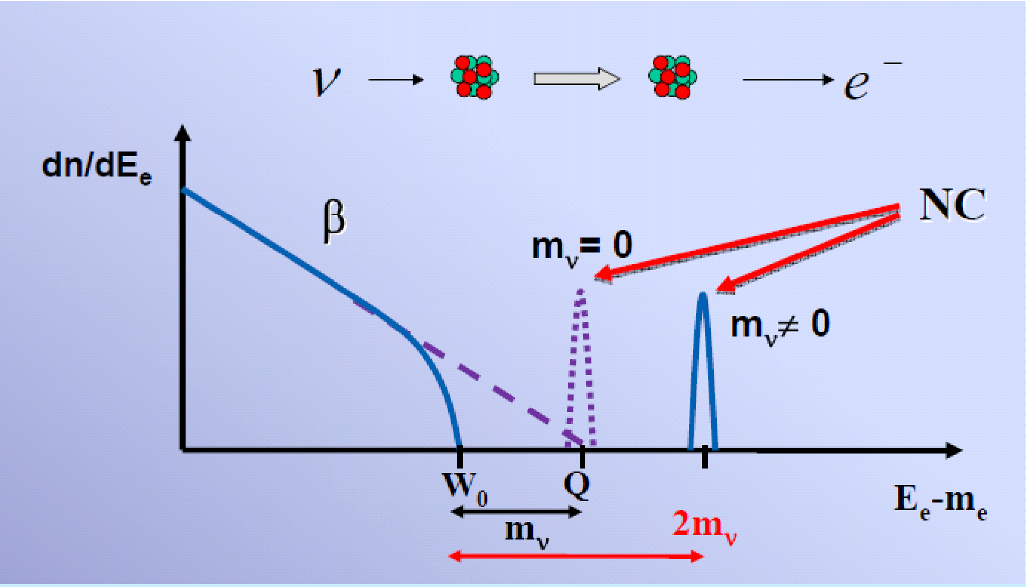
- Neutrinos of CνB are non-relativistic and weakly-clustered
- If they are heavy enough, such that their velocities become less than the escape velocity of a massive object, the RNs fall into the potencial wells of the latter – and are clustered today
- Massive neutrinos, $m_\nu \sim 1$ eV, will be gravitationally clustered on the scale of ~Mpc (~3×10^{19}km), that is on the scale of galaxy clusters
- Overdensities of the order of 10^3–10^4

R. Lazauskas, P. Vogel, C. Volpe, J. Phys. G: Nucl. Part. Phys. 35 (2008)

Detection of relic neutrinos by KATRIN experiment

$$\nu + {}^3H((1/2)^+) \rightarrow {}^3He((1/2)^+) + e^-$$

$$\Gamma^\nu({}^3H) = \frac{1}{\pi} G_\beta^2 \, F_0(2,p) \, p \, p_0 \, \left(|M_F|^2 + g_A^2 \, |M_{GT}|^2\right) \, \frac{\eta_\nu}{<\eta_\nu>} \, <\eta_\nu>$$



Assuming M_F=1, M_{GT}=√3 and η_ν=<η_ν> the capture rate

$$\Gamma^\nu({}^3H) = 4.2 \; 10^{-25} \; y^{-1}$$

KATRIN will use ~50 μg of ^{3}H

$$N^\nu_{capt}(KATRIN) \approx 4.2 \; 10^{-6} \; \frac{\eta_\nu}{<\eta_\nu>} \; y^{-1}$$

Even considering effect of clustering of ν, η_ν/<η_ν> ~ 10^3-10^4 :

N^ν_{capt}(KATRIN) < 1 y^{-1}

Heavy relic neutrino detection using ν_h capture on ^3He

$$\nu_h + {}^3He \rightarrow {}^3H^- + e^+$$

Mixing of neutrinos

$$\nu_e = \sum_{i=1}^{3} U_{ei}\nu_i + U_{eh}\nu_h$$

Capture rate per atom

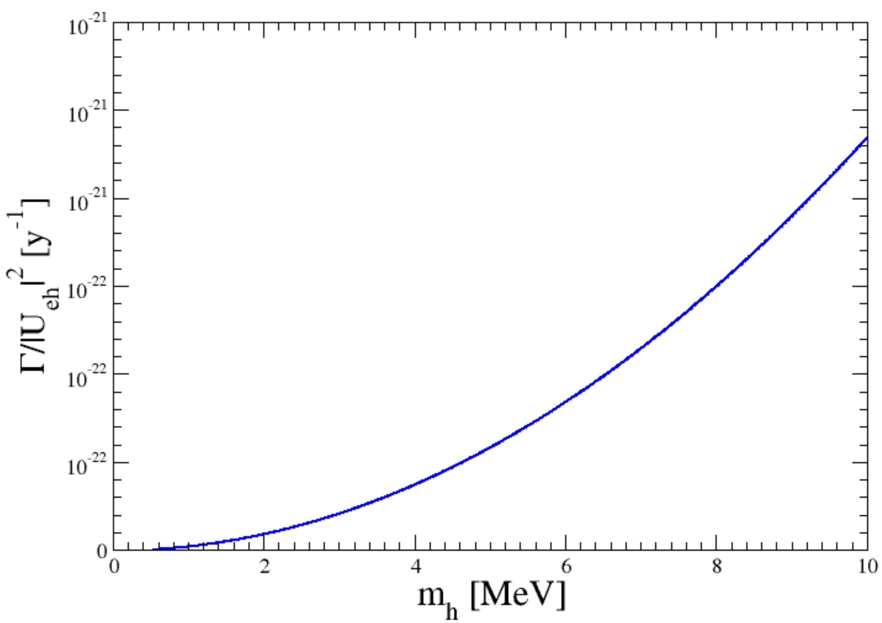
$$\Gamma^{\nu_h}({}^3He) = |U_{eh}|^2 \frac{1}{\pi} G_\beta^2 . F_0(1,p) . p_{e^+} . p_0 \left(|M_F|^2 + g_A^2 |M_{GT}|^2 \right) \frac{\eta_\nu}{\langle \eta_\nu \rangle} \langle \eta_\nu \rangle$$

$$p_{e^+} = \sqrt{E_e^2 - m_e^2} = \sqrt{(m_{\nu_h} - 18.6keV)^2 - m_e^2}$$

Production rate for 1kg of ^3He

m_{ν_h} [MeV]	$N_{\nu_h}/(\lvert U_{eh}\rvert^2 . \frac{\eta_\nu}{\langle \eta_\nu \rangle})$ [y^{-1}]
1.0	$7.9\ 10^2$
5	$2.3\ 10^4$
10	$9.4\ 10^4$

$$m_{\nu_h} > 18.6keV + m_{e^+}$$



Signatures in experiment:
ion $^3H^-$, annihilation of e^+

Fedor

Detection of relic neutrinos by MARE experiment

$$\nu + {}^{187}Re((5/2)^+) \rightarrow {}^{187}Os((1/2)^-) + e^-$$

The capture rate

$$\Gamma^{\nu}({}^{187}Re) = \frac{1}{\pi} G_\beta \, F_1(76,p) \, \frac{1}{3} \, (p\,R)^2 \, \mathscr{B} \, p \, p_0 \; \frac{\eta_\nu}{<\eta_\nu>} \; <\eta_\nu>$$

The strength

$$\mathscr{B} = \frac{g_A^2}{6} \, |< {}^{187}Os(1/2^-) \, || \, \sqrt{\frac{4\pi}{3}} \sum_n \tau_n^+ \frac{r_n}{R} \, \{\sigma_n \otimes Y_1(\Omega_{r_n})\}_2 \, || \, {}^{187}Re(5/2^+) > |^2$$

$T_{1/2} = 4.35 \times 10^{10}$ y $\Rightarrow$

$$\mathscr{B} = 3.57 \times 10^{-4}$$

$$\Gamma^{\nu}({}^{187}Re) = 2.75 \; 10^{-32} \; y^{-1}$$

Ratio of capture and decay rates

$$\frac{\Gamma^{\nu}({}^{187}Re)}{\Gamma^{\beta}({}^{187}Re)} = 1.7 \; 10^{-21}$$

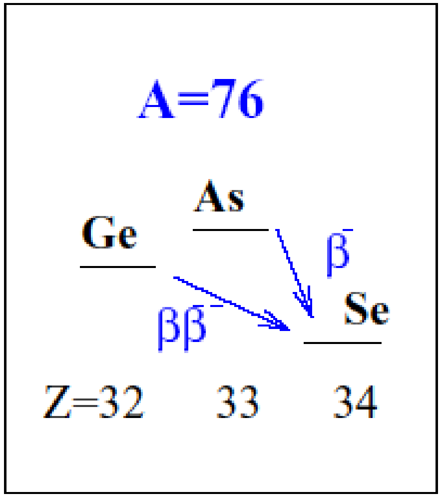
MARE: 760 g of $AgReO_4$ bolometers $\Rightarrow$

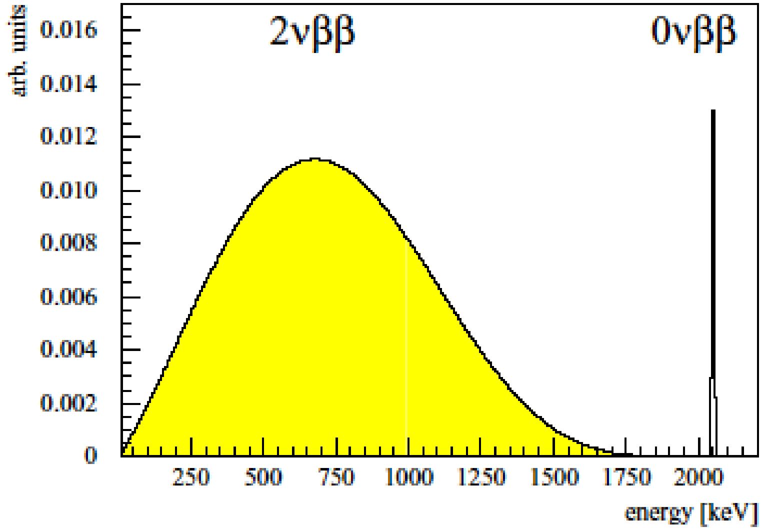
200 larger as for ^{3}H

$$N^{\nu}_{capt}(MARE) \simeq 7.6 \; 10^{-8} \; \frac{\eta_\nu}{<\eta_\nu>} \; y^{-1}$$

50 smaller as for ^{3}H

The double β-decay



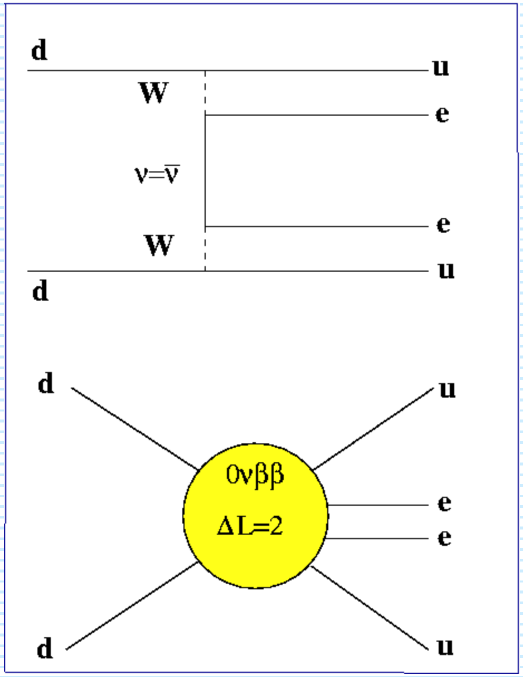


$$(A,Z) \rightarrow (A,Z+2) + 2e^- + 2\bar{\nu}_e$$

Observed for 10 isotopes: ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo. ^{116}Cd. ^{128}Te, ^{130}Te, ^{150}Nd, ^{238}U, $T_{1/2} \approx 10^{18}$-10^{24} years

1967: ^{130}Te, Kirsten et al, Takaoka et al, (geochemical)
1987: ^{82}Se, Moe et al. (direct observation)
2006: ^{100}Mo, NEMO 3 coll. ~ 300 00 events

$$(A,Z) \rightarrow (A,Z+2) + 2e^-$$

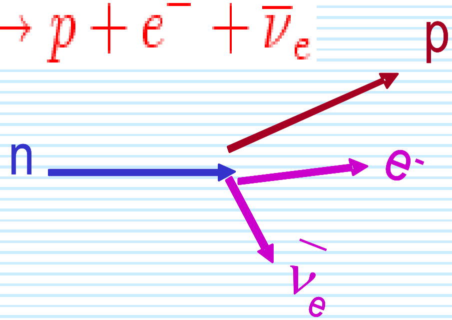


SM forbidden ,not observed yet: $T_{1/2}$ (^{76}Ge)>10^{25} years

1934 Fermi theory of beta decay

Fermi, Z. Physik 88 (1934) 161

$$n \to p + e^- + \overline{\nu}_e$$



Fermi 4-fermion contact interaction, Lagrangian of interaction (in analogy with electrodynamics):

$$\mathcal{L}(x) = -\frac{G_F}{\sqrt{2}}\left[\overline{\phi}_p(x)\gamma^\mu\phi_n(x)\right]\left[\overline{\phi}_e(x)\gamma^\mu\phi_\nu(x)\right]$$

G_F = Fermi coupling constant = (1.16637±0.000001) 10^{-5} GeV^{-2}

$$(A,Z) \to (A,Z+2) + 2e^- + 2\overline{\nu}_e$$

1935
Q-value about 10 MeV
$T_{1/2} \approx 10^{17}$ years

Eugene Wigner

Maria-Goeppert Mayer

History of Double Beta Decay I

The early period (1935-1957)

- **1935 Goepper-Mayer suggested the 2νββ-decay**
- **1937 Dirac** $\nu \neq \overline{\nu}$ **or Majorana** $\nu \equiv \overline{\nu}$
- **1939 Furry proposed the 0νββ-decay**
- **till 1957 Observation of 0νββ more favored (phase space)**

$$n \to p + e^- + \overline{\nu}_e \quad \nu_e + n \to p + e^-$$

Period of scepticism (1957-1970)

- **1957 Wu, weak interaction violates parity, Majorana or Dirac – open question**

$$n \to p + e^- + \overline{\nu}_e{}^{RH} \quad \nu_e{}^{LH} + n \to p + e^-$$

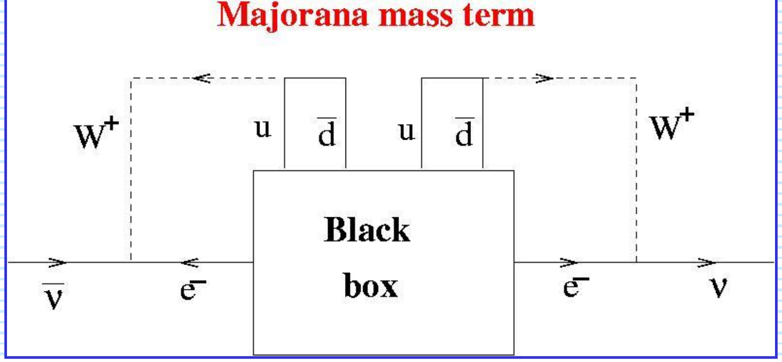
Declined interest to 0νββ-decay

- **1968 Pontecorvo proposed** $\pi^- \to \pi^+ + 2e^-$**, superweak int.**

Period of GUT (1970-1998)

- **1975 Primakoff and Rosen – Right handed current mech.**
- **1981 Doi, Kotani, Takasugi ν-mech. within gauge theories**
- **1981 Wolfenstein: cancellation mech. possible**

$$< m_\nu >= \sum_k |U_{ek}|^2 \; \eta_{CP} \; m_k, \quad \eta_{CP} = \pm i$$



History of Double Beta Decay II

- 1982 Scheckter-Valle theorem
  The observation of 0νββ-decay implies the existence of Majorana mass term
- 1986 Vogel, Zirnbauer – quenching mech. of 2νββ-decay
- 1987 Elliott, Hahn, Moe -first detection of 2νββ-decay (^{82}Se)
- 1987 Mohapatra, Vergados, R-parity breaking SUSY mech.
- 1997 Feassler, Kovalenko, Simkovic, dominance of pion-exchange SUSY mech.
- 1997 Kovalenko, Hirsch, Klapdor,
  leptoquark mech.

Period of massive ν (1998→20??)

•1998- neutrino oscillations (SK, SNO, Kamland) convin. evid.

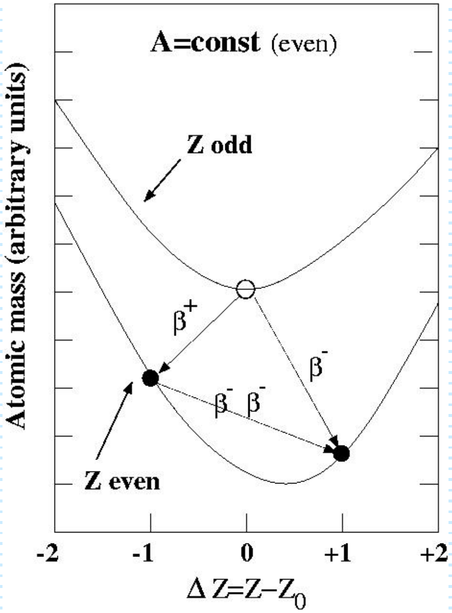
- 2001 Klapdor-Kleingrothaus, Dietz, Krivosheina, first claim for observation of the 0νββ-decay
- Many works on neutrino mass pattern, absolute mass scale, CP phases, extra dim. mech.
- Many works on future large (tons) 0νββ-decay experiments

Quo vadis 0νββ-decay?

Majorana period (2???→)

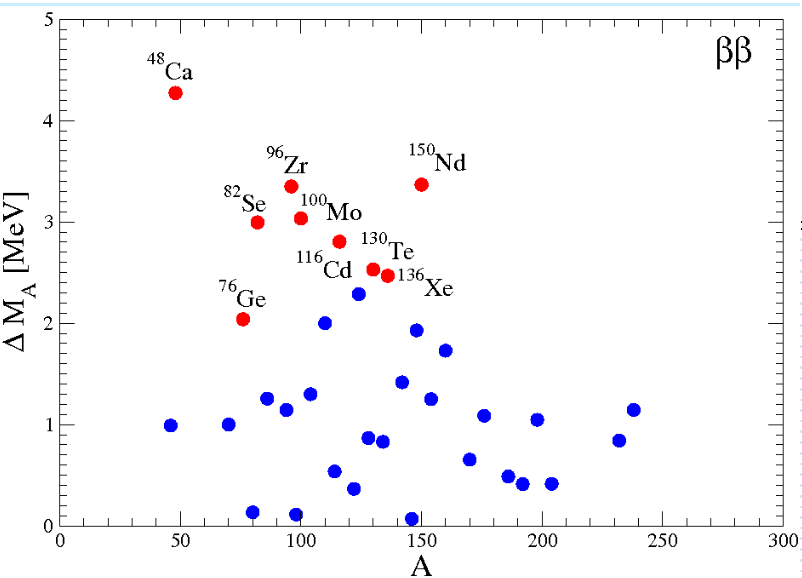
- 2??? Observation of 0νββ-decay
- 2??? …

The double beta decay process can be observed due to nuclear pairing interaction that favors energetically the even-even nuclei over the odd-odd nuclei



$$\frac{1}{T^{0\nu}_{1/2}} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 \; G^{01}(E_0, Z) \; \left|M^{0\nu}\right|^2$$

transition	$G^{01}(E_0,Z)$ $\times 10^{14} y$	$Q_{\beta\beta}$ [MeV]	Abund. (%)	$\lvert M^{0\nu}\rvert^2$
$^{150}Nd \to {}^{150}Sm$	26.9	3.667	6	?
$^{48}Ca \to {}^{48}Ti$	8.04	4.271	0.2	?
$^{96}Zr \to {}^{96}Mo$	7.37	3.350	3	?
$^{116}Cd \to {}^{116}Sn$	6.24	2.802	7	?
$^{136}Xe \to {}^{136}Ba$	5.92	2.479	9	?
$^{100}Mo \to {}^{100}Ru$	5.74	3.034	10	?
$^{130}Te \to {}^{130}Xe$	5.55	2.533	34	?
$^{82}Se \to {}^{82}Kr$	3.53	2.995	9	?
$^{76}Ge \to {}^{76}Se$	0.79	2.040	8	?



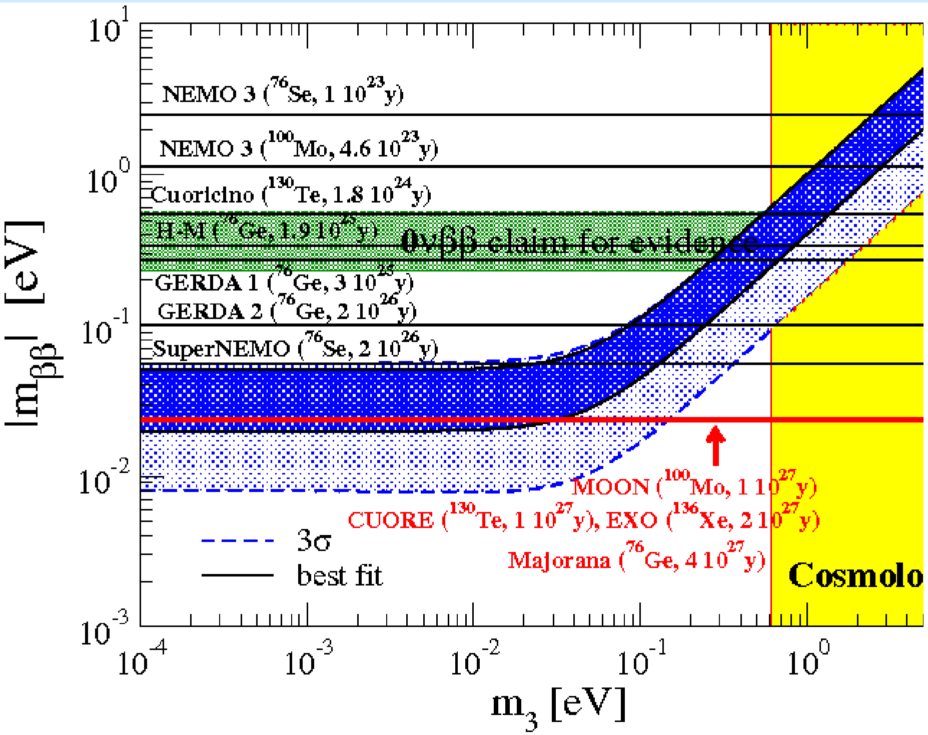
The NMEs for 0νββ-decay must be evaluated using tools of nuclear theory

Neutrino mass spectrum
And perspectives of the 0νββ-decay search

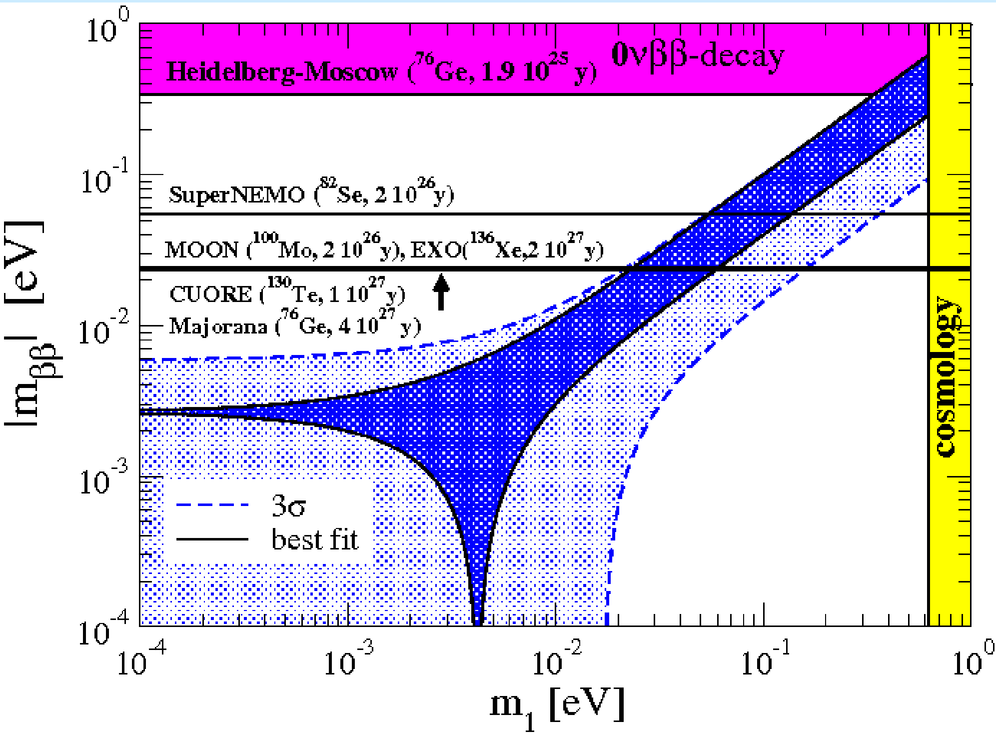
What is the absolute mass scale of neutrinos: Limits from cosmology, tritium beta decay, neutrinoless double beta decay

What are the Majorana CP phases? ...

Inverted hierarchy



Normal hierarchy



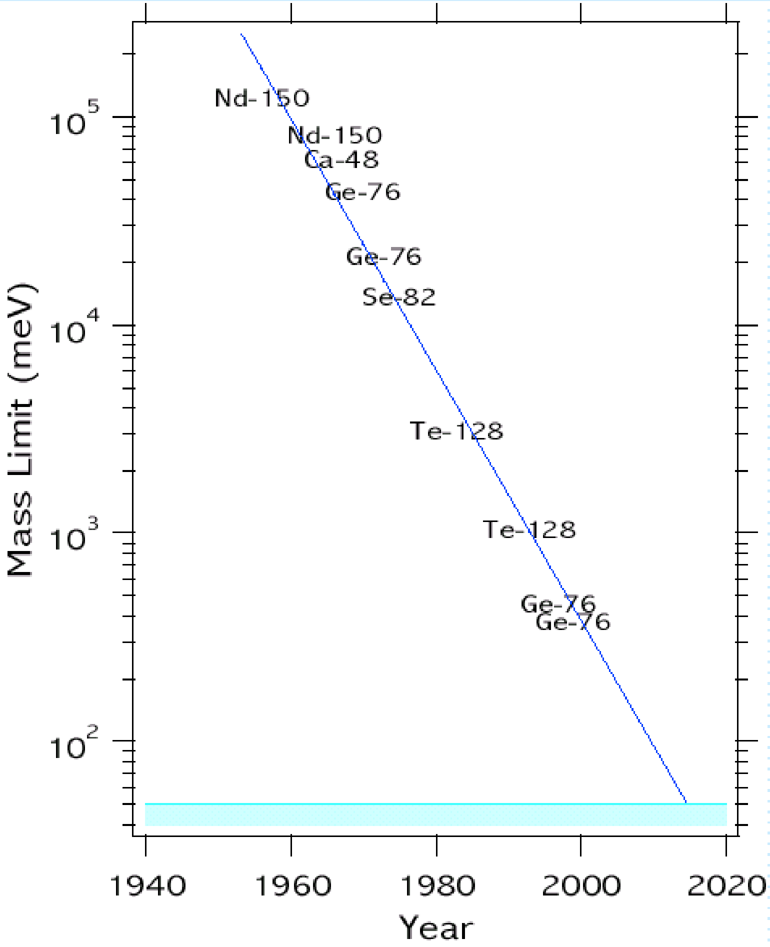
Bilenky, Faessler, Šimkovic, PRD 70 (2004) 033003

If (or when) the $0\nu\beta\beta$ decay is observed two theoretical problems must be resolved

S.R. Elliott, P. Vogel, Ann.Rev.Nucl.Part.Sci. 52, 115 (2002)

1) What is the mechanism of the decay, i.e., what kind of virtual particle is exchanged between the affected nucleons (quarks).

2) How to relate the observed decay rate to the fundamental parameters, i.e., what is the value of the corresponding nuclear matrix elements.



The 0νββ-decay is a particle physics problem

The 0νββ-decay mechanisms

Two basic categories are
long-range
(exchange of light Majorana ν)
and
short-range
(exchange of heavy ν, squarks, gluinos …)
contributions to the 0νββ-decay

Light ν-exchange 0νββ–decay mechanism

S.M. Bilenky, S. Petcov, Rev. Mod. Phys. 59, 671 (1987)

Majorana condition

$$C\,\overline{\chi_k}^T(x) \;=\; \xi_k\,\chi_k(x)$$

Majorana particle propagator

$$\begin{aligned} < \chi_\alpha(x_1)\overline{\chi}_\beta(x_2) > \;&=\; \frac{-1}{(2\pi)^4}\int \left(\frac{1}{\gamma p - im}\right)_{\alpha\beta} e^{ip(x_1-x_2)}dp \\ &=\; S_{\alpha\beta}(x_1-x_2) \end{aligned}$$

$$\begin{aligned} < \chi(x_1)\chi^T(x_2) > \;&=\; -\xi S(x_1-x_2)C \\ < \overline{\chi}^T(x_1)\overline{\chi}(x_2) > \;&=\; \xi C^{-1}S(x_1-x_2) \end{aligned}$$

Weak β-decay Hamiltonian

$$\mathcal{H}^\beta_W = \frac{G_F}{\sqrt{2}}\,\overline{e}\gamma_\alpha(1+\gamma_5)\nu_e\; j_\alpha + h.c.$$

Neutrino mixing

$$\nu_{eL} \;=\; \sum_k U^L_{lk}\,\chi_{kL}$$

S-matrix term

$$S^{(2)} = -\frac{(-i)^2}{2} 4 \left(\frac{G_F}{\sqrt{2}}\right)^2 \int N\left[\overline{e_L}(x_1)\gamma_\alpha < \nu_{eL}(x_1)\nu_{eL}^T(x_2) > \gamma_\beta^T \overline{e_L}^T(x_2)\right] \times$$
$$T\left(j_\alpha(x_1) j_\beta(x_2) e^{-i\int \mathcal{H}_{str}(x)dx}\right) dx_1 dx_2$$

Contraction of ν-fields

$$< \nu_{eL}(x_1){\nu_{eL}}^T(x_2) > = -\sum_k \left(U_{ek}^L\right)^2 \xi_k \frac{1+\gamma_5}{2} S_k(x_1 - x_2) \frac{1+\gamma_5}{2} C$$
$$= \frac{i}{(2\pi)^4} \sum_k \left(U_{ek}^L\right)^2 \xi_k m_k \int \frac{e^{iq(x_1-x_2)}dq}{q^2 + m_k^2} \frac{1+\gamma_5}{2} C$$

Effective mass of Majorana neutrinos

$$m_{\beta\beta} = \sum_k \left(U_{ek}^L\right)^2 \xi_k m_k$$

0νββ-decay matrix element

$$
\begin{aligned}
< f|S^{(2)}|i > \;&=\; m_{\beta\beta}\left(\frac{G_F}{\sqrt{2}}\right)^2 N_{p_1}N_{p_2}\overline{u}(p_1)\gamma_\alpha(1+\gamma_5)\gamma_\beta C\overline{u}^T(p_2)\times \\
&\int e^{-ip_1x_1}e^{-ip_2x_2}\frac{-i}{(2\pi)^4}\int\frac{e^{iq(x_1-x_2)}dq}{q^2}\times \\
&< A'|T\left[J_\alpha(x_1)J_\beta(x_2)\right]|A > dx_1dx_2 - (p_1\leftrightarrow p_2)
\end{aligned}
$$

Use of completness **1=Σ$_n$|n><n|**

$$
\begin{aligned}
< A'|J_\alpha(x_1)J_\beta(x_2)|A > \;&=\; \sum_n < A'|J_\alpha(0,\vec{x}_1)|n >< n|J_\beta(0,\vec{x}_2)|A > \times \\
&e^{-i(E'-E_n)x_{10}}e^{-i(E_n-E)x_{20}}
\end{aligned}
$$

After integration over time variables

$$
\begin{aligned}
< f|S^{(2)}|i > \;&=\; im_{\beta\beta}\left(\frac{G_F}{\sqrt{2}}\right)^2 N_{p_1}N_{p_2}\overline{u}(p_1)\gamma_\alpha(1+\gamma_5)\gamma_\beta C\overline{u}^T(p_2) \\
&\times\int d\vec{x}_1d\vec{x}_2e^{-i\vec{p}_1\cdot\vec{x}_1}e^{-i\vec{p}_2\cdot\vec{x}_2}\frac{1}{(2\pi)^3}\int\frac{e^{i\vec{q}\cdot(\vec{x}_1-\vec{x}_2)}d\vec{q}}{\vec{q}^2}\times \\
&\sum_n\left(\frac{< A'|J_\alpha(0,\vec{x}_1)|n >< n|J_\beta(0,\vec{x}_2)|A >}{E_n+q_0+p_{20}-E}+\right. \\
&\left.\frac{< A'|J_\beta(0,\vec{x}_1)|n >< n|J_\alpha(0,\vec{x}_2)|A >}{E_n+q_0+p_{10}-E}\right) \\
&\times\; 2\pi\delta(E'+p_{10}+p_{20}-E)
\end{aligned}
$$

Approximations and simplifications

1) Non-relativistic impulse approx. for nuclear current

$$J_\alpha(0,\vec{x}) = \sum_n \tau_n^+ (\delta_{\alpha 4} + i g_A (\vec{\sigma})_k \delta_{\alpha k}) \delta(\vec{x} - \vec{x}_n)$$

2) Long-wave approximation for lepton wave functions

$$e^{-i\vec{p}_1\cdot\vec{x}_1 - i\vec{p}_2\cdot\vec{x}_2} \longrightarrow 1$$

3) Closure approximation

$$E_n \ \rightarrow \ < E_n >$$

$$< f|S^{(2)}|i > = \overline{u}(p_1)\gamma_\alpha(1+\gamma_5)\gamma_\beta C\overline{u}^T(p_2) A_{\alpha\beta}, \quad A_{\alpha\beta} = A_{\beta\alpha}$$

Hadron part is symmetric

$$J_\alpha(0,\vec{x}_1) J_\beta(0,\vec{x}_2) = J_\beta(0,\vec{x}_2) J_\alpha(0,\vec{x}_1)$$

contribute

$$\gamma_\alpha\gamma_\beta = \delta_{\alpha\beta} + \frac{1}{2}(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha)$$

0νββ-decay matrix element

$$\begin{aligned} < f|S^{(2)}|i > \quad &= \quad i\, m_{\beta\beta} \left(\frac{G_F}{\sqrt{2}}\right)^2 N_{p_1} N_{p_2} \overline{u}(p_1)(1-\gamma_5) C \overline{u}^T(p_2) \frac{1}{R} \\ &\times \left(M_F - g_A^2 M_{GT}\right) \delta(p_{10} + p_{20} + M' - M) \end{aligned}$$

Nuclear matrix elements

$$M_F = < A'|\sum_{n,m} \tau_n^+ \tau_m^+ h(|\vec{x}_n - \vec{x}_m|)|A >$$

$$M_F = < A'|\sum_{n,m} \tau_n^+ \tau_m^+ h(|\vec{x}_n - \vec{x}_m|)\vec{\sigma}_n \cdot \vec{\sigma}_m|A >$$

Neutrino exchange potential

$$h(|\vec{x}_n - \vec{x}_m|) = \frac{1}{2\pi^2}\int \frac{e^{i\vec{q}\cdot\vec{x}}d\vec{q}}{q_0(q_0 + < E_n > -(E+E')/2)} \approx \frac{1}{|\vec{x}|}$$

Differential 0νββ-decay rate

$$d\Gamma_{0\nu} = \frac{1}{2}\frac{G_F^4 m_e^5}{(2\pi)^5}|m_{\beta\beta}|^2 \frac{1}{R^2}|M_F - g_A^2 M_{GT}|^2(1-\cos\theta) F^2(Z)(\varepsilon_0 - \varepsilon + 1)^2(\varepsilon+1)d\varepsilon \sin\theta d\theta$$

$$F(Z) = \frac{2\pi\alpha(Z+2)}{1 - exp[-2\pi\alpha(Z+2)]}$$

$$\varepsilon_0 = \frac{1}{m_e}(M - M' - 2m_e)$$

Full 0νββ-decay rate

$$\Gamma_{0\nu} = \frac{1}{2}\frac{G_F^4 m_e^5}{(2\pi)^5}|m_{\beta\beta}|^2 \frac{1}{R^2}|M_F - g_A^2 M_{GT}|^2 F^2(Z) \times \frac{1}{15}\left(\varepsilon_0^5 + 10\varepsilon_0^4 + 40\varepsilon_0^3 + 60\varepsilon_0^2 + 30\varepsilon_0\right)$$

Assumption $M_R \gg m_D$

$$\begin{pmatrix}\overline{\nu}_L & \overline{(\nu_R)^c}\end{pmatrix}\begin{pmatrix}0 & m_D \\ m_D & M_R\end{pmatrix}\begin{pmatrix}(\nu_L)^c \\ \nu_R\end{pmatrix}$$

Eigenvalues and eigenvectors

$m_1 = m_D^2/M_R \ll m_D \quad m_2 \approx M_R$

$\nu_1 = \nu_L - m_D/M_R\,(\nu_R)^c \quad \nu_2 = \nu_R + m_D/M_R\,(\nu_L)^c$

Left-right symmetric models SO(10)

Two-charged vector bosons

$W_1^{\pm} = \cos\zeta\; W_L^{\pm} + \sin\zeta\; W_R^{\pm}$

$W_2^{\pm} = -\sin\zeta\; W_L^{\pm} + \cos\zeta\; W_R^{\pm}$

Parameters

$-2\; 10^{-4} \leq \zeta \leq 3.3\; 10^{-3}$ (superallowed β–decay)

$M_1 = 81$ GeV, $M_2 > 715$ GeV, $(M_1/M_2)^2 < 10^{-2}$

See-saw scenario

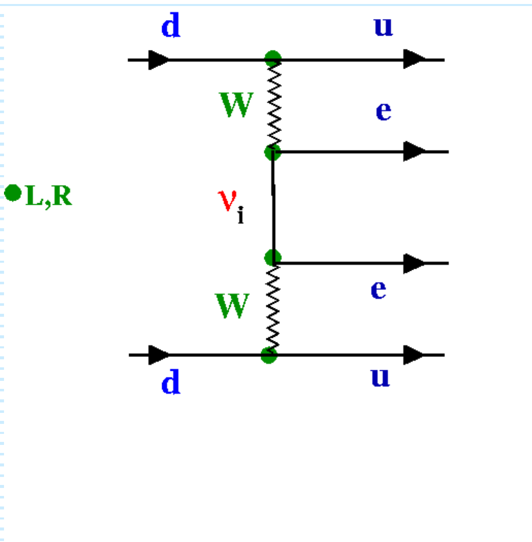
$$\nu_{eL} = \sum_{i=1}^{light} U_{ei}\chi_{iL} + \sum_{i=1}^{heavy} U_{ei} N_{iL}$$

large small

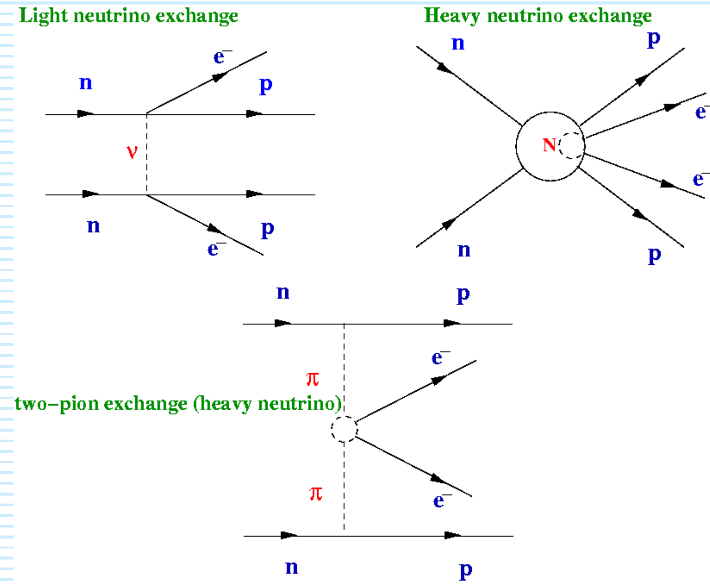
$$(\nu_{eR})^c = \sum_{i=1}^{light} V_{ei}\chi_{iL} + \sum_{i=1}^{heavy} V_{ei} N_{iL}$$

small large

quark level



nucleon level



$$P_L \frac{\hat{q}+im}{q^2+m^2} P_L \quad \Rightarrow \quad \frac{im}{q^2}$$

$$P_L \frac{\hat{q}+im}{q^2+m^2} P_R \quad \Rightarrow \quad \frac{i\hat{q}}{q^2}$$

$$P_{L,R} \frac{\hat{q}+iM}{q^2+M^2} P_{L,R} \quad \Rightarrow \quad \frac{i}{M}$$

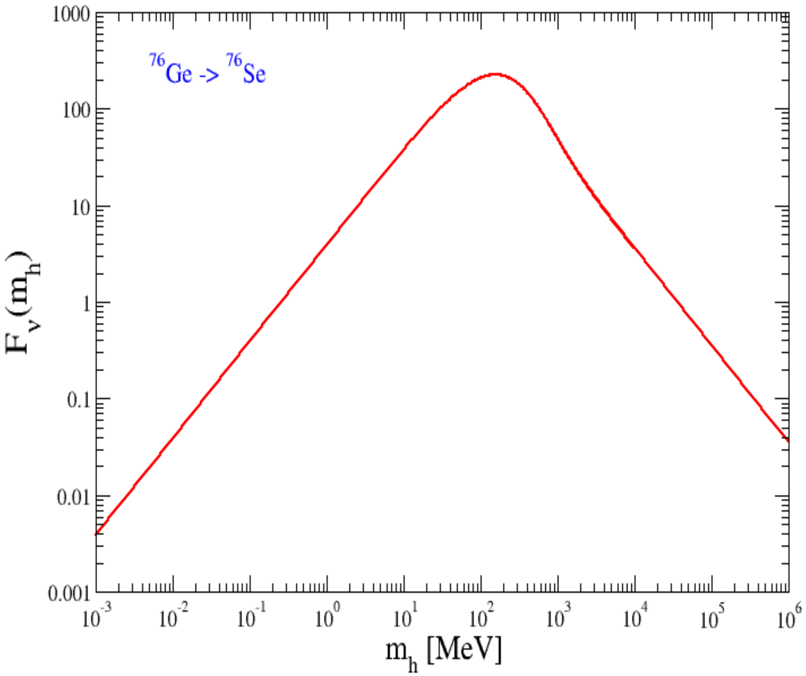
Mechanisms

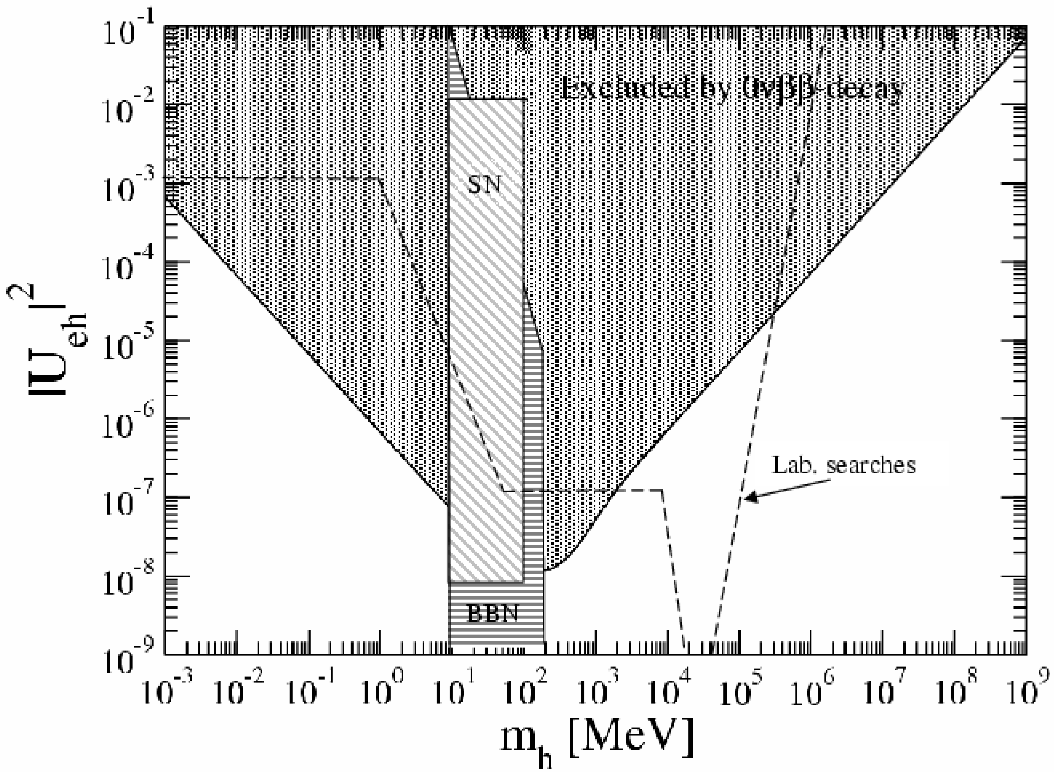
neutrino	*lept.v.*	*quark v.*	*hadr.m.*	*supp.f.*	*LNV p.*	*limit*
light	LL	LL	$2n$		$\sum^{light} UUm$	$m_{\beta\beta} \leq 0.5\ eV$
	LR	LR	$2n$	$(M_1/M_2)^2$	$\sum^{light} UV$	$<\lambda> \leq 7\ 10^{-7}$
	LR	LL	$2n$	$\tan\zeta$	$\sum^{light} UV$	$<\eta> \leq 4\ 10^{-9}$
heavy	LL	LL	$2n$	$-$	$\sum^{heavy} U\ U\ m_p/M$	$\eta_N \leq 8\ 10^{-8}$
	RR	RR	$2n$	$(M_1/M_2)^4$	$\sum^{heavy} V\ V\ m_p/M$	
	RR	LL	$2n$	$(\tan\zeta)^4$	$\sum^{heavy} V\ V\ m_p/M$	
	RR	RL	2π	$\tan\zeta\ (M_1/M_2)^2$	$\sum^{heavy} V\ V\ m_p/M$	

Heavy neutrino in 0νββ-decay

$$\nu_e = \sum_{i=1}^{3} U_{ei}\nu_i + U_{eh}\nu_h$$

$$[T_{1/2}^{0\nu}]^{-1} = G_{01}\left|\frac{\langle m_\nu\rangle_{ee}}{m_e}M_\nu^{light} + U_{eh}^2\frac{m_h}{m_e}M^{0\nu}(m_h)\right|^2.$$





$$F_\nu(m_h) = \frac{m_h}{m_e}M^{0\nu}(m_h)$$

$$|U_{eh}|^2 \le \frac{1}{|F_\nu(m_h)|}\frac{1}{\sqrt{T_{1/2}^{0\nu-exp}G_{01}}},$$

Beneš, Faesler, F.Š., Kovalenko, PRD 71 (2005) 077901

Minimal Supersymmetric Standard Model

Normal particles / fields		Supersymmetric particles / fields			
		Interaction eigenstates		Mass eigenstates	
Symbol	Name	Symbol	Name	Symbol	Name
$q = d,c,b,u,s,t$	quark	$\tilde{q}_L, \tilde{q}_R$	squark	$\tilde{q}_1, \tilde{q}_2$	squark
$l = e, \mu, \tau$	lepton	$\tilde{l}_L, \tilde{l}_R$	slepton	$\tilde{l}_1, \tilde{l}_2$	slepton
$\nu = \nu_e, \nu_\mu, \nu_\tau$	neutrino	$\tilde{\nu}$	sneutrino	$\tilde{\nu}$	sneutrino
g	gluon	$\tilde{g}$	gluino	$\tilde{g}$	gluino
$W^\pm$	W-boson	$\tilde{W}^\pm$	wino	$\tilde{\chi}^\pm_{1,2}$	chargino
$H^\mp$	Higgs boson	$\tilde{H}^\mp_{1/2}$	Higgsino		
B	B-field	$\tilde{B}$	bino	$\tilde{\chi}^0_{1,2,3,4}$	neutralino
W^3	W^3-field	$\tilde{W}^3$	wino		
H_1^0	Higgs boson	$\tilde{H}_1^0$	Higgsino		
H_2^0	Higgs boson	$\tilde{H}_2^0$	Higgsino		
H_{31}^0	Higgs boson				

R=+1 R=–1

R-parity: $R=(-1)^{3B+L+2S}$

R-parity Breaking MSSM
(neutralino is not dark matter candidate)

$\lambda_{ij<k}\, LLE + \lambda'_{ijk}\, LQD + \lambda''_{ij<k}\, UDD$

$9 + 27 + 9 = 45$ coupling constants

R-parity breaking terms In superpotential

$\lambda'_{11k} * \lambda''_{11k} < 10^{-22}$ proton decay

$\lambda < 10^{-3}$ to 10^{-1} with $\lambda_{133} < 0.003$ limit on ν_e mass

$\lambda' < 10^{-2}$ to 10^{-1} with $\lambda'_{111} < 4.10^{-4}$ neutrinoless beta decay

Neutrino-Neutralino mixing matrix (see-saw structure)

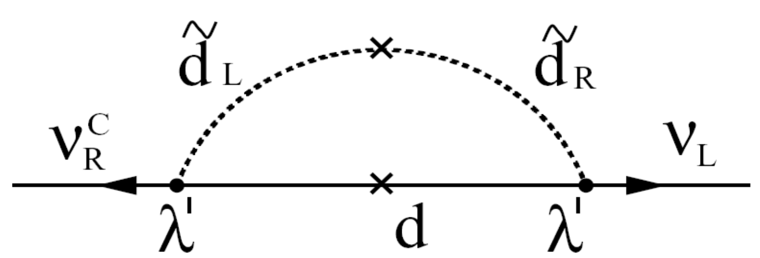
$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m \\ m^T & M_\chi \end{pmatrix}$$

$$\Psi'^{T}_{(0)} = (\nu_e, \nu_\mu, \nu_\tau, -i\lambda', -i\lambda_3, \tilde{H}^0_1, \tilde{H}^0_2),$$

Radiative corrections to neutrino mass

$$\mathcal{M}_\nu = \mathcal{M}^{tree} + \mathcal{M}^l + \mathcal{M}^q$$

Gozdz, Kaminski, Šimkovic, PRD 70 (2004) 095005



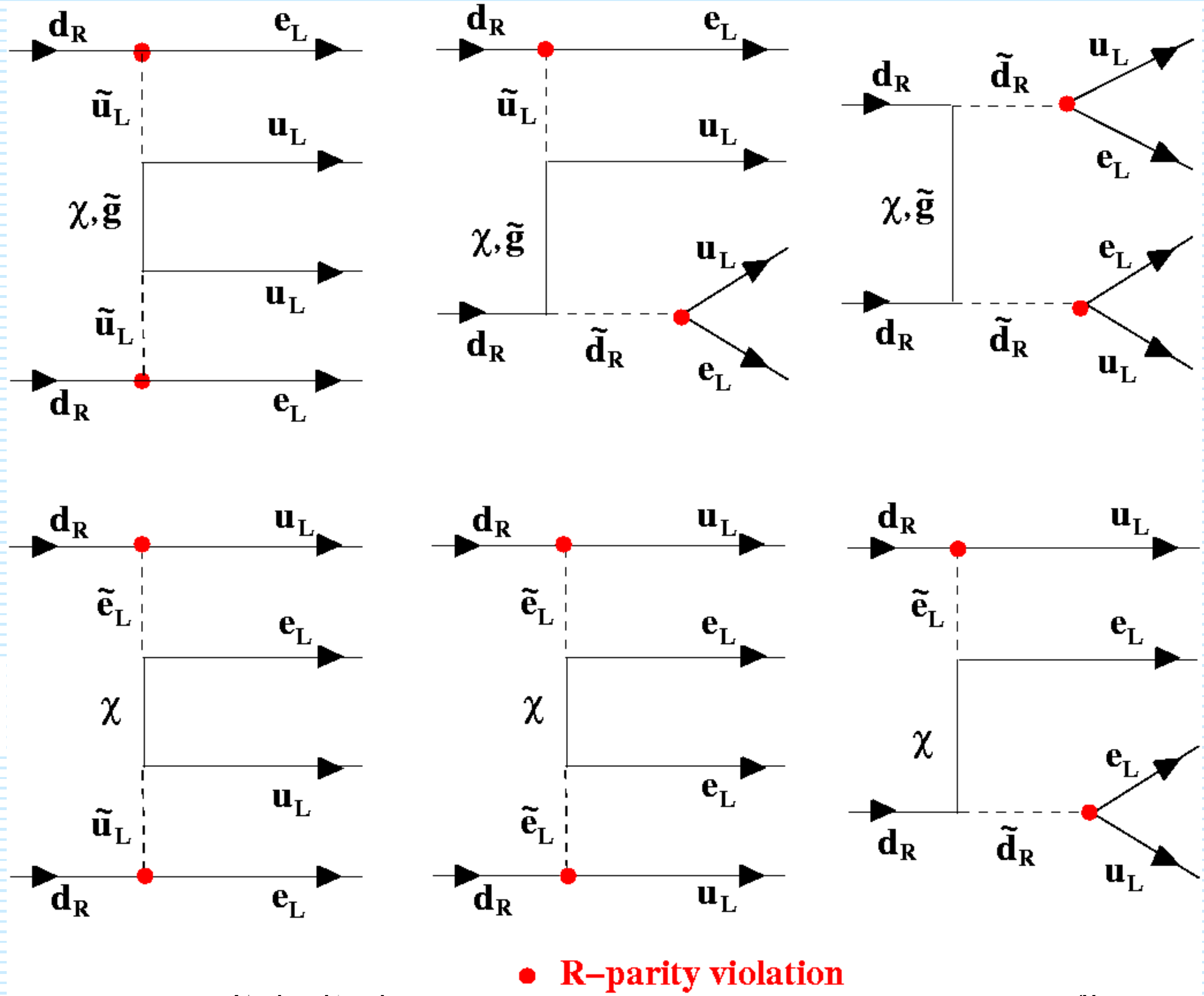
gluino/neutralino exchange R-parity breaking SUSY mechanism of the $0\nu\beta\beta$–decay

quark-level diagrams

$d + d \rightarrow u + u + e^- + e^-$

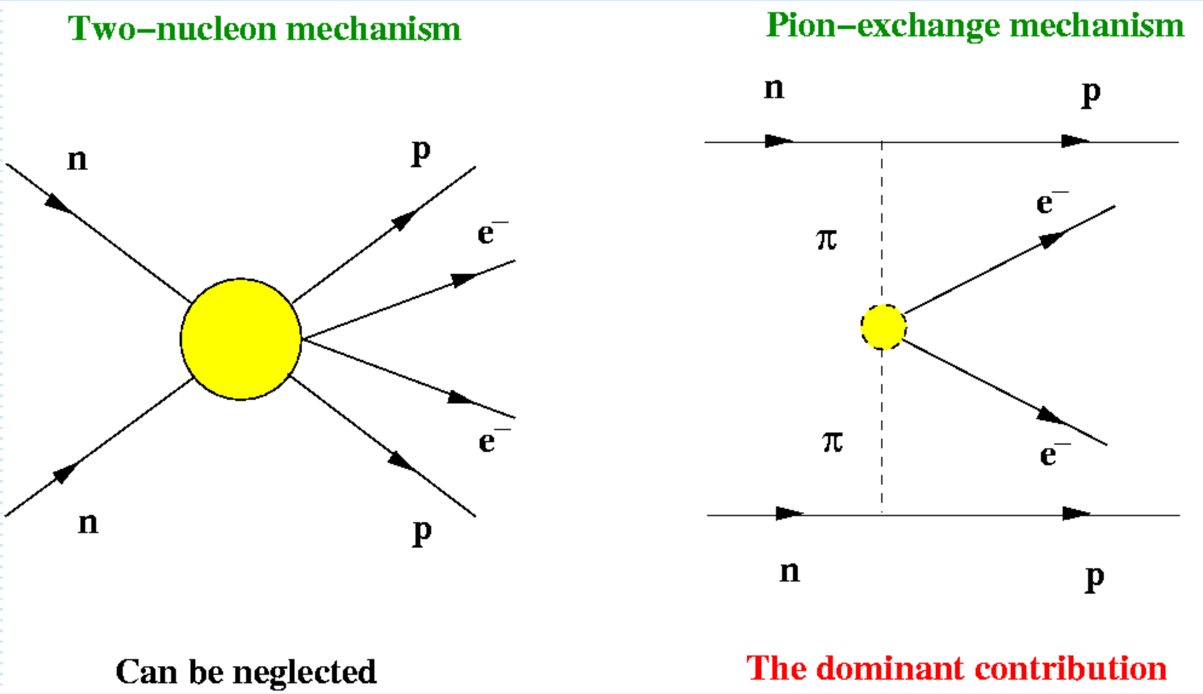
exchange of **squarks**, **neutralinos** and **gluinos**

$(\lambda'_{111})^2$ mechanism



1968 Pontecorvo proposed $\pi^- \to \pi^+ + 2e^-$, superweak int.
We identified with R-parity breaking SUSYmechanism

$$\mathcal{L}_{qe} = \frac{G_F^2}{2m_p}\,\bar{e}(1+\gamma_5)e^c\left[\eta^{PS}\,J_{PS}J_{PS} - \frac{1}{4}\eta^T\,J_T^{\mu\nu}J_{T\mu\nu}\right].$$



Hadron-level diagrams

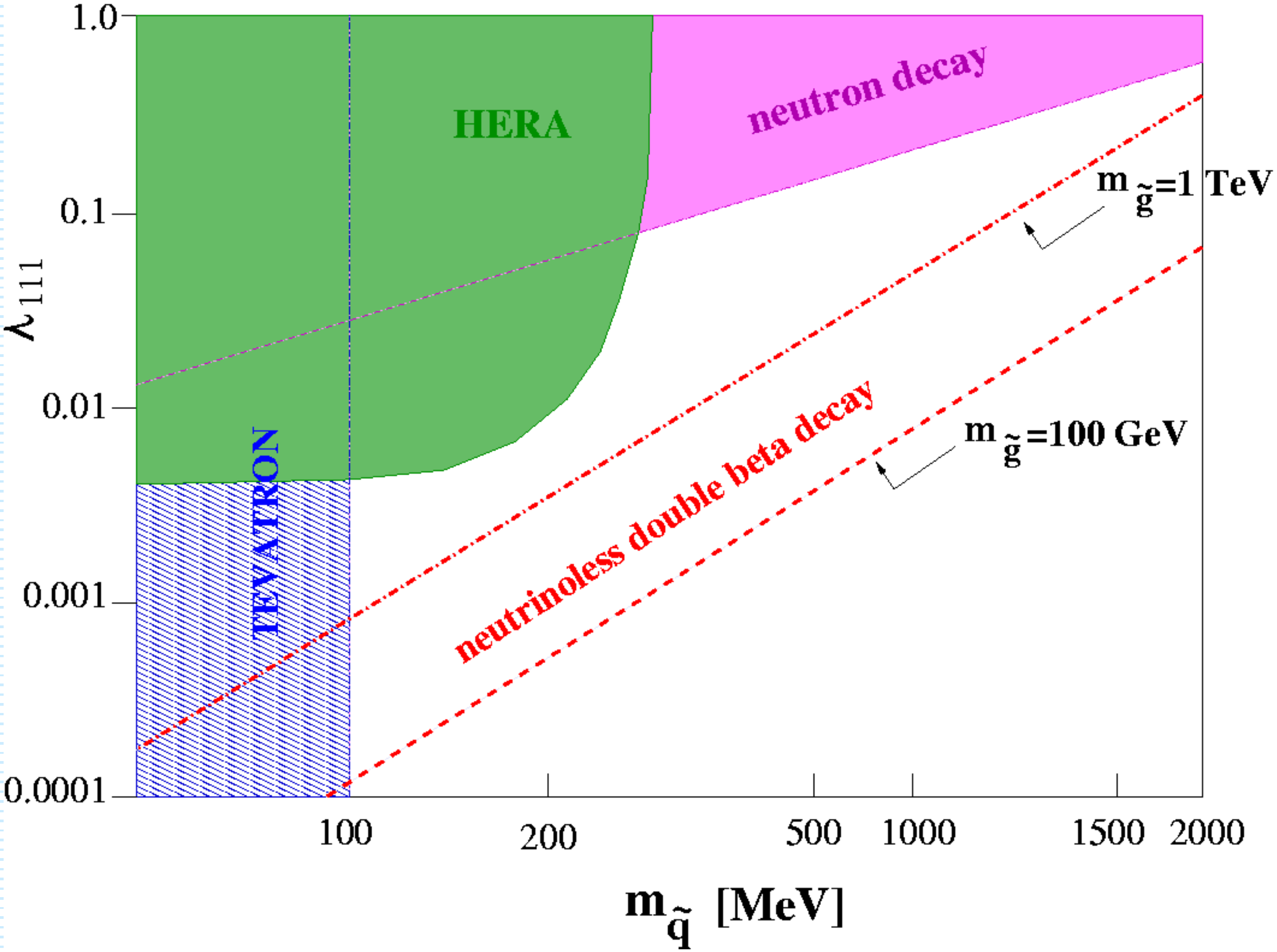
Faessler, Kovalenko, Šimkovic
PRL 78 (1998) 183
Wodecki, Kaminski, Šimkovic,
PRD 60 (1999) 11507

$$\langle 0|\bar{u}\gamma_5 d|\pi^-\rangle = i\sqrt{2}f_\pi \frac{m_\pi^2}{m_u+m_d}, \quad (m_\pi/(m_u+m_d) \approx 13)$$

$$\langle 0|\bar{u}\gamma_\alpha\gamma_5 d|\pi^-\rangle = i\sqrt{2}f_\pi k_\alpha$$

Limit on R-parity breaking parameter λ'_{111}

Faessler, F.Š , Kovalenko, PRD 58 (1998) 115004

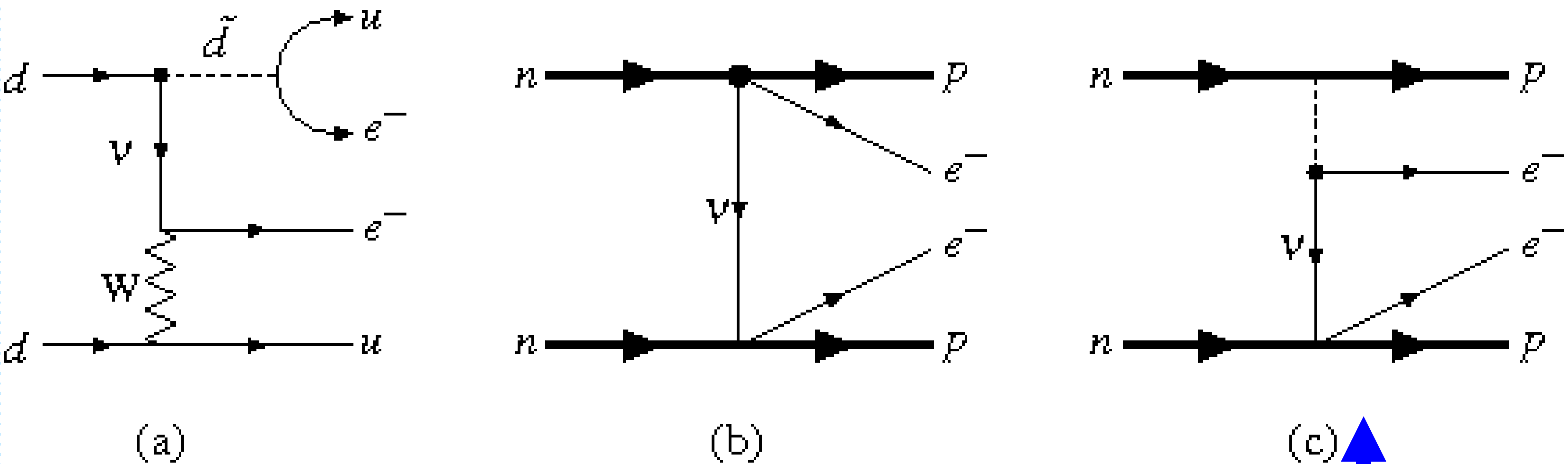


$$\acute{\lambda}_{111} = 1.3\ 10^{-4} \left(\frac{m_{\tilde{q}}}{100\ GeV}\right)^2 \left(\frac{m_{\tilde{g}}}{100\ GeV}\right)^{1/2}$$

Squark mixing SUSY mechanism

Mixing between scalar superpartners of the left- and right-handed fermions

$$M^2_{\tilde{d}^k} = \begin{pmatrix} m^2_{\tilde{d}^k_L} + m^2_{d^k} - \frac{1}{6}(2m^2_W + m^2_Z)\cos 2\beta & -m_{d^k}((\mathbf{A}_D)_{kk} + \mu \tan\beta) \\ -m_{d^k}((\mathbf{A}_D)_{kk} + \mu \tan\beta) & m^2_{\tilde{d}^k_R} + m^2_{d^k} + \frac{1}{3}(m^2_W - m^2_Z)\cos 2\beta \end{pmatrix}$$



Hirsch,
Klapdor-Kleingrothaus,
Kovalenko
PLB 372 (1996) 181

A. Faessler,
Th. Gutsche,
S. Kovalenko,
F.Š.,
PRD 77, 113012 (2008)

Effective SUSY ν-e Lagrangian

Neutrino vertex

$$\mathcal{L}^{LH} = \frac{G_F}{\sqrt{2}} \sum_i U_{ei} \, (\bar{e}\gamma_\alpha(1-\gamma_5)\nu) \, (\bar{u}\gamma^\alpha(1-\gamma_5)d) + h.c. \quad (V-A)$$

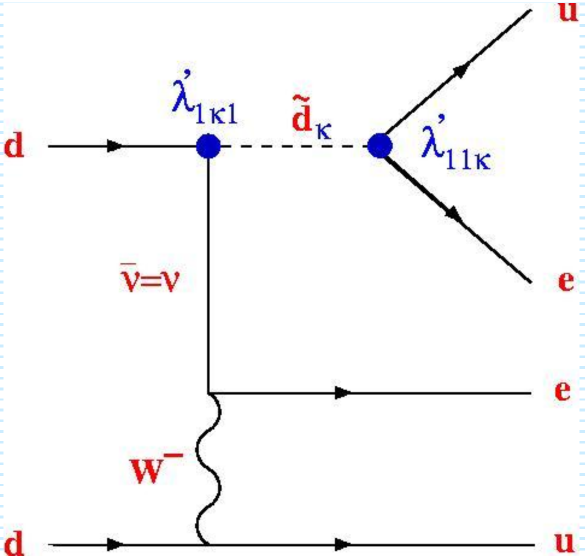
R-parity violating SUSY vertex

Hirsch, Klapdor-Kleingrothaus, Kovalenko
PLB 372 (1996) 181

$$\mathcal{L}^{eff}_{SUSY} = \frac{G_F}{\sqrt{2}} \left(\frac{1}{4}\eta_{(q)LR} \sum_i U^*_{ei} \, (\bar{\nu}(1+\gamma_5)e) \, (\bar{u}(1+\gamma_5)d) \right. \quad (S,P)$$

$$\left. + \frac{1}{8}\eta_{(q)LR} \sum_i U^*_{ei} \, (\bar{\nu}\sigma_{\alpha\beta}(1+\gamma_5)e) \, (\bar{u}\sigma^{\alpha\beta}(1+\gamma_5)d) + h.c. \right) \quad (Tensor)$$

Paes, Hirsch, Klapdor-Kleingrothaus,
PLB 459 (1999) 450



LN-violating parameter

$$\eta_{(q)LR} = \sum_k \frac{\lambda'_{11k}\lambda'_{1k1}}{8\sqrt{2}G_F} \sin 2\theta^d_{(k)} \left(\frac{1}{m^2_{\tilde{d}_1(k)}} - \frac{1}{m^2_{\tilde{d}_2(k)}} \right)$$

Limits on R-breaking parameters

TABLE II: Nuclear matrix elements (NMEs) of the squark-neutrino $\not{R}_p$ SUSY mechanism of $0\nu\beta\beta$-decay. The NMEs of the 2N-mode are calculated for the two cases of the nucleon form factors: Quark Bag Model (QBM) and Non-Relativistic Quark Model (NRQM). The quantities M_{2N}, M_π are the 2N and pion mode nuclear matrix elements averaged over small, medium and large model spaces (see the text) with their variance σ given in parentheses.

	QBM				NRQM				
nucl.	$M^{\tilde q}_{VT}$	$M^{\tilde q}_{MT}$	$M^{\tilde q}_{AP}$	$M^{\tilde q}_{2N}$	$M^{\tilde q}_{VT}$	$M^{\tilde q}_{MT}$	$M^{\tilde q}_{AP}$	$M^{\tilde q}_{2N}$	$M^{\tilde q}_{\pi}$
^{76}Ge	-46.2	61.5	14.8	27.8 (4.6)	-25.5	64.6	15.6	52.4 (2.7)	302. (37)
^{100}Mo	-54.9	61.0	16.5	22.9 (1.8)	-30.3	64.1	17.4	51.0 (0.3)	297. (40)
^{130}Te	-44.9	51.6	14.2	19.3 (3.4)	-24.8	54.2	14.9	42.4 (2.6)	257. (16)

2n mode

Pion mode

TABLE III: Upper bounds on the $\not{R}_p$ SUSY parameter $\eta^{11}_{(q)LR}$ as well as on the related products of the trilinear $\not{R}_p$-couplings $\lambda'_{11k}\lambda'_{1k1}$ (k=1,2,3) for $\Lambda_{SUSY} = 100$ GeV (see scaling law in Eq. (37)) deduced from the current lower bounds on the half-life of $0\nu\beta\beta$-decay for ^{76}Ge, ^{100}Mo and ^{130}Te.

nucl.	$T^{0\nu-exp}_{1/2}$ [Ref.] (years)	$\eta^{11}_{(q)LR}$	$\lambda'_{111}\lambda'_{111}$	$\lambda'_{112}\lambda'_{121}$	$\lambda'_{113}\lambda'_{131}$
^{76}Ge	$\geq 1.9\ 10^{25}$ [2]	$8.5\ 10^{-9}$	$1.5\ 10^{-5}$	$8.0\ 10^{-7}$	$3.3\ 10^{-8}$
^{100}Mo	$\geq 5.8\ 10^{23}$ [4]	$1.8\ 10^{-8}$	$3.2\ 10^{-5}$	$1.7\ 10^{-6}$	$7.0\ 10^{-8}$
^{130}Te	$\geq 3.0\ 10^{24}$ [5]	$9.5\ 10^{-9}$	$1.7\ 10^{-5}$	$9.0\ 10^{-7}$	$3.7\ 10^{-8}$

A. Faessler,
Th. Gutsche,
S. Kovalenko,
F.Š.,
PRD 77, 113012 (2008)

The $0\nu\beta\beta$-decay is a nuclear physics problem

In double beta decay two neutrons bound in the ground state of an initial even-even nucleus are simultaneously transformed into two protons that are bound in the ground state or excited (0^+, 2^+) states of the final nucleus

It is necessary to evaluate, with a sufficient accuracy, wave functions of both nuclei, and evaluate the matrix element of the $0\nu\beta\beta$-decay operator connecting them

This can not be done exactly, some approximation and/or truncation is always needed. Moreover, there is no other analogues observable that can be used to judge directly the quality of the result.

The 0νββ-decay NME (light ν exchange mech.)

The 0νββ-decay half-life

$$\frac{1}{T_{1/2}} = G^{0\nu}(E_0, Z)|M'^{0\nu}|^2|\langle m_{\beta\beta}\rangle|^2 \ ,$$

NME= sum of Fermi, Gamow-Teller and tensor contributions

$$M'^{0\nu} = \left(\frac{g_A}{1.25}\right)^2 \ \langle f| - \frac{M_F^{0\nu}}{g_A^2} + M_{GT}^{0\nu} + M_T^{0\nu}|i\rangle$$

Neutrino potential (about 1/r$_{12}$)

$$H_K(r_{12}) = \frac{2}{\pi g_A^2} R \int_0^\infty \ f_K(q r_{12}) \ \frac{h_K(q^2) q dq}{q + E^m - (E_i + E_f)/2}$$

$$f_{F,GT}(q r_{12}) = j_0(q r_{12}), \qquad f_T(q r_{12}) = -j_2(q r_{12})$$

Form-factors:
finite nucleon size

$$\begin{aligned} h_F &= g_V^2(q^2) \\ h_{GT} &= g_A^2\left[1 - \frac{2}{3}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2} + \frac{1}{3}\left(\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\right)^2\right] \\ h_T &= g_A^2\left[\frac{2}{3}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2} - \frac{1}{3}\left(\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\right)^2\right] \end{aligned}$$

Induced pseudoscalar coupling
(pion exchange)

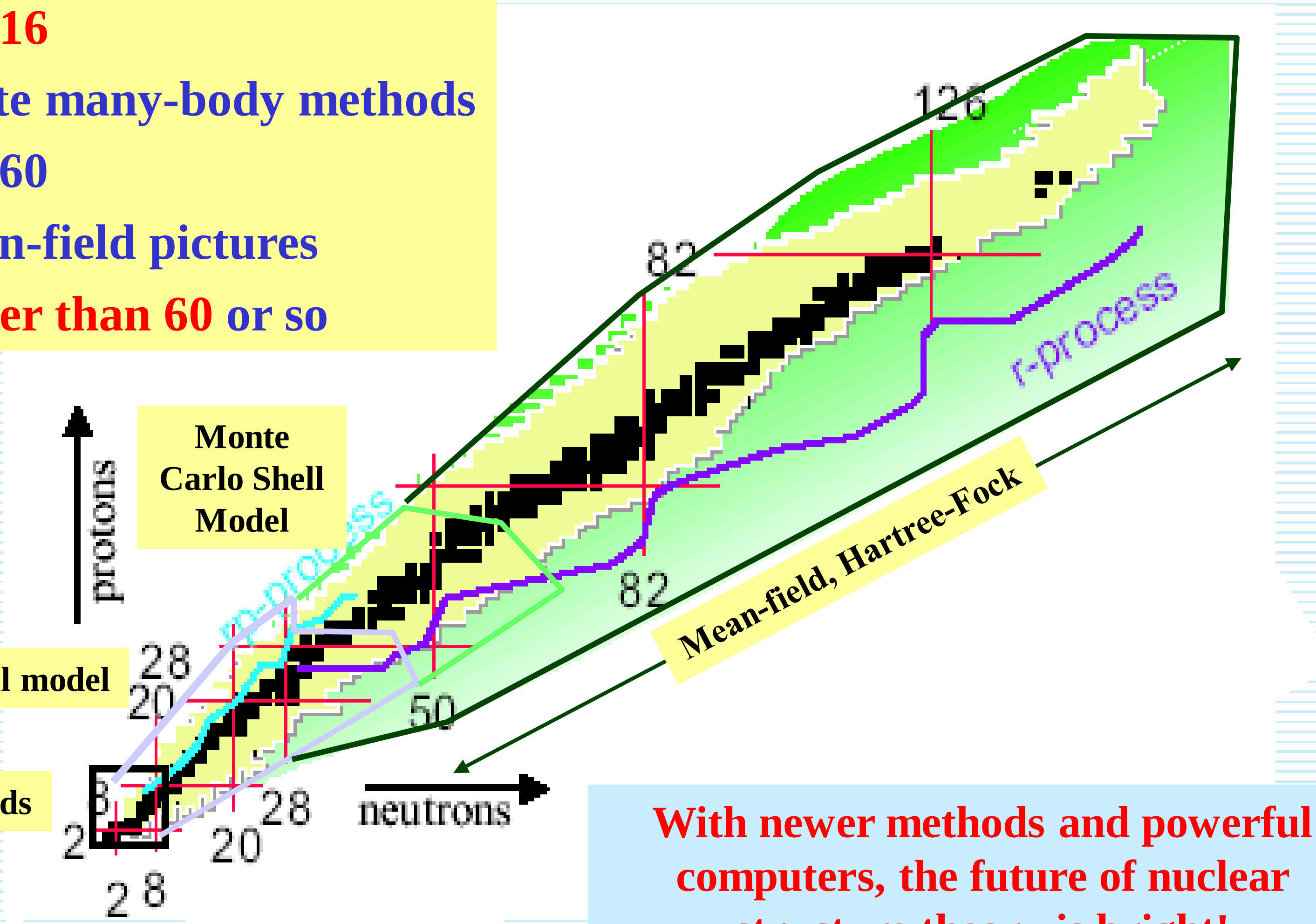
$$M_{K=F,GT,T} = \sum_{J^\pi,k_i,k_f,\mathcal{J}} \ \sum_{pnp'n'} (-1)^{j_n+j_{p'}+J+\mathcal{J}} \sqrt{2\mathcal{J}+1} \left\{ \begin{array}{ccc} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{array} \right\}$$
$$\langle p(1), p'(2); \mathcal{J} \parallel f(r_{12}) O_K f(r_{12}) \parallel n(1), n'(2); \mathcal{J}\rangle$$
$$\times \langle 0_f^+ || [c_{p'}^+ \tilde{c}_{n'}]_J || J^\pi k_f\rangle \langle J^\pi k_f | J^\pi k_i\rangle \langle J^\pi k_f || [c_p^+ \tilde{c}_n]_J || 0_i^+\rangle$$

Jastrow f. s.r.c.

J$^\pi$ =
$0^+, 1^+, 2^+ \ldots$
$0^-, 1^-, 2^- \ldots$

Nuclear Structure

- Exact methods exist up to A=4
- Computationally exact methods for A up to 16
- Approximate many-body methods for A up to 60
- Mostly mean-field pictures for A greater than 60 or so

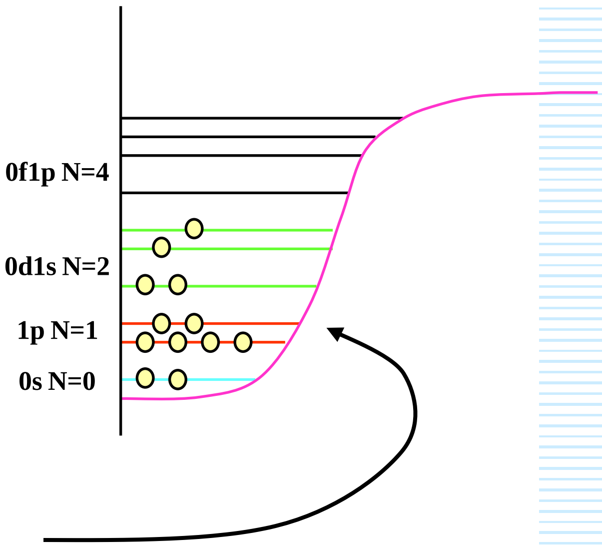


With newer methods and powerful computers, the future of nuclear structure theory is bright!

Many-body Hamiltonian

- **Start with the many-body Hamiltonian**

$$H = \sum_i \frac{\vec{p}_i^{\,2}}{2m} + \sum_{i<j} V_{NN}\left(\vec{r}_i - \vec{r}_j\right)$$



- **Introduce a mean-field U to yield basis**

$$H = \sum_i \left(\frac{\vec{p}_i^{\,2}}{2m} + U(\vec{r}_i) \right) + \underbrace{\sum_{i<j} V_{NN}\left(\vec{r}_i - \vec{r}_j\right) - \sum_i U(\vec{r}_i)}_{\text{Residual interaction}}$$

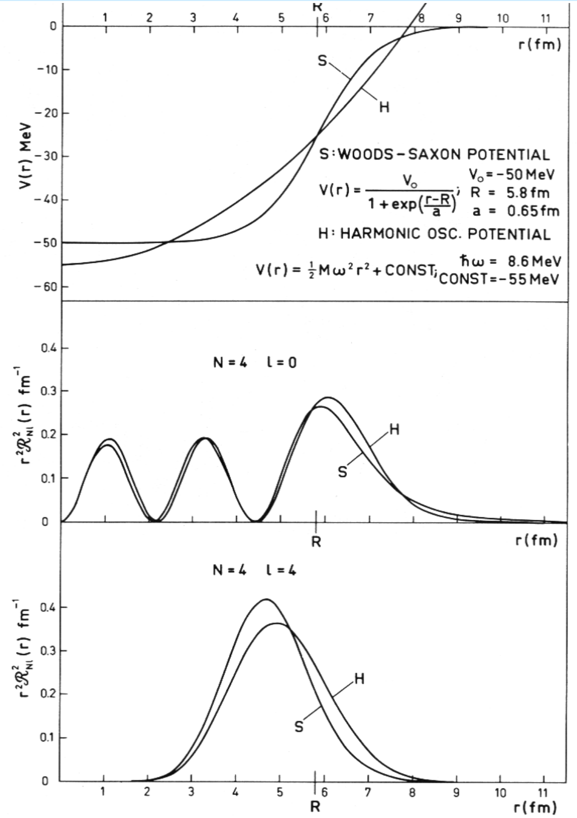
The success of any nuclear structure calculation depends on the choice of the mean-field basis and the residual interaction!

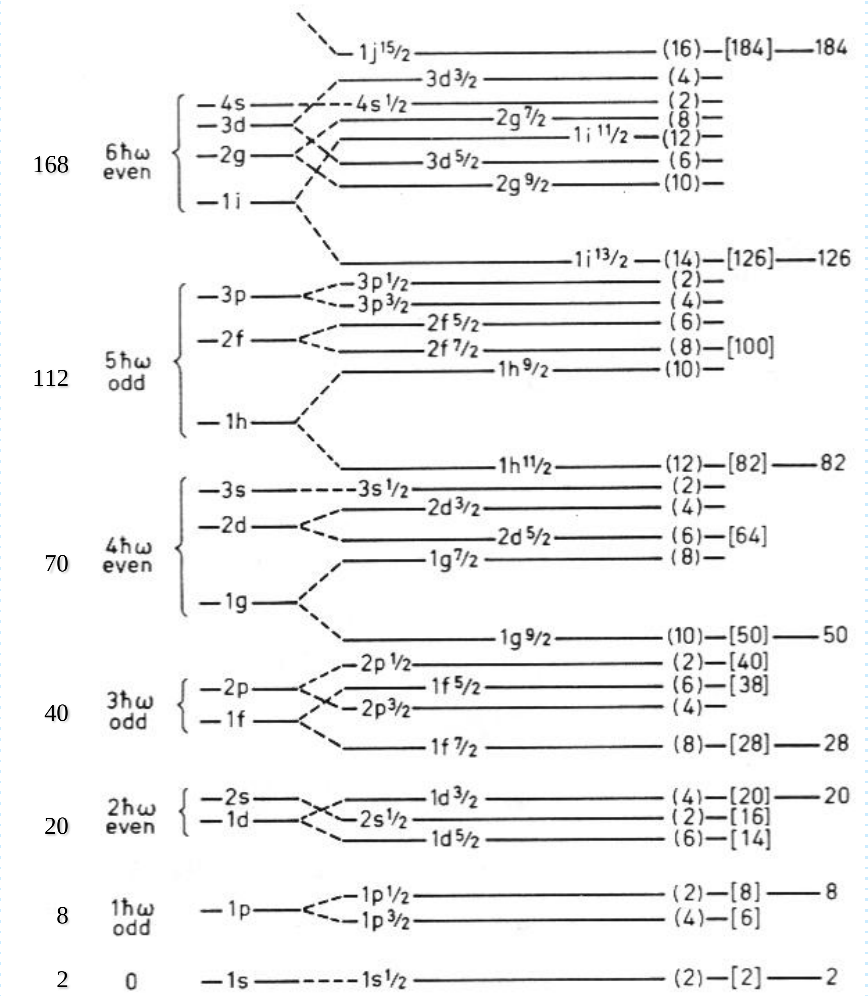
- **The mean field determines the shell structure**
- **In effect, nuclear-structure calculations rely on perturbation theory**

Goeppert-Mayer and Haxel, Jensen, and Suess proposed the independent-particle shell model to explain the magic numbers

Origin of the shell model

Harmonic oscillator with spin-orbit is a reasonable approximation to the nuclear mean field





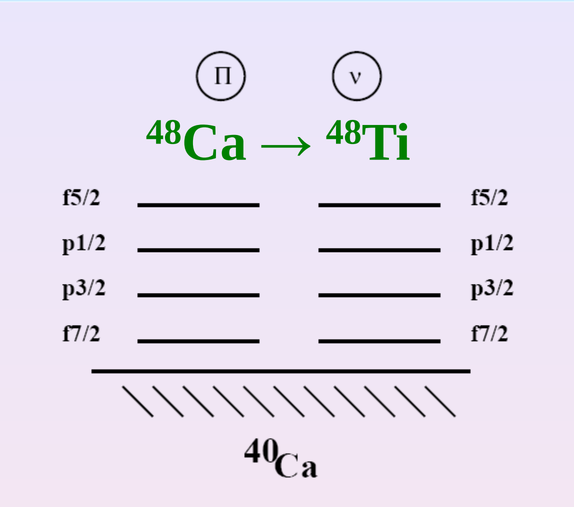
M.G. Mayer and J.H.D. Jensen, *Elementary Theory of Nuclear Shell Structure*, p. 58, Wiley, New York, 1955

Nuclear Shell Model

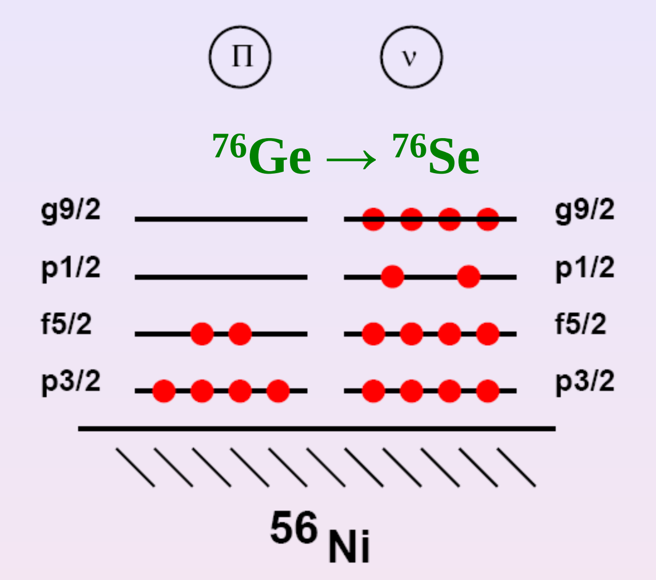
In NSM a limited valence space is used but all configurations of valence nucleons are included. Describes well properties of low-lying nuclear states. Technically difficult, thus only few $0\nu\beta\beta$-decay calculations

- **Define a valence space**
- **Derive an effective interaction $H\,\Psi = E\,\Psi \rightarrow H_{eff}\,\Psi_{eff} = E\,\Psi_{eff}$**
- **Build and diagonalize Hamiltonian matrix (10^{10})**
- **Transition operator $<\Psi_{eff}\,|\,O_{eff}\,|\,\Psi_{eff}>$**
- **Phenomenological input: Energies of states, systematics of B(E2) and GT trans.**

$$H = \sum_a \varepsilon_a a_a^+ a_a - \sum_{abcd} \frac{\langle j_a j_b; JT|V|j_c j_d; JT\rangle_A}{\sqrt{(1+\delta_{ab})(1+\delta_{cd})}} \left[\left[a_a^+ \otimes a_b^+\right]^{JT} \otimes \left[\tilde{a}_c \otimes \tilde{a}_d\right]^{JT}\right]^{00}_{00}$$



Small calculations

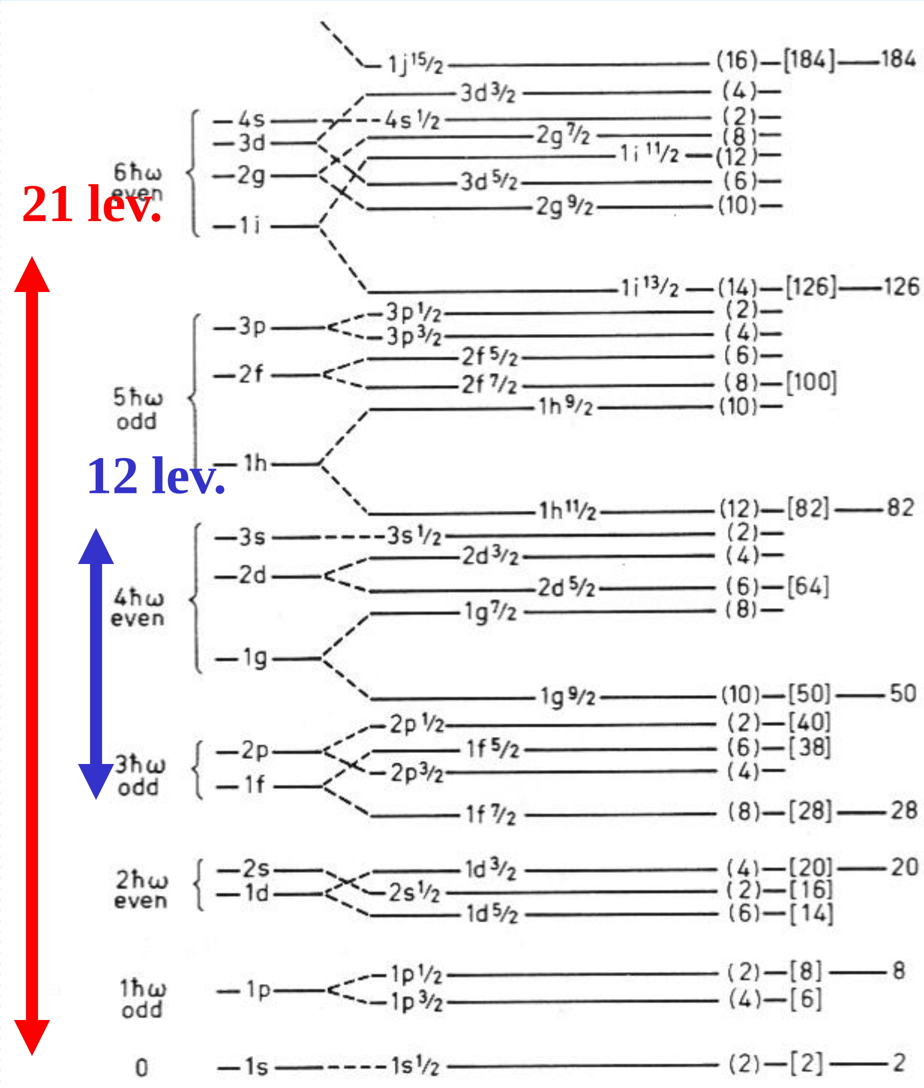


$^{76}Se_{42}$ in the valence

6 protons and 14 neutrons

Quasiparticle Random Phase Approximation (QRPA)

In QRPA a large valence space is used, but only a class of configurations is included. Describe collective states, but not details of dominantly few particle states. Relative simple, thus more 0nbb-decay calculations



- **Large model space** (up 23 s.p.l, ^{150}Nd – 60 active prot. and 90 neut.)
- **Spin-orbit partners included**
- **Possibility to describe all multipolarities of the intermed. nucl. J^{π} ($\pi=\pm1$, J=0…9)**

$$H = H_0 + g_{ph} V_{ph} + g_{pp} V_{pp}$$

quasiparticle mean field

Residual interaction

The Interacting Boson Model¶

- The low-lying states of the nucleus, composed by n and z valence nucleons, are modeled in terms of (n+z)/2 bosons.
- The bosons have either L = 0 (*s* boson) or L = 2 (*d* boson).
- The bosons can interact through one-body and two-body forces giving rise to bosonic wave functions.
- Any observable can be calculated using these wave functions provided that the relevant operator is employed.

¶ F. Iachello and A. Arima, *The Interacting Boson Model*, Cambridge University Press, 1987

Projected Hartree-Fock-Bogoliubov Model

PHFB Model

States of good angular momentum J

$$\left|\Psi_M^J\right\rangle = \frac{2J+1}{8\pi^2 a_J}\int d\Omega\, D_{MK}^J(\Omega)\hat{R}(\Omega)\left|\Phi_K\right\rangle$$

Axially symmetric HFB intrinsic state

$$\left|\Phi_0\right\rangle = \prod_{im}\left(u_{im} + v_{im} b_{im}^{+} b_{i\bar{m}}^{+}\right)$$

where

$$b_{im}^{+} = \sum_m C_{i\alpha m} a_{im}^{+} \qquad b_{i\bar{m}}^{+} = \sum_m (-1)^{l+j-m} C_{i\alpha m} a_{i-m}^{+}$$

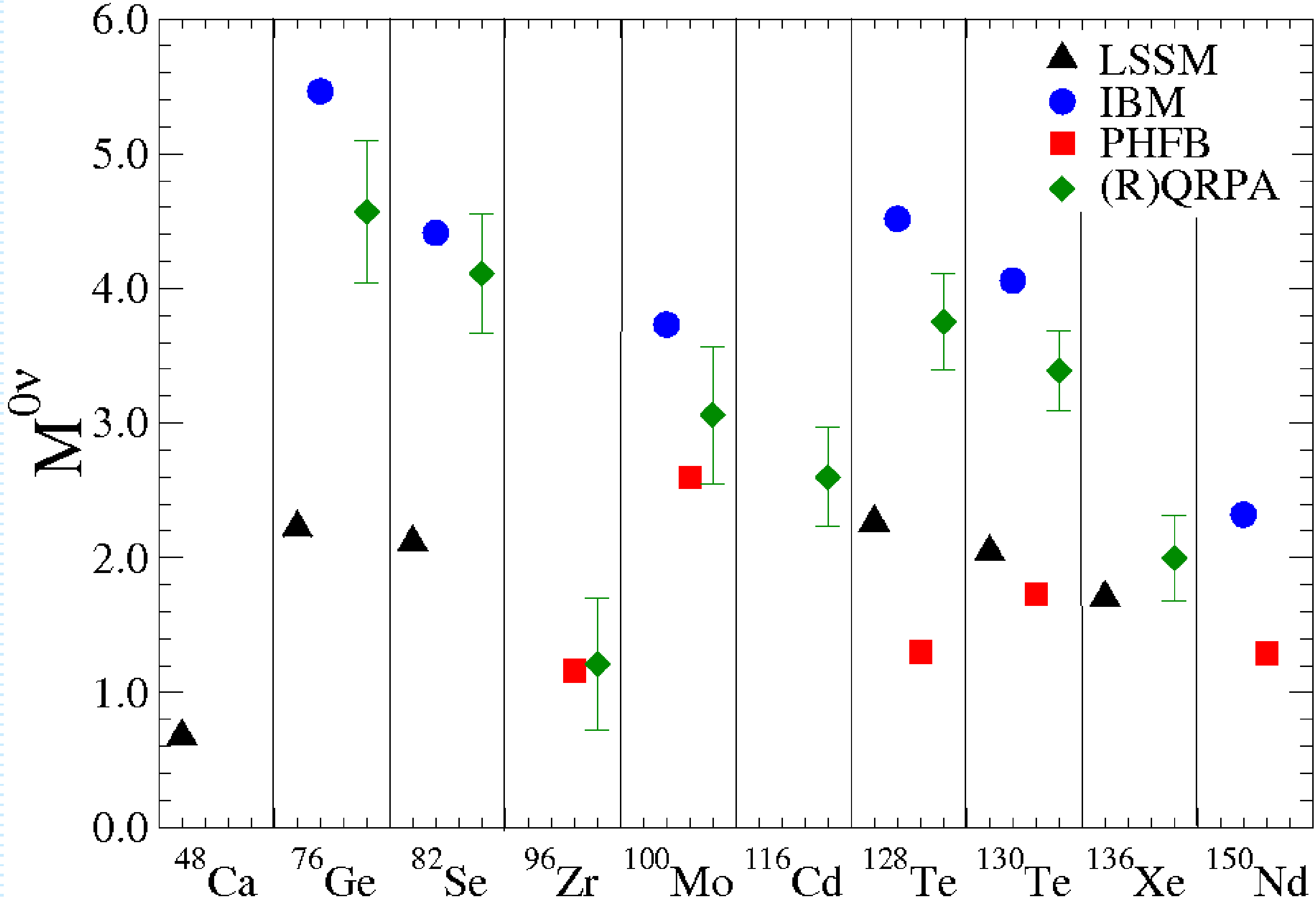
Hamiltonian:

$$H = H_{sp} + V(P) + \zeta_{qq} V(QQ)$$

Only quadrupole interaction,
GT interaction is missing

The 0νββ-decay NMEs (Status:2010)

Differences:
i) mean field;
ii) residual int.;
iii) size of the m.s.
iv) many-body appr.



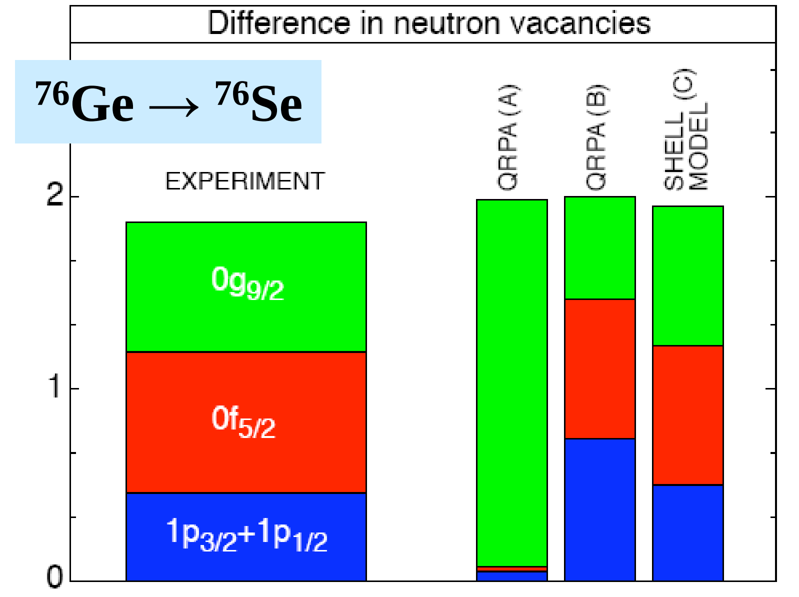
Nobody is perfect:

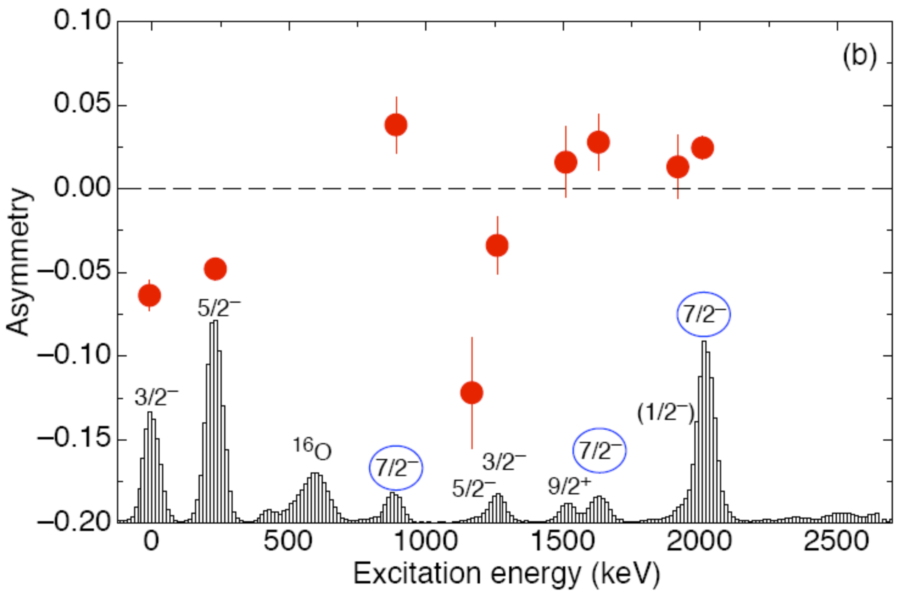
LSSM (small m.s., negative parity states)
PHFB (GT force neglected)
IBM (Hamiltonian truncated)
(R)QRPA (g.s. correlations not accurate enough)

Constraining the mean field with proton, neutron removing transfer reaction

Schiffer et al., PRL 100, 112501 (2008)

$$n_j^{exp} = \langle 0^+_{init} | \Sigma_m c^+_{j,m} c_{j,m} | 0^+_{init} \rangle$$





Adjusted WS mean field

	^{76}Ge				^{76}Se			
neut.	BCS	Q	S	exp	BCS	Q	S	exp
p	5.65	5.27	4.64	4.9±0.2	5.57	5.05	4.12	4.4±0.2
$f_{5/2}$	5.54	5.12	4.34	4.6±0.4	5.53	5.00	3.63	3.8±0.4
$f_{7/2}$	7.91	7.67	7.62	-	7.90	7.54	7.37	-
$s_{1/2}$	0.01	0.05	0.07	-	0.01	0.04	0.08	-
$d_{3/2}$	0.03	0.14	0.15	-	0.02	0.14	0.16	-
$d_{5/2}$	0.09	0.30	0.36	-	0.07	0.27	0.39	-
$g_{7/2}$	0.14	0.53	0.48	-	0.12	0.56	0.58	-
$g_{9/2}$	4.63	4.78	6.35	6.5±0.3	2.78	3.55	5.66	5.8±0.3
prot.								
p	2.23	2.34	1.75	1.77±0.15	2.77	2.76	2.28	2.08±0.15
$f_{5/2}$	1.61	2.27	2.08	2.04±0.25	2.95	2.97	3.03	3.16±0.25
$f_{7/2}$	7.83	7.19	7.13	-	7.76	7.12	7.06	-
$s_{1/2}$	0.00	0.02	0.03	-	0.00	0.03	0.04	-
$d_{3/2}$	0.01	0.07	0.07	-	0.01	0.09	0.09	-
$d_{5/2}$	0.01	0.12	0.15	-	0.02	0.17	0.18	-
$g_{7/2}$	0.02	0.19	0.16	-	0.03	0.31	0.27	-
$g_{9/2}$	0.29	0.85	0.62	0.23±0.25	0.46	1.15	1.04	0.84±0.25

Reduction of NME within the SRQRPA

$^{76}Ge \to {}^{76}Se$	prev.	new
Jastrow s.r.c.	4.24(0.44)	3.49(0.23)
UCOM s.r.c.	5.19(0.54)	4.60(0.39)

F.Š., A. Faessler, P. Vogel, PRC 79, 015502 (2009)

Realistic NN-interactions used in the QRPA calculations

Modern *(phase-shift equivalent) NN* potentials

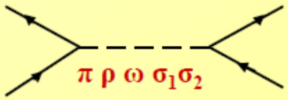
Nijmegen I - (P_D = 5.66%) - 41 parameters - χ^2/N_{data} = 1.03

Nijmegen II - (P_D = 5.64%) - 47 parameters - χ^2/N_{data} = 1.03

Argonne V_{18} - (P_D = 5.76%) - 40 parameters - χ^2/N_{data} = 1.09

CD Bonn - (P_D = 4.85%) - 43 parameters - χ^2/N_{data} = 1.02

based upon the OBE model



(1999 *NN* Database: 5990 *pp* and *np* scattering data)

Brueckner G-matrices from Tuebingen (H. Muether group)

Renormalization of the *NN* interaction

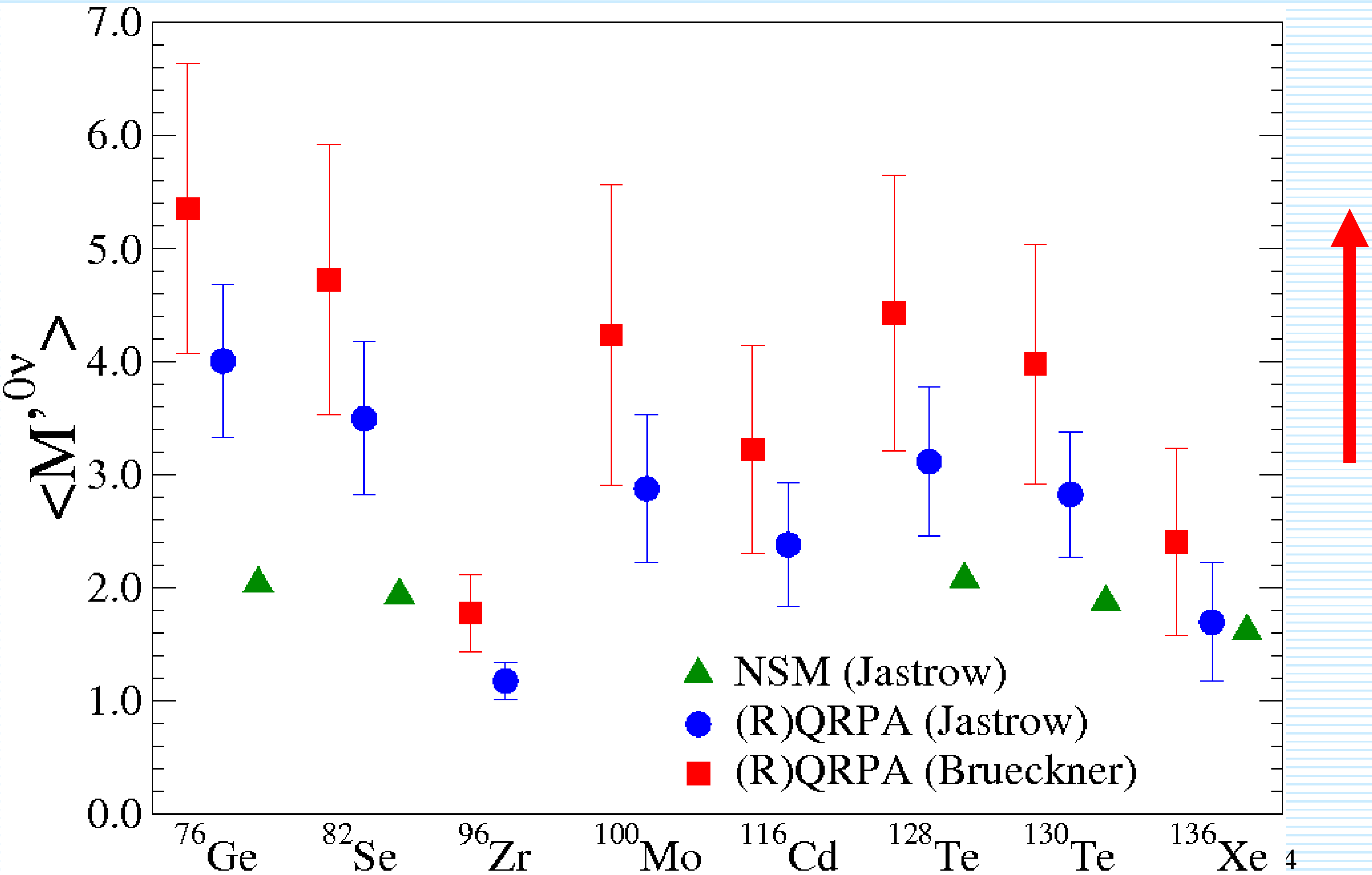
Difficulty in the derivation of V_{eff} from any modern *NN* potential: existence of a strong repulsive core which prevents its direct use in nuclear structure calculations.

Traditional approach to this problem: Brueckner *G*-matrix method. The *G* matrix is model-space dependent as well as energy dependent.

Bethe-Goldstone equation

$$G = V + V\frac{Q}{W - H_0 + i\varepsilon}G$$

The value of the $0\nu\beta\beta$-decay NME calculated with consistent treatment of s.r.c. is increased



4

F.Š., Faessler, Muether, Rodin, Stauf, PRC 79, 055501 (2009)

There is a need for supporting experiments

Nuclear matrix elements:

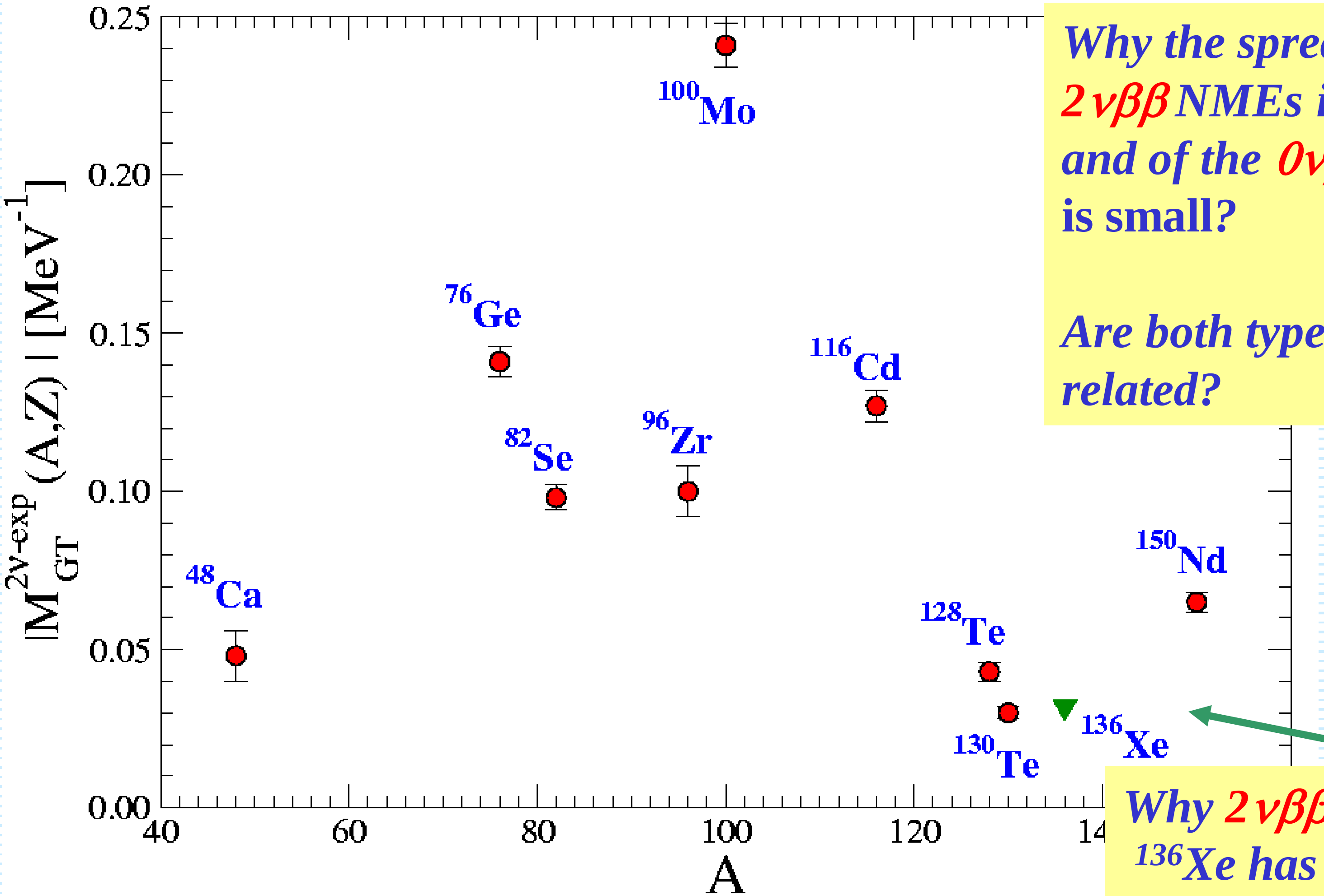
• *Mean field*	*p and n removing transfer reactions*
• *β^- and β^+ strengths*	*Charge-changing reactions and muon capture*
• *deformation*	*Exp. to remeasure deformetion needed*
• *$2\nu\beta\beta$-decay*	*Double beta decay experiments*

On the relation between $0\nu\beta\beta$-decay and $2\nu\beta\beta$-decay (GT) NMEs

$$M^{0\nu} = M^{0\nu}_{GT}\left(1 + \frac{1}{g_A^2}\frac{M^{0\nu}_F}{M^{0\nu}_{GT}} + \frac{M^{0\nu}_T}{M^{0\nu}_{GT}}\right)$$

2νββ-decay NMEs

$$\frac{1}{T_{1/2}^{2\nu-exp}} = G^{2\nu}(E_0, Z)\ g_A^4\ |M_{GT}^{2\nu}|^2$$



Why the spread of the 2νββ NMEs is large and of the 0νββ* NMEs *is small?

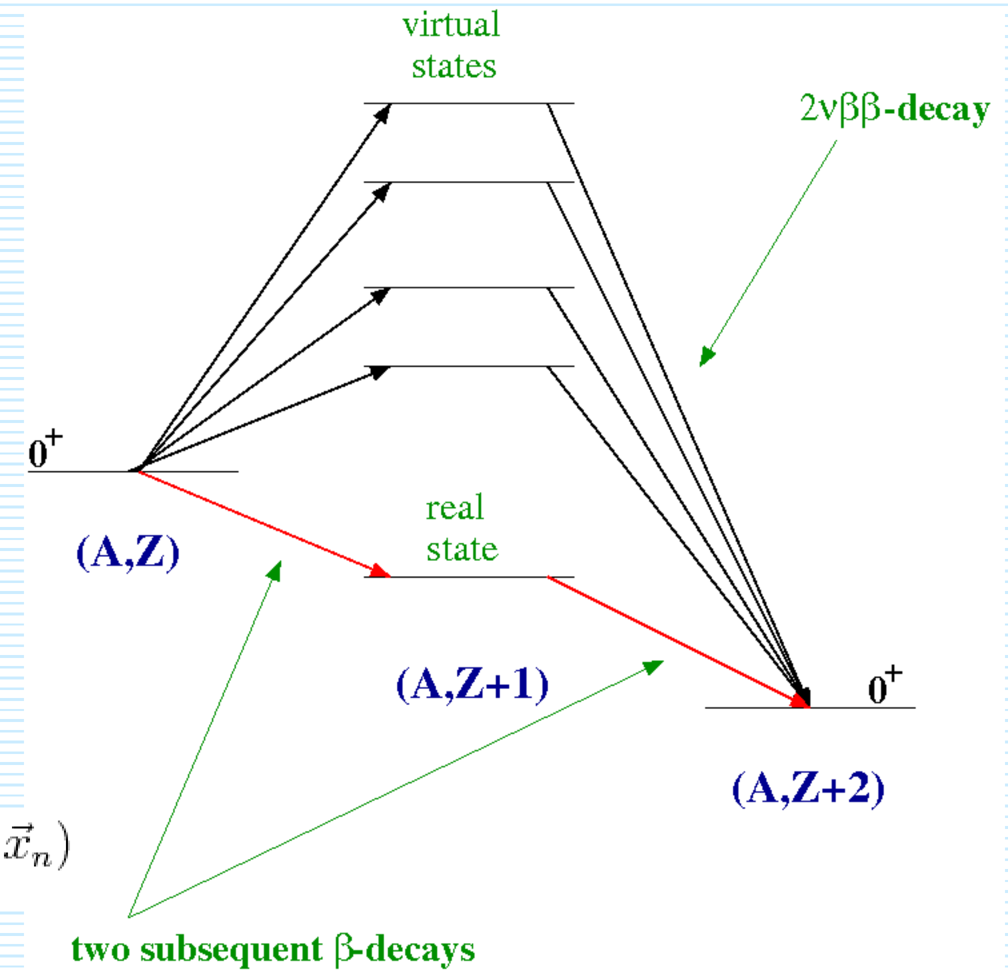
Are both type of NMEs related?

***Why 2νββ of ^{136}Xe has been not observed yet? Do this affect the value of 0νββ* NME**

Differencies among 2νββ-decay NMEs: up to factor 10

$$T(J_\mu(x_1)J_\nu(x_2)) = J_\mu(x_1)J_\nu(x_2) \qquad (two\ \beta - decays)$$
$$+ \ \Theta(x_{20}-x_{10})[J_\nu(x_2), J_\mu(x_1)] \qquad (2\nu\beta\beta - decay)$$

A sum over intermediate nuclear states represents a sum over all meson and gamma exchange correlations of two beta decaying nucleons



$$J_\alpha(0,\vec{x}) = \sum_n \tau_n^+(\delta_{\alpha 4} + i g_A(\vec{\sigma})_k \delta_{\alpha k})\delta(\vec{x}-\vec{x}_n)$$

$$J^{2\beta 2\nu}_{\mu\nu}(p_1,p_2,k_1,k_2) = -i2M_{GT}\delta_{\mu k}\delta_{\nu k}$$
$$\times 2\pi\delta(E_f - E_i + p_{10} + k_{10} + p_{20} + k_{20}), \ k = 1,2,3,$$

Integral representation of M_{GT}

$$M_{GT} = \frac{i}{2}\int_0^\infty \left(e^{i(p_{10}+k_{10}-\Delta)t} + e^{i(p_{20}+k_{20}-\Delta)t}\right) M_{AA}(t)\,dt$$

$with$

$$M_{AA}(t) = < 0_f^+ | \frac{1}{2}[A_k(t/2), A_k(-t/2)] | 0_i^+ >$$

$$A_k(t) = e^{iHt} A_k(0) e^{-iHt}, \qquad A_k = \sum_i \tau_i^+ (\vec{\sigma}_i)_k, \;\; k = 1, 2, 3.$$

$$A_k(t) = e^{itH} A_k(0) e^{-itH} = \sum_{n=0}^{\infty} \frac{(it)^n}{n!} \overbrace{[H[H...[H}^{n\ times}, A_k(0)]...]]$$

Completeness:
Σ_n |n><n|=1

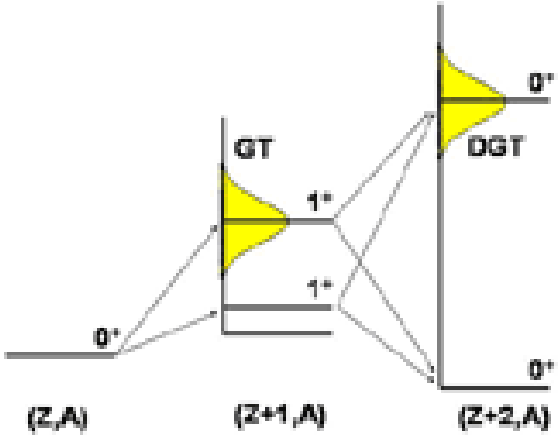
$$< A'|J_\alpha(x_1) J_\beta(x_2)|A > \;=\; \sum_n < A'|J_\alpha(0,\vec{x}_1)|n >< n|J_\beta(0,\vec{x}_2)|A > \times e^{-i(E'-E_n)x_{10}} e^{-i(E_n-E)x_{20}}$$

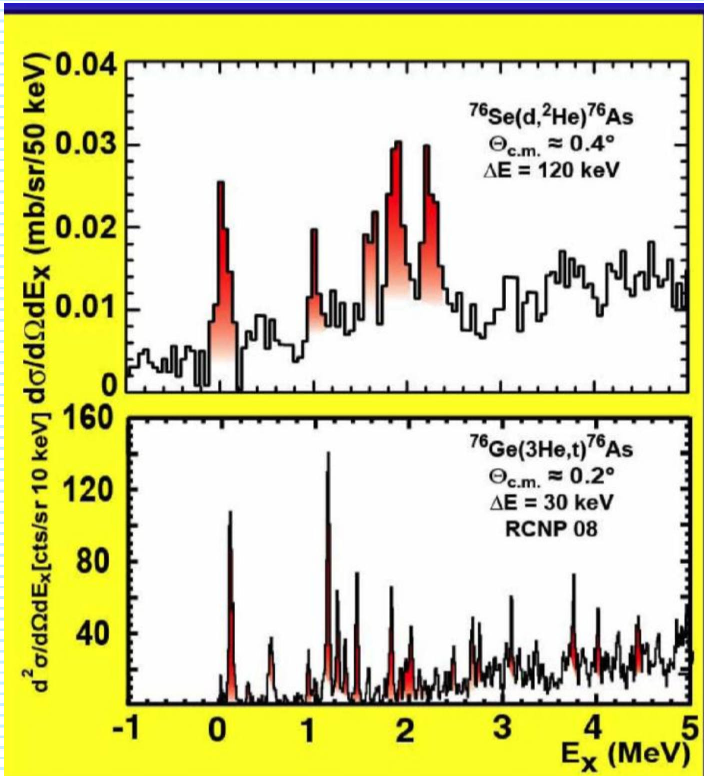
$$\int_0^\infty e^{-iat} dt \Rightarrow \lim_{\epsilon\to 0} \int_0^\infty e^{-i(a-i\epsilon)t} dt = \lim_{\epsilon\to 0} \frac{-i}{a - i\epsilon}$$

$$M_{GT} = \sum_n \frac{< 0_f^+ |A(0)_k| 1_n^+ >< 1_n^+ |A(0)_k| 0_i^+ >}{E_n - E_i + \Delta}$$

***The cross sections of* (t,^{3}He) *and* (d,^{2}He) *reactions give* B(GT$^\pm$) *for* β^+ *and* β^-*, product of the amplitudes* (B(GT)$^{1/2}$) *entering the numerator of* $M^{2\nu}_{GT}$**

$$M^{2\nu}_{GT} = \sum_m \frac{M^{(+)}_{GT}(m)\; M^{(-)}_{GT}(m)}{Q_{\beta\beta}/2 + m_e + E_x(1^+_m) - E_0}$$





2νββ-matrix element

0.16 ± 0.04 MeV^{-1}

with

G$^{(2\nu)}$ = 3.4x 10^{-20} MeV^2 a^{-1}

2νββ - half-life

(1.1 ± 0.2) x 10^{21} a

recommended. exp. value:

(1.5 ± 0.1) x 10^{21} a

Closure 2νββ-decay NME

$$M^{2\nu}_{GT-cl} = \sum_m M^{(+)}_{GT}(m)\; M^{(-)}_{GT}(m)$$

SSD hypothesis

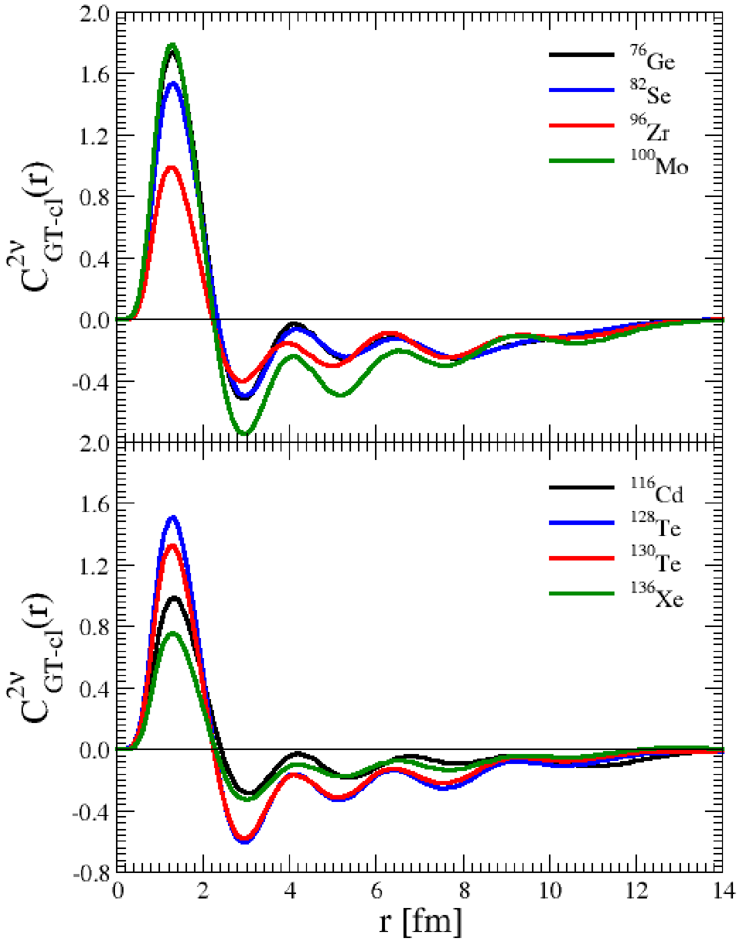
$$g_A^2\; M^{2\nu}_{GT-cl} = \frac{3\;D}{\sqrt{ft_{EC}\,ft_{\beta^-}}}$$

Grewe, …Frekers at al, PRC 78, 044301 (2008)

Going to relative coordinates:

$$M^{2\nu}_{GT-cl} = \int_0^\infty C^{2\nu}_{GT-cl}(r)dr$$

***r**- relative distance of two nucleons*

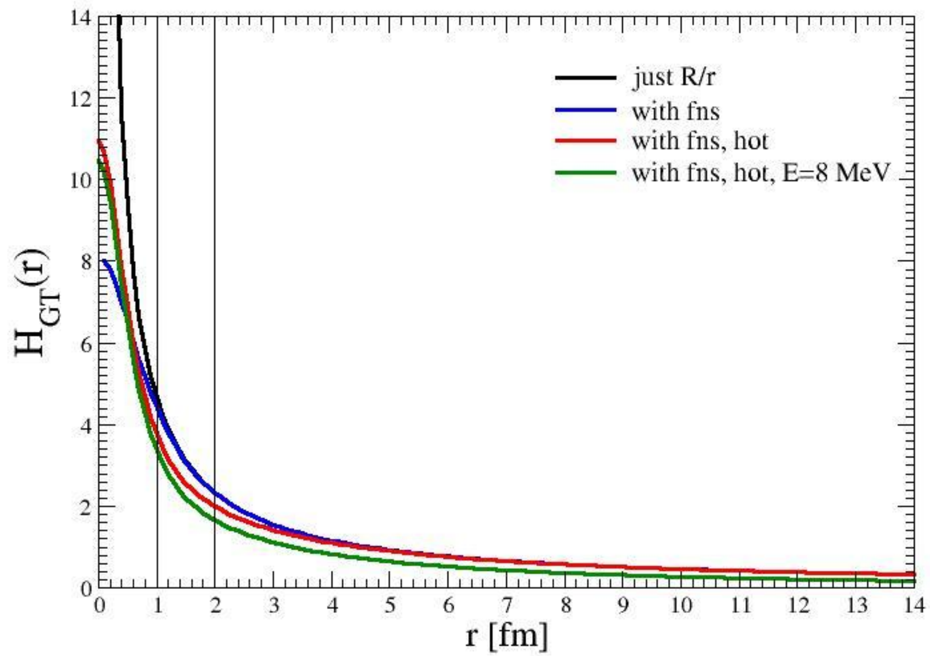


A connection between closure $2\nu\beta\beta$ and $0\nu\beta\beta$ GT NMEs

$$M^{0\nu}_{GT} = \int_0^\infty H^{0\nu}_{GT}(r)C^{2\nu}_{GT-cl}(r)dr$$

Neutrino potential

$$H(r) = R\frac{2}{\pi}\int_0^\infty j_0(qr)\frac{q}{q+\overline{E}}f^2_{FNS}(q^2)g_{HOT}(q^2)dr$$



Neutrino potential prefer short distances

Phenomenological estimation of $M^{0\nu}_{GT}$

Nucleus	$T^{2\nu-exp}_{1/2}$ [y] [years]	$\lvert g_A^2 M^{2\nu-exp}_{GT}\rvert$ $[MeV^-1]$	SSD $\lvert g_A^2 M^{2\nu}_{GT-cl}\rvert$	SSD $\lvert g_A^2 M^{0\nu-ph}\rvert$	ChER $\lvert M^{2\nu}_{GT}\rvert$ $[MeV^-1]$	ChER $\lvert M^{2\nu}_{GT-cl}\rvert$	ChER $\lvert M^{0\nu-ph}\rvert$
^{48}Ca	4.4×10^{19}	0.0735	-	-	0.083	0.355	3.19
^{76}Ge	1.5×10^{21}	0.219	-	-	0.159	0.840	8.80
^{96}Zr	2.3×10^{19}	0.145	-	-	-	0.357	4.04
^{100}Mo	7.1×10^{18}	0.373	0.564	6.47	-	-	-
^{116}Cd	2.8×10^{19}	0.203	0.562	6.78	0.064	0.491	5.92

Neutrino potential

$$H(r) = R\frac{2}{\pi}\int_0^\infty j_0(qr)\frac{q}{q+\overline{E}}f^2_{FNS}(q^2)g_{HOT}(q^2)dr$$

$$\begin{aligned} M^{0\nu}_{GT} &= H_{GT}(r=0)\ M^{2\nu}_{GT-cl} \\ &\quad - \int_0^\infty \mathcal{F}(r)C^{2\nu}_{GT-cl}(r)dr \\ &= M^{0\nu-ph}_{GT} - M^{0\nu-rest}_{GT} \end{aligned}$$

with Taylor expansion

$$\begin{aligned} j_0(qr) &= 1 - \frac{1}{6}(qr)^2 + \frac{1}{120}(qr)^4 - \cdots \\ &= 1 - \mathcal{F}(r) \end{aligned}$$

A: Phenomen. prediction:
Too large
(~ factor 2)

B: Need to be calculated
Not negligable

There is no proportionality between $M^{0\nu}_{GT}$ and $M^{2\nu}_{GT\text{-}cl}$

Closure $2\nu\beta\beta$ GT NME

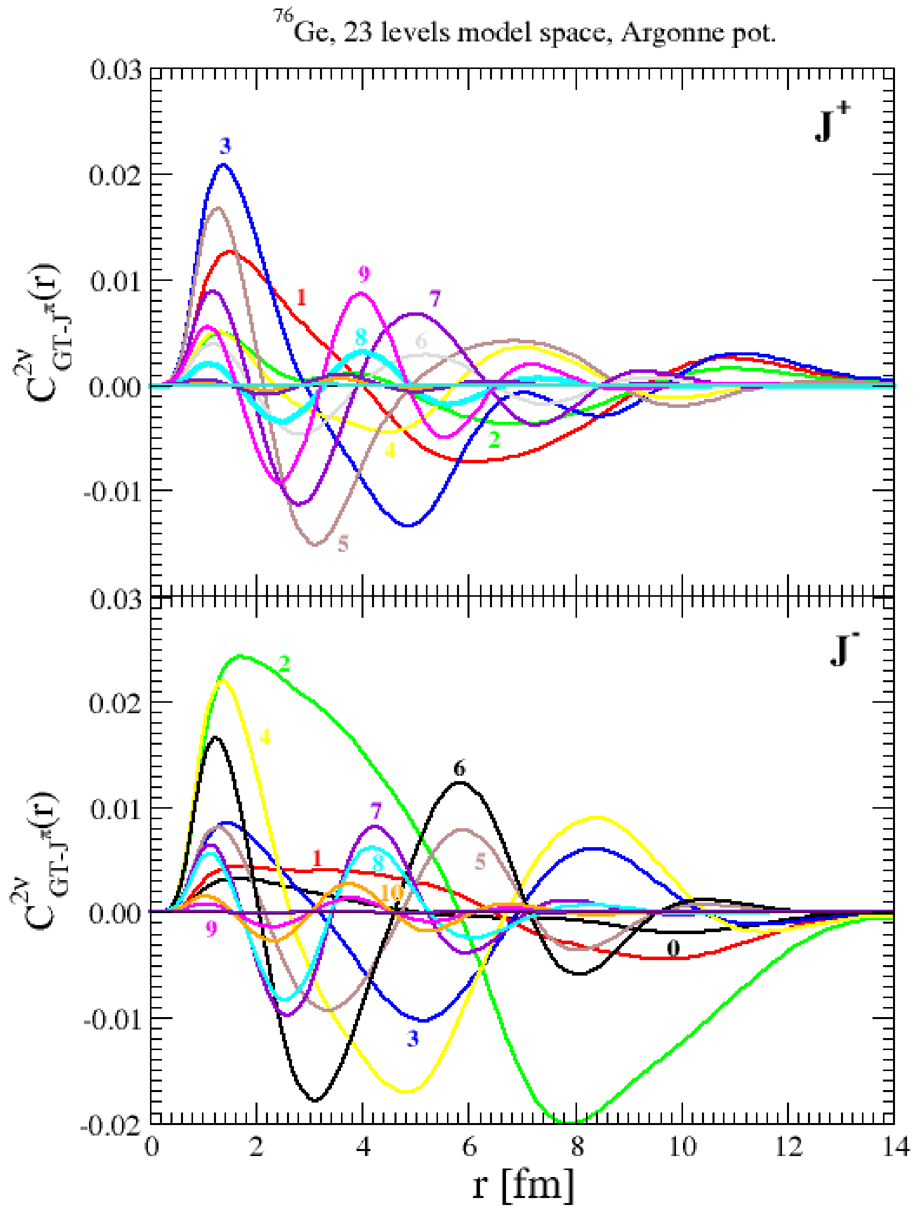
The only non-zero contribution from J^π=1$^+$

$$M^{2\nu}_{GT-cl} =$$

$$\sum_{J^\pi,m} < 0_f^+|\tau^+\vec{\sigma}|J^\pi, m > \cdot < J^\pi, m|\tau^+\vec{\sigma}|0_i^+ >$$

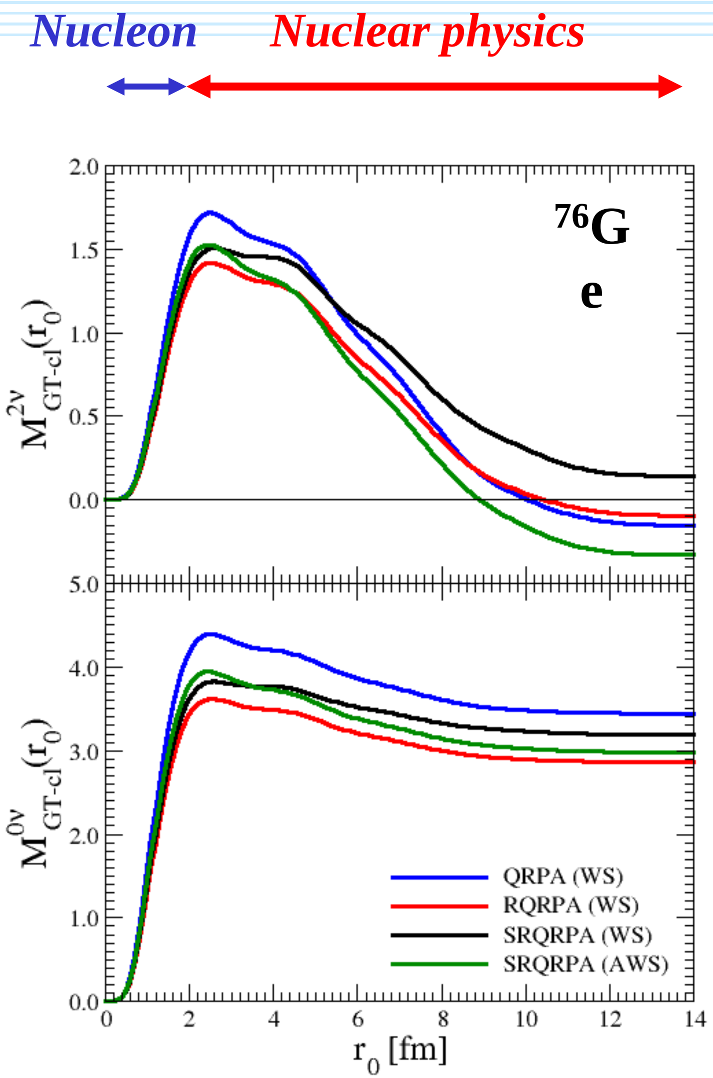
$$=\sum_{m} < 0_f^+|\tau^+\vec{\sigma}|1^+, m > \cdot < 1^+, m|\tau^+\vec{\sigma}|0_i^+ >$$

$$M^{2\nu}_{GT-cl} = \sum_{J^\pi} \int_0^\infty C^{2\nu}_{GT-J^\pi}(r)dr$$



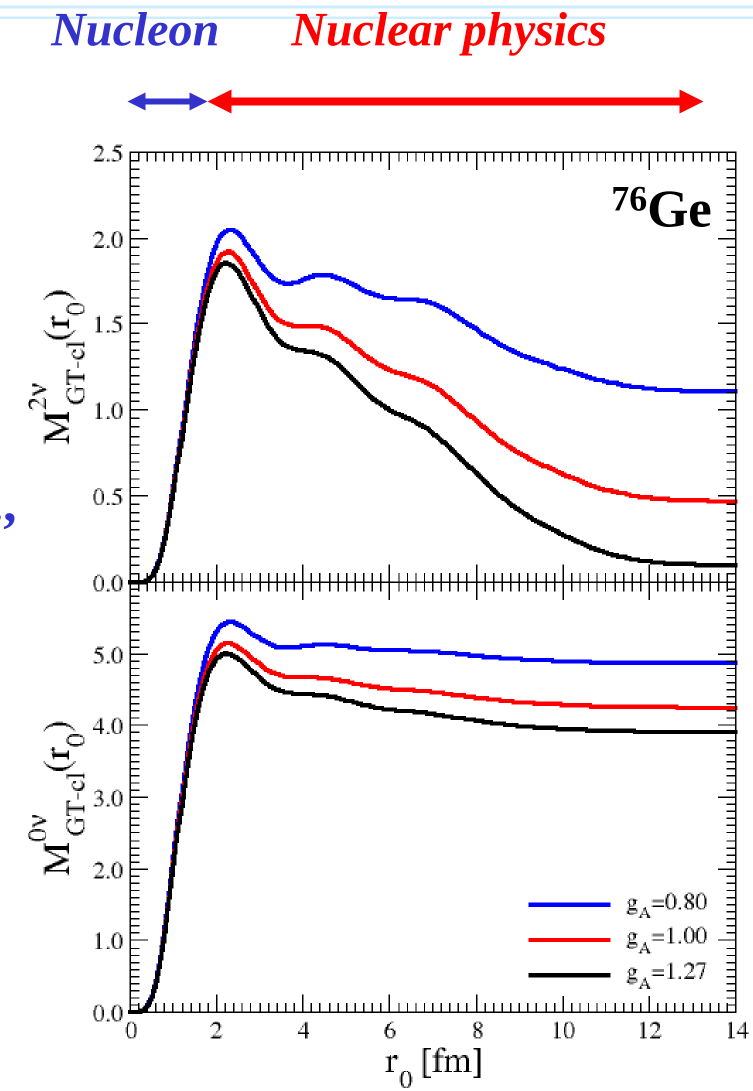
$M^{0\nu}_{GT}$ depends weakly on g_A/g_{pp} and QRPA approach unlike $M^{2\nu}_{GT}$

$$M^{0\nu}_{GT}(r_0) = \int_0^{r_0} H^{0\nu}_{GT}(r) C^{2\nu}_{GT-cl}(r) dr$$



Different QRPA-like approches

F.Š.,



Dependence on axial-vector coupling

Co-existence of few mechanisms of the $0\nu\beta\beta$-decay

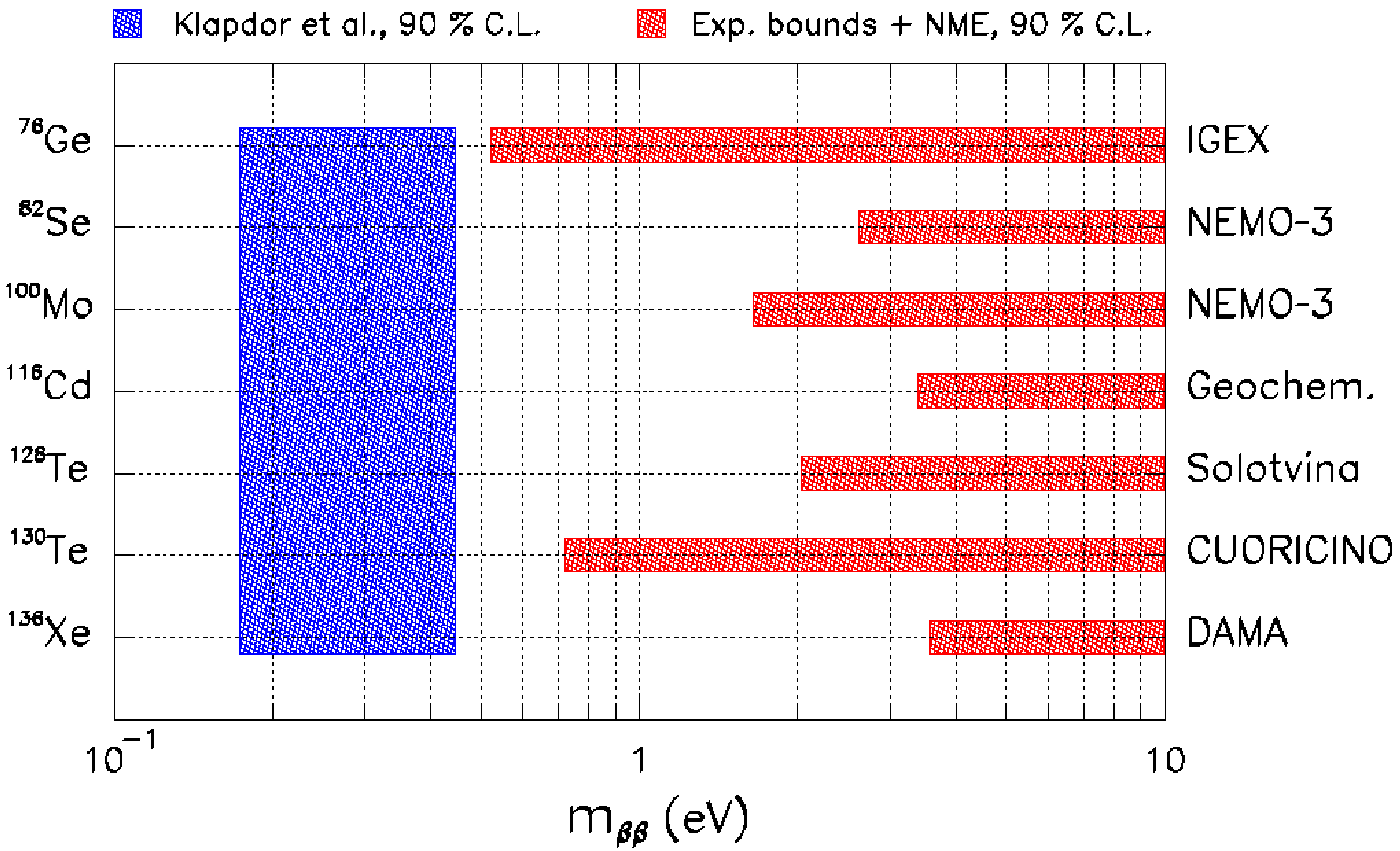
It may happen that in year 201? (or 2???) the $0\nu\beta\beta$-decay will be detected for 2-3 or more isotopes …

Dominance of light ν mass mechanism of the 0νββ-decay

$T^{0\nu}_{1/2}(^{76}Ge) = 2.23^{+0.44}_{-0.31}\ 10^{25}\ years$

H.V. Klapdor, I. Krivosheina,
Mod. Phys. Lett. A 21, 1547 (2006)

$$|m_{\beta\beta}| = \frac{m_e}{|M_1^{0\nu}|\sqrt{T_1\ G_1}} = \frac{m_e}{|M_2^{0\nu}|\sqrt{T_2\ G_2}} = \cdots$$



Faessler, Fogli, Lisi, Rodin, Rotunno, F.Š., PRD 79, 053001 (2009)

Co-existence of 2, 3 or more mechanisms of the $0\nu\beta\beta$-decay

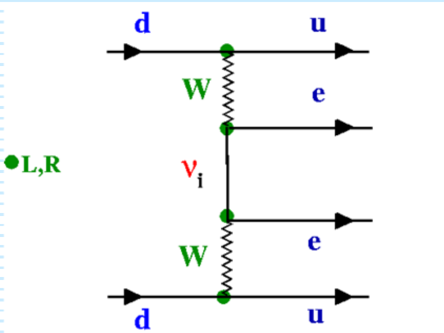
It is well-known that there exist many mechanisms that may contribute to the $0\nu\beta\beta$. Let consider 3 mechanisms: i) light ν-mass mechanism, ii) heavy ν-mass mechanism, iii) R-parity breaking SUSY mechanism with gluino exchange and CP conservation

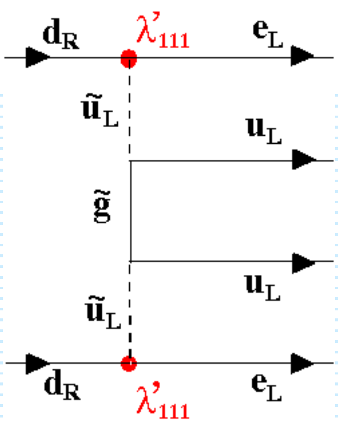
$$\frac{1}{T^{0\nu}_{1/2}} = G^{0\nu}(E_0, Z)\left|\frac{m_{\beta\beta}}{m_e} M^{0\nu}_{\nu} + \eta^L_N M^{0\nu}_N + \eta_{\lambda'_{111}} M^{0\nu}_{\lambda'_{111}} \ldots\right|^2$$

$$m_{\beta\beta} = \sum_k \left(U^L_{ek}\right)^2 \xi_k m_k$$

$$\eta^L_N = \sum_{k=4}^{6} |U^L{}_{ek}|^2\, \xi'_k\, \frac{m_p}{M_k}, \qquad \eta^R_N = \sum_{k=4}^{6} |U^R{}_{ek}|^2\, \xi'_k\, \frac{m_p}{M_k}.$$

$$\eta_{\lambda'_{111}} = \frac{\pi\alpha_s}{6}\frac{\lambda'^2_{111}}{G_F^2 m^4_{\tilde{d}_R}}\frac{m_p}{m_{\tilde{g}}}\left[1+\left(\frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}}\right)^2\right]^2.$$





Claim of evidence:

Klapdor-Kleingrothaus, Krivosheina, Mod. Phys. A 21, 1547 (2009)

$$T^{0\nu}_{1/2}(^{76}Ge) = 2.23^{+0.44}_{-0.31} \times 10^{25}\ \mathrm{y}$$

$$T^{0\nu}_{1/2}(^{100}\mathrm{Mo}) \geq 5.8 \times 10^{23}\ \mathrm{y}$$

$$T^{0\nu}_{1/2}(^{130}\mathrm{Te}) \geq 3.0 \times 10^{24}\ \mathrm{y}$$

$\xi_{Te} < 1.2$
$\xi_{Mo} <$ 2.6

We introduce

$$\xi = \frac{|M^{\nu}_1|\sqrt{T_1\ G_1}}{|M^{\nu}_2|\sqrt{T_2\ G_2}}$$

ξ=0, non-observation ($T_2 \to \infty$)
ξ=1, solution for single active mech. is reproduced

4 sets of two linear eq.

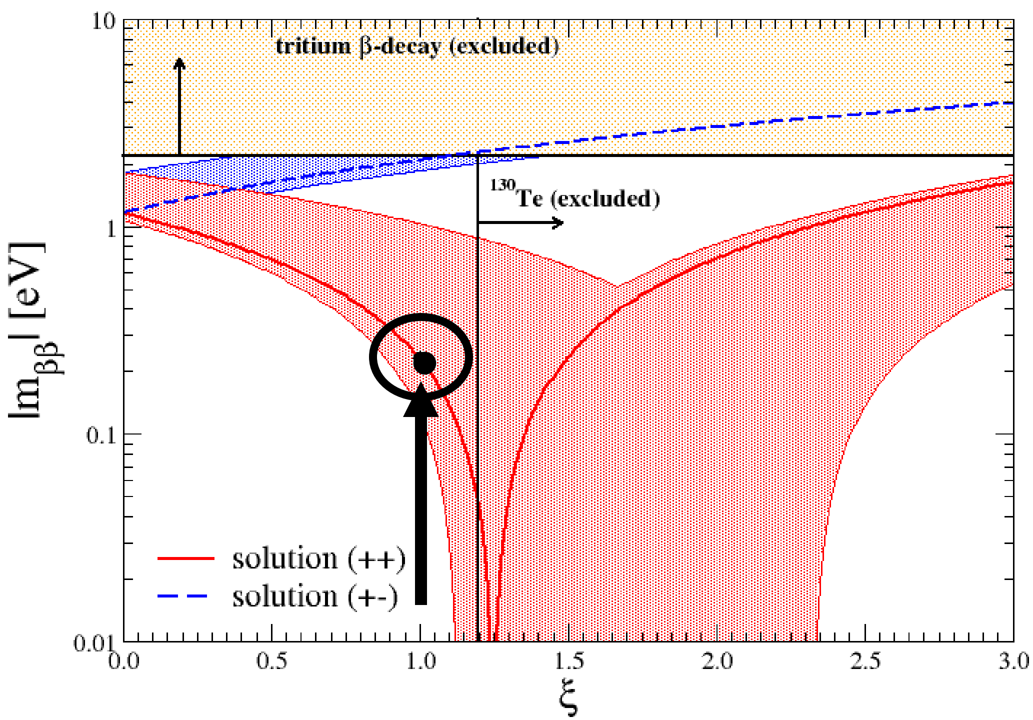
$$\frac{\pm 1}{\sqrt{T_1\ G_1}} = \frac{m_{\beta\beta}}{m_e} M_1^\nu + \eta M_1^\eta$$
$$\frac{\pm 1}{\sqrt{T_2\ G_2}} = \frac{m_{\beta\beta}}{m_e} M_2^\nu + \eta M_2^\eta$$

2 different solutions ***CP-conservation assumed***

$$|m_{\beta\beta}| = \left| \frac{m_e}{M_1^\nu \sqrt{T_1\ G_1}} \frac{M_1^\nu\ M_2^\eta}{(M_1^\nu M_2^\eta - M_2^\nu M_1^\eta)} \pm \frac{m_e}{M_2^\nu \sqrt{T_2\ G_2}} \frac{M_2^\nu\ M_1^\eta}{(M_1^\nu M_2^\eta - M_2^\nu M_1^\eta)} \right|$$

2 active mechanisms of the 0νββ-decay: Light and heavy ν-mass mechanism

Non-observation of the 0νββ-decay for some isotopes might be in agreement with non- zero $m_{\beta\beta}$



Non-observation for ^{130}Te

- ***Single solution for light ν-mass mech.***

10/26/2010

3 active mechanisms of the 0νββ-decay

$$\frac{\pm 1}{\sqrt{T_i\, G_i}} = \frac{m_{\beta\beta}}{m_e} M_i^{\nu} + \eta_N M_i^{\eta} + \eta_{\lambda'} M_i^{\lambda'}$$

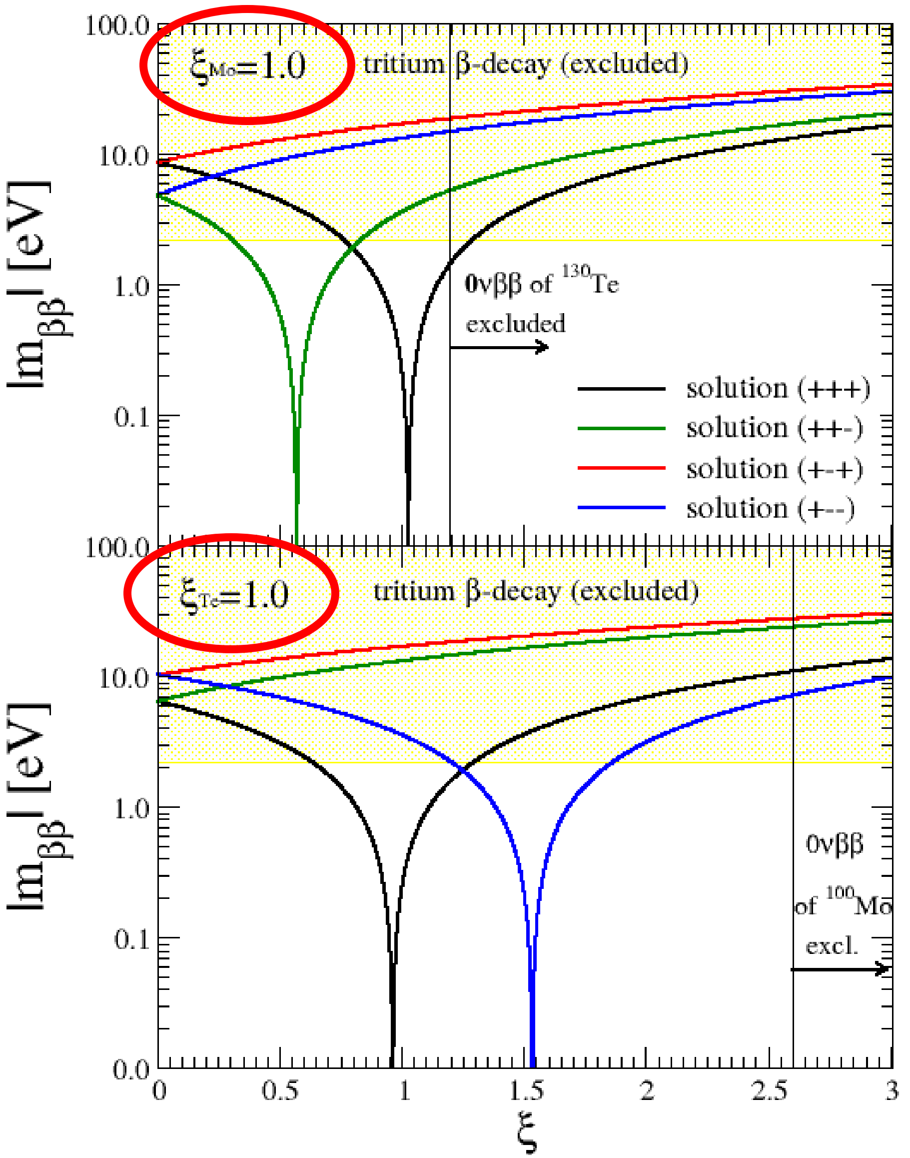
$$i = 1, 2, 3$$

assuming evidence

$$T_{1/2}^{0\nu}(^{76}Ge) = 2.23^{+0.44}_{-0.31} \times 10^{25}\ \mathrm{y}$$

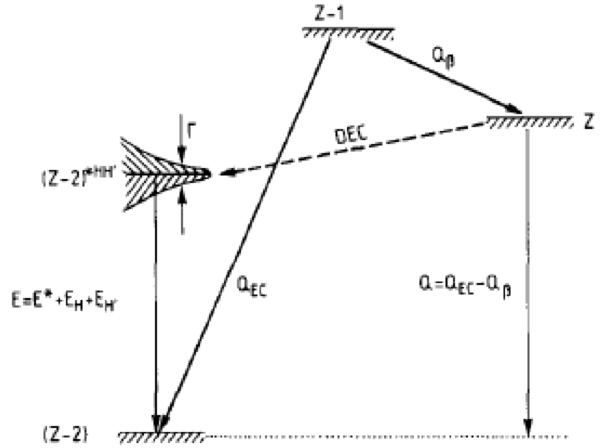
current limits

ξ_{Te} < 1.2
ξ_{Mo} < 2.6



The 0νECEC is a problem of particle, nuclear and atomic physics

Oscillations of atoms



Neutrinoless double electron capture (resonance transitions)

$(A,Z)\rightarrow(A,Z\text{-}2)^{*HH'}$

J. Bernabeu, A. DeRujula, C. Jarlskog, Nucl. Phys. B 223, 15 (1983)

Atom mixing amplitude ΔM

$$E \simeq E^* + E_{\rm H} + E_{\rm H'},$$

$$\Gamma \simeq \Gamma^* + \Gamma_{\rm H} + \Gamma_{\rm H'}.$$

Decay rate

$$\frac{1}{\tau} \simeq \frac{(\Delta M)^2}{(Q-E)^2 + \frac{1}{4}\Gamma^2}\,\Gamma,$$

2νECEC-background depends strongly on Q-value

DEC transitions, abundance, daughter nuclear excitation, atomic vacancies and figure of merit of some isotopes [10]

Transition $Z \rightarrow Z-2$	Z-natural abundance in %	Nuclear excitation E^* (in MeV), J^P	Atomic vacancies H, H′	Figure of merit $Q-E$ (in keV)
$^{74}_{34}$Se → $^{74}_{32}$Ge	0.87	1.204 (2^+)	2S(P), 2S(P)	2 ± 3
$^{78}_{36}$Kr → $^{78}_{34}$Se	0.36	2.839 (2^+) 2.864 (?)	1S, 1S	$^{19}_{-6} \pm 10$
$^{102}_{46}$Pd → $^{102}_{44}$Ru	1	1.103 (2^+) 1.107 (4^+)	1S, 1S	$^{29}_{25} \pm 9$
$^{106}_{48}$Cd → $^{106}_{46}$Pd	1.25	2.741 (?)	1S, 1S	-8 ± 10
$^{112}_{50}$Sn → $^{112}_{48}$Cd	1.01	1.871 (0^+)	1S, 1S	-3 ± 10
$^{130}_{56}$Ba → $^{130}_{54}$Xe	0.11	2.502 (?) 2.544 (?)	1S, 1S 1S, 2S(P)	$^{8}_{-6} \pm 13$
$^{152}_{64}$Gd → $^{152}_{62}$Sm	0.20	0 (0^+)	1S, 2S	4 ± 4
$^{162}_{68}$Er → $^{162}_{66}$Dy	0.14	1.783 (2^+)	1S, 2S	1 ± 6
$^{164}_{68}$Er → $^{164}_{66}$Dy	1.56	0 (0^+)	2S, 2S	9 ± 5
$^{168}_{70}$Yb → $^{168}_{68}$Er	0.14	1.355 (1^-) 1.393 (?)	1S, 2S 2S, 2S	$^{-1}_{8} \pm 4$
$^{180}_{74}$W → $^{180}_{72}$Hf	0.13	0 (0^+) 0.093 (2^+)	1S, 1S 1S, 3S	$^{26}_{-4} \pm 17$
$^{196}_{80}$Hg → $^{186}_{78}$Pt	0.15	0.689 (2^+)	1S, 2S	26 ± 9

Modes of the 0νECEC-decay:

$$e_b + e_b + (A,Z) \rightarrow (A,Z-2) + \gamma$$
$$+\ 2\gamma$$
$$+\ e^+e^-$$
$$+\ M$$

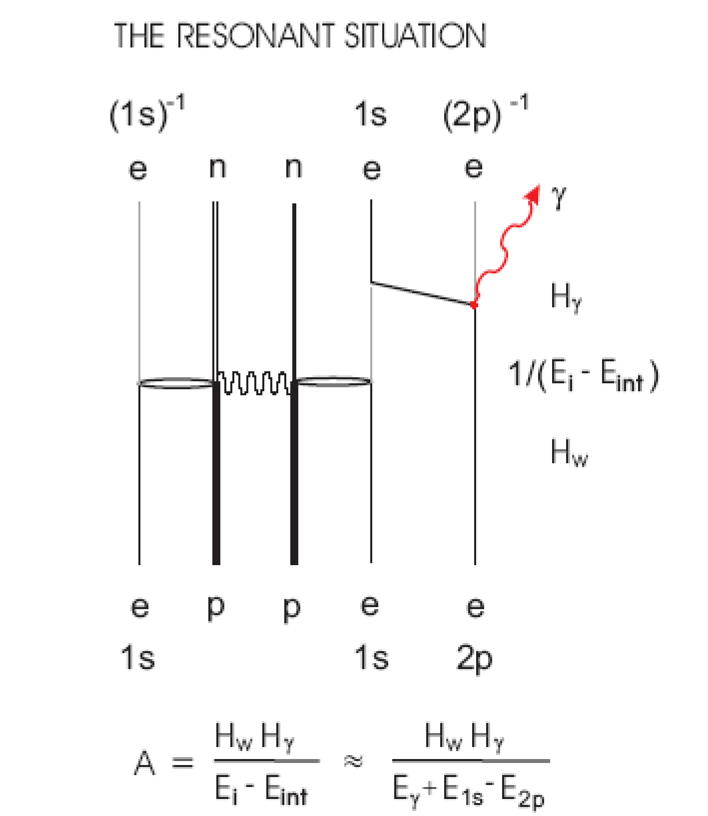
Neutrinoless double electron capture

(perturbation theory approach)

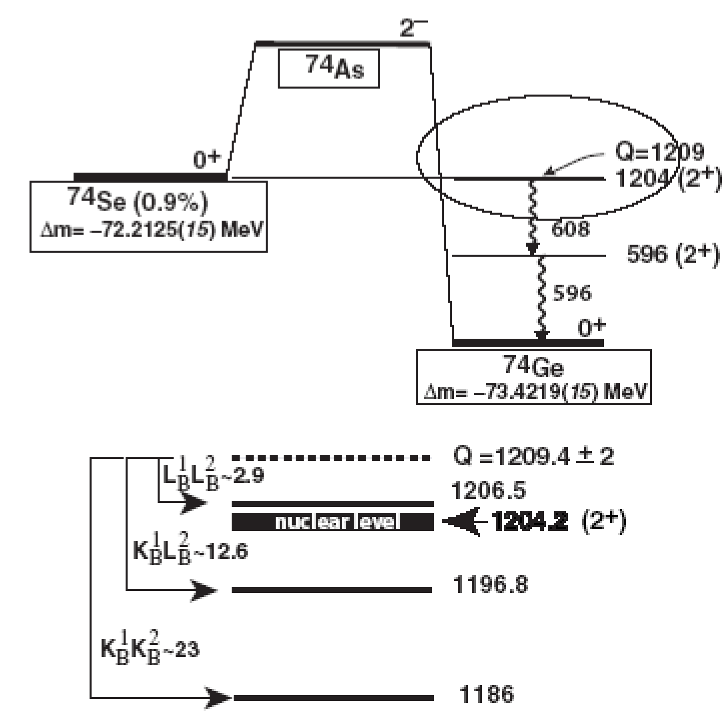
Theoretically, not well understood yet:

- **which mechanism is important?**
- **which transition is important?**

$$e_b + e_b + (A,Z) \rightarrow (A,Z-2) + \gamma$$



⇒



$$\Gamma^{0\nu\gamma} = \frac{\Gamma^r(2p \rightarrow 1s)}{[E_\gamma - Q_{res}]^2 + [\Gamma^r/2]^2} |R^{cc}_{0\nu}|^2$$

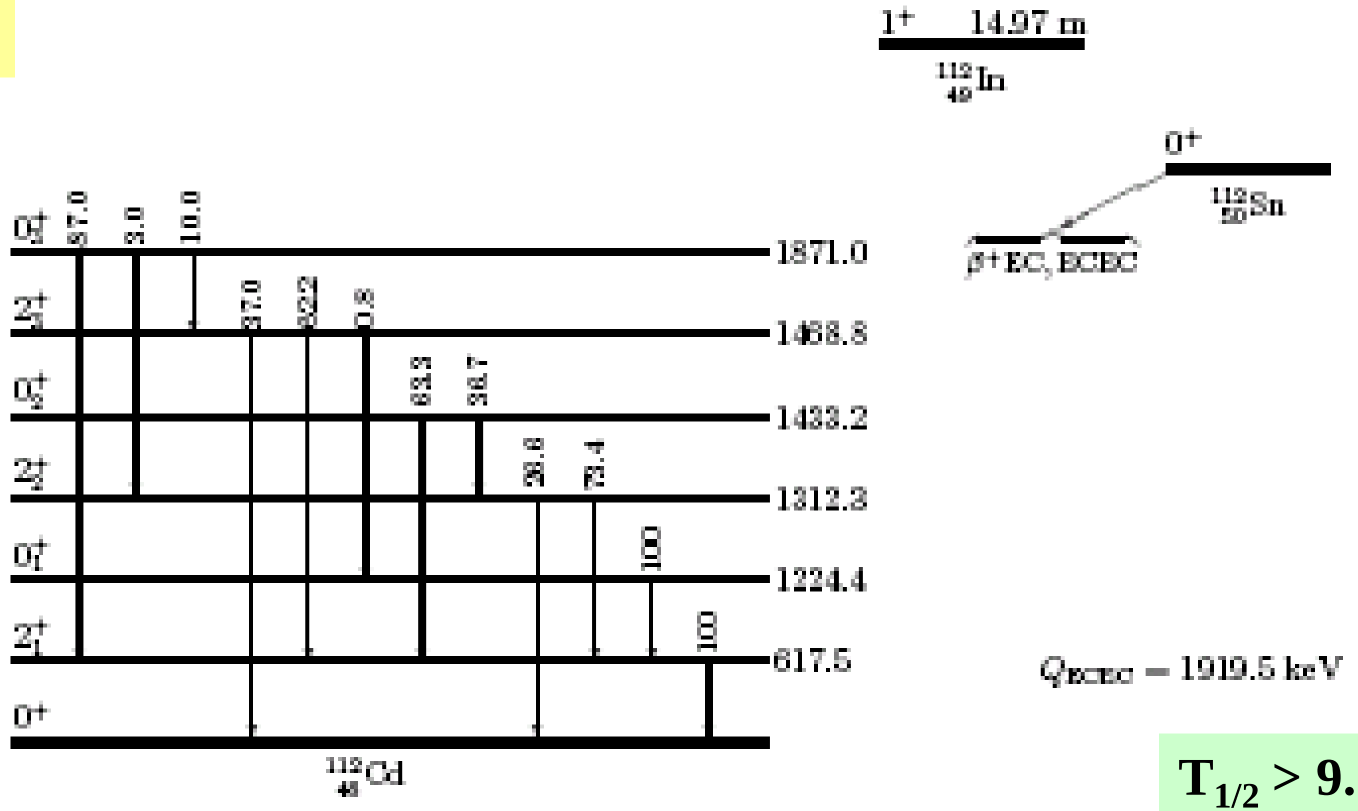
Fed $Q_{res} = E_{s_{1/2}} - E_{p_{1/2}}$

Sujkowski, Wycech, PRC 70, 052501 (2004)

D.Frekers, hep-ex/0506002

Experimental activities (^{112}Sn)

^{112}Sn



$T_{1/2}$ > 9.2 10^{19} years

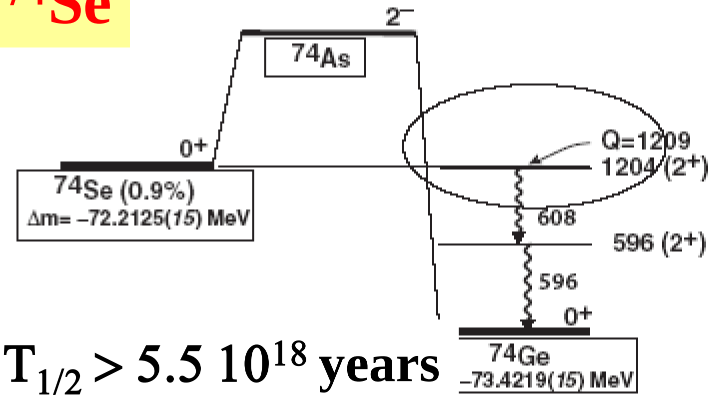
In comparison with the 0νββ-decay disfavoured due:

- process in the 3-rd (4th) order in electroweak theory
- bound electron wave functions

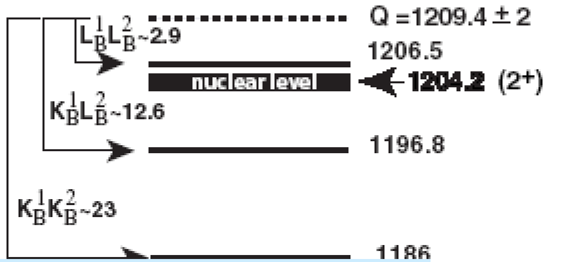
favoured: resonant enhancement ?

A.S. Barabash et al., NPA 807 (2008) 269

^{74}Se



$T_{1/2} > 5.5\ 10^{18}$ years

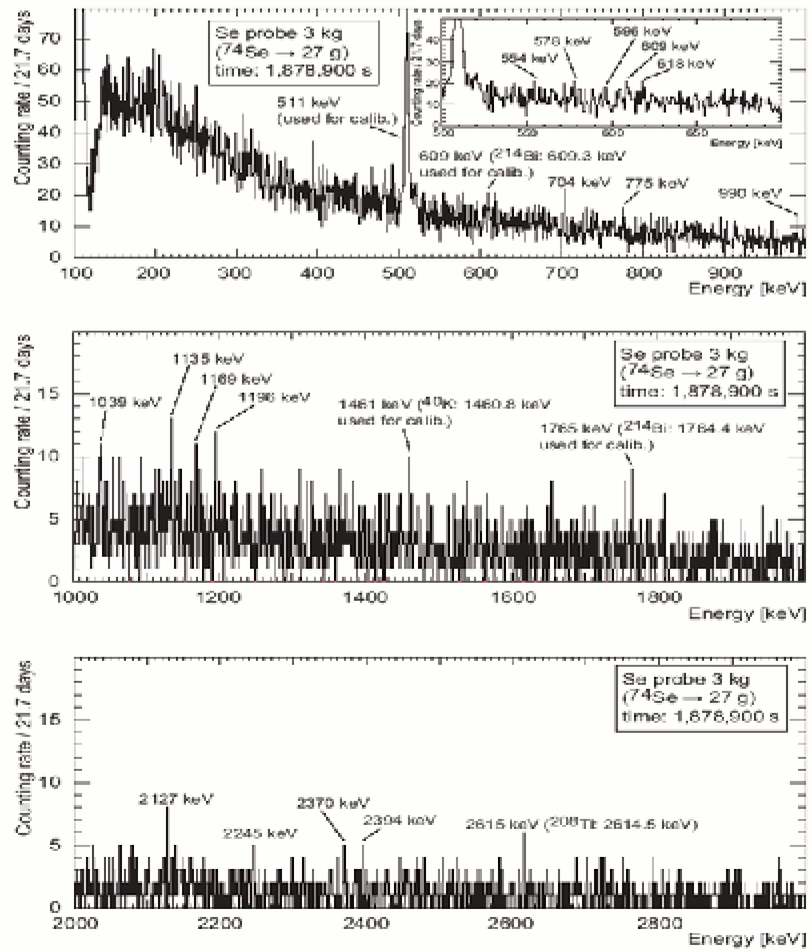


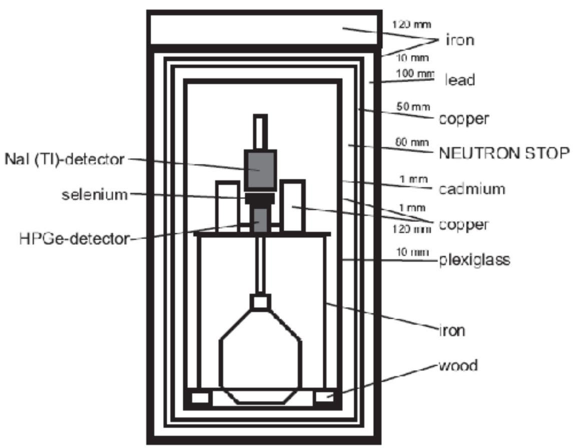
A.S. Barabash et al., NPA 785 (2007) 371

Experimental activities (^{74}Se)

Muenster and Bratislava groups exp. in Bratislava

Frekers et al., to be submitted





Different types of Oscillations (Effective Hamiltonian)

$$H_{eff}^{K_0\overline{K_0}} = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \Gamma_{12} \\ M_{12}^* - \Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillations of ν_l-$\nu_{l'}$
(lepton flavor)

Oscillation of K_0-anti{K_0}
(strangeness)

$$H_{eff}^{n\bar{n}} = \begin{pmatrix} M & V^{BNV} \\ V^{BNV} & M - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillation of n-anti{n}
(baryon number)

$$H_{eff}^{atom} = \begin{pmatrix} M_i & V^{LNV} \\ V^{LNV} & M_f - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillation of Atoms (OoA)
(total lepton number)

F.Š., M. Krivoruchenko, Phys.Part.Nucl.Lett. 6 (2009) 485.

Full width of unstable atom/nucleus

Eigenvalues

$$\lambda_+ = M_i + \Delta M - \frac{i}{2}\Gamma_1,$$
$$\lambda_- = M_f - \frac{i}{2}\Gamma - \Delta M + \frac{i}{2}\Gamma_1$$

$$\Delta M = \frac{V^2(M_i - M_f)}{(M_i - M_f)^2 + \frac{1}{4}\Gamma^2},$$
$$\Gamma_1 = \frac{V^2\Gamma}{(M_i - M_f)^2 + \frac{1}{4}\Gamma^2}.$$

Oscillations of atoms

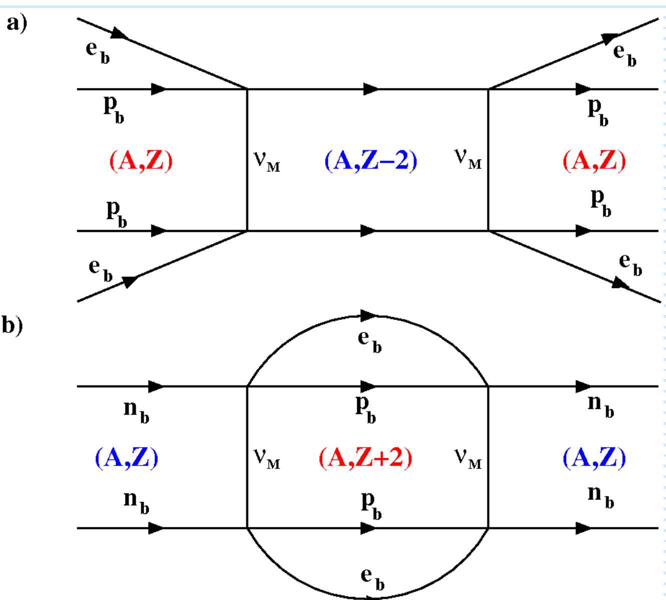
$$H_{eff}^{atom} = \begin{pmatrix} M_i & V^{LNV} \\ V^{LNV} & M_f - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillation of atoms (lepton number violation)

F.Š., M. Krivoruchenko, Phys.Part.Nucl.Lett. 6 (2009) 485.

In analogy with oscillations of n-anti{n} (baryon number violation)

$$H_{eff}^{n\bar{n}} = \begin{pmatrix} M & V^{BNV} \\ V^{BNV} & M - \frac{i}{2}\Gamma \end{pmatrix}$$



Oscillations of stable atoms **(Γ=0)**

$$|< f|e^{-iH_{eff}t}|i>|^2 = \frac{4V^2}{(M_i - M_f)^2}\sin^2\left[t\ (M_i - M_f)/2\right]$$

$$^{164}_{68}Er \rightarrow {}^{164}_{66}Dy$$

$$(M_i - M_f) = 24.1\ keV$$

$$|< f|e^{-iH_{eff}t}|i>|^2 \leq 3\ 10^{-55}$$

Oscillations of unstable atoms **(Γ≠0)**

Double electron capture

(resonant enhancement)

Light ν-exchange potential for the 0νEEC

$$\Gamma^{0\nu ECEC}(J^\pi) = \frac{|V_{\alpha\beta}(J^\pi)|^2}{(M_i - M_f)^2 + \Gamma^2_{\alpha\beta}/4}\ \Gamma_{\alpha\beta}$$

β-decay Hamiltonian

$$\mathcal{H}^\beta(x) = \frac{G_\beta}{\sqrt{2}}\bar{e}(x)\gamma^\mu(1-\gamma_5)\nu_e(x)j_\mu(x) + \mathrm{h.c.}$$

ν-mixing decay

$$\nu_{eL}(x) = \sum_{i=1}^{3} U_{ek}\chi_{kL}(x)$$

Potential

$$\begin{aligned}
V_{\alpha\beta} \;=\;& i m_{\beta\beta}\left(\frac{G_\beta}{\sqrt{2}}\right)^2 \frac{1}{\sqrt{1+\delta_{\alpha\beta}}} \sum_{m_\alpha m_\beta} C^{JM}_{j_\alpha m_\alpha j_\beta m_\beta} \int d\vec{x}_1 d\vec{x}_2 \\
\times\;& \Psi_{\alpha m_\alpha}{}^T(\vec{x}_1) C\gamma^\mu\gamma^\nu(1-\gamma_5)\Psi_{\beta m_\beta}(\vec{x}_2) \int \frac{e^{-i\vec{q}\cdot(\vec{x}_1-\vec{x}_2)}}{2q_0}\frac{d\vec{q}}{(2\pi)^3} \\
\times\;& \sum_n \Bigg[\frac{< A, Z-2|J_\mu(\vec{x}_1)|n >< n|J_\nu(\vec{x}_2)|A, Z >}{q_0 + E_n - M_i - \varepsilon_\beta} \\
&+ \frac{< A, Z-2|J_\nu(\vec{x}_2)|n >< n|J_\mu(\vec{x}_1)|A, Z >}{q_0 + E_n - M_i - \varepsilon_\alpha} - (\alpha \leftrightarrow \beta)
\end{aligned} \qquad (2)$$

0νEEC potential - approximations

Non-relativistic impulse approximation for nucleon current

$$J^\mu(0,\vec{x}) = \sum_{n=1}^{A} \tau_n^- [g_V g^{\mu 0} + g_A (\sigma_k)_n g^{\mu k}] \delta(\vec{x} - \vec{x}_n)$$

Closure approximation

$$E_n - M_i \Rightarrow < E > \approx 8\ MeV$$

$$\sum_n |n><n| = 1$$

$$V^{\alpha\beta}(J_f^\pi) = \frac{1}{4\pi}\ G_\beta^2 m_{\beta\beta} \frac{g_A^2}{R} \sqrt{2J_f + 1} \mathcal{M}_{\alpha\beta}(J_f^\pi)$$

Factorization of atomic and nuclear part

$$\mathcal{M}_{\alpha\beta}(J_f^\pi) \approx \mathcal{A}_{\alpha\beta}\ M^{0\nu}(J_f^\pi)$$

Similar form as for 0νββ-decay

$$M^{0\nu}(0_f^+) \ = \ < 0_f^+ \parallel \sum_{nm} \tau_n^- \tau_m^- h(r_{nm})[-\frac{g_V^2}{g_A^2} + (\vec{\sigma}_n \cdot \vec{\sigma}_m)] \parallel 0_i^+ >,$$

$$M^{0\nu}(0_f^-) \ = \ < 0_f^- \parallel \sum_{nm} \tau_n^- \tau_m^- h(r_{nm})(\hat{r}_n - \hat{r}_m) \cdot [\frac{g_V}{g_A}(\vec{\sigma}_n - \vec{\sigma}_m) - i(\vec{\sigma}_n \times \vec{\sigma}_m)] \parallel 0_i^+ >$$

$$h(r_{nm}) = \frac{2}{\pi} R \int_0^\infty j_0(q r_{nm}) \frac{q_0}{q_0 + < E > - m} dq.$$

Capture of $s_{1/2}$ and $p_{1/2}$ atomic electrons is prefered

$$\Psi_{\alpha m_\alpha}(\vec{x}) = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} f_\alpha(r)\ \chi_{m_\alpha} \\ -i g_\alpha(r)\ (\vec{\sigma}\cdot\hat{r})\ \chi_{m_\alpha} \end{pmatrix} \qquad (\alpha = n_\alpha s_{1/2})$$

$$\Psi_{\alpha m_\alpha}(\vec{x}) = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} -i f_\alpha(r)\ (\vec{\sigma}\cdot\hat{r})\ \chi_{m_\alpha} \\ -g_\alpha(r)\ \chi_{m_\alpha} \end{pmatrix} \qquad (\alpha = n_\alpha p_{1/2})$$

$J^\pi = 0^+, 0^-, 1^+, 1^-$

Shell		^{78}Se	^{112}Cd	^{124}Te	^{130}Xe	^{156}Gd
$1s_{1/2}$	$<f>$	3.45×10^3	6.80×10^3	8.83×10^3	1.09×10^4	1.33×10^4
	$<g>$	-4.34×10^2	-1.23×10^3	-1.81×10^3	-2.47×10^3	-3.30×10^3
$2s_{1/2}$	$<f>$	1.25×10^3	2.54×10^3	3.35×10^3	4.19×10^3	5.20×10^3
	$<g>$	-1.58×10^2	-4.59×10^2	-6.87×10^2	-9.48×10^2	-1.29×10^3
$3s_{1/2}$	$<f>$	6.83×10^2	1.39×10^3	1.83×10^3	2.29×10^3	2.85×10^3
	$<g>$	-8.60×10^1	-2.51×10^2	-3.76×10^2	-5.18×10^2	-7.05×10^2
$4s_{1/2}$	$<f>$	4.43×10^2	8.99×10^2	1.19×10^3	1.48×10^3	1.84×10^3
	$<g>$	-5.58×10^1	-1.63×10^2	-2.43×10^2	-3.36×10^2	-4.57×10^2
$2p_{1/2}$	$<f>$	-1.72×10^1	-7.22×10^1	-1.23×10^2	-1.87×10^2	-2.78×10^2
	$<g>$	-1.37×10^2	-3.99×10^2	-5.97×10^2	-8.25×10^2	-1.12×10^3
$2p_{3/2}$	$<f>$	8.06×10^{-1}	2.38×10^0	3.48×10^0	4.62×10^0	6.31×10^0
	$<g>$	-5.02×10^{-2}	-2.10×10^{-1}	-3.46×10^{-1}	-5.03×10^{-1}	-7.47×10^{-1}

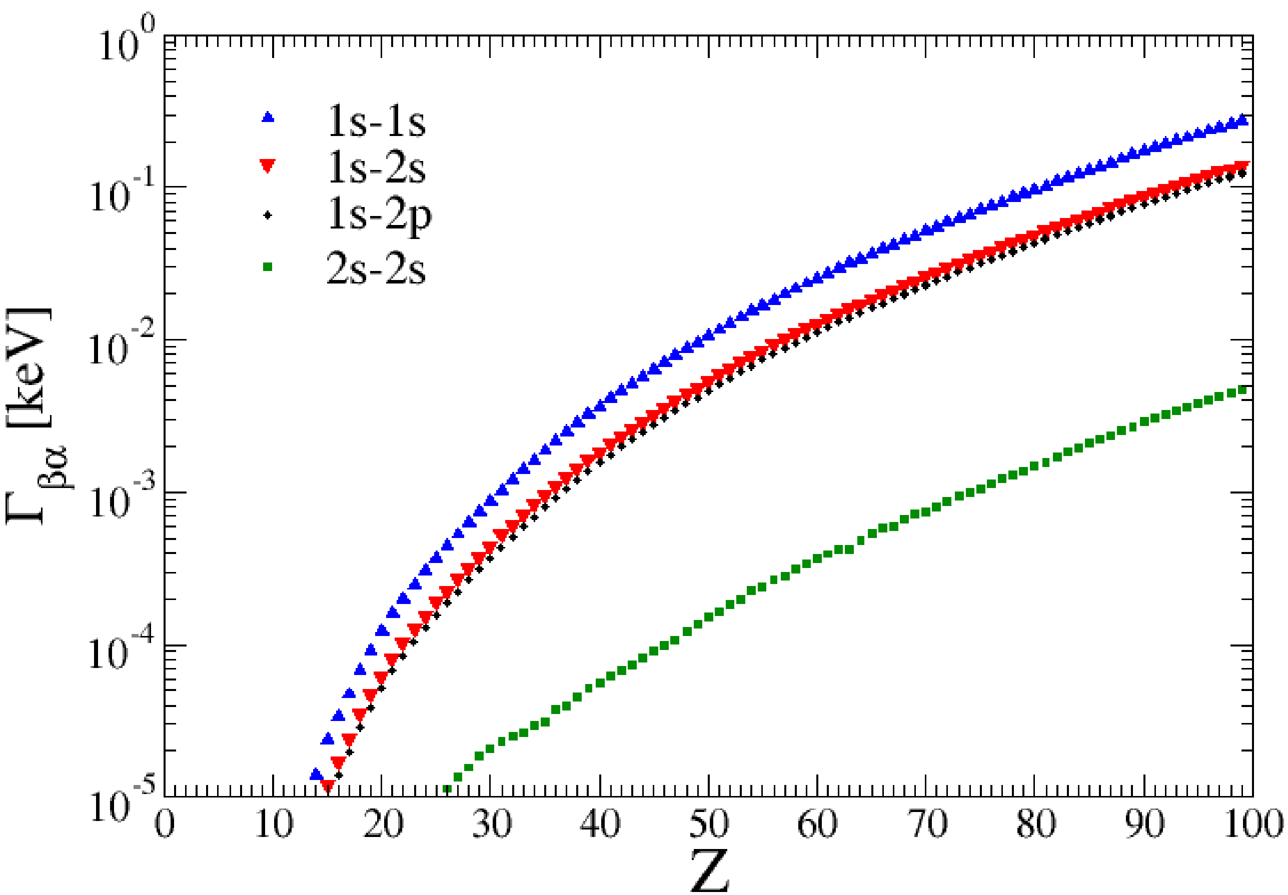
Widths of atomic excited states (2 holes)

$$\Gamma^*_{\beta\gamma} = \sum_\alpha \frac{\nu_\alpha - \delta_{\alpha\gamma}}{2(2l_\beta+1)}\eta_{\beta\alpha}\Gamma_{\beta\alpha} + \sum_\alpha \frac{\nu_\alpha - \delta_{\alpha\beta}}{2(2l_\gamma+1)}\eta_{\gamma\alpha}\Gamma_{\gamma\alpha}$$

$$\eta_{\beta\alpha} = \frac{\omega^{*3}_{\beta\alpha}}{\omega^3_{\beta\alpha}}\frac{d^{*2}_{\beta\alpha}}{d^2_{\beta\alpha}}$$

$$d^*_{\beta\alpha} = \int \frac{1}{a_\beta^{3/2}}R_{nl}(\frac{r}{a_\beta})\frac{1}{a_\alpha^{3/2}}R_{n'l'}(\frac{r}{a_\alpha})r^3dr$$

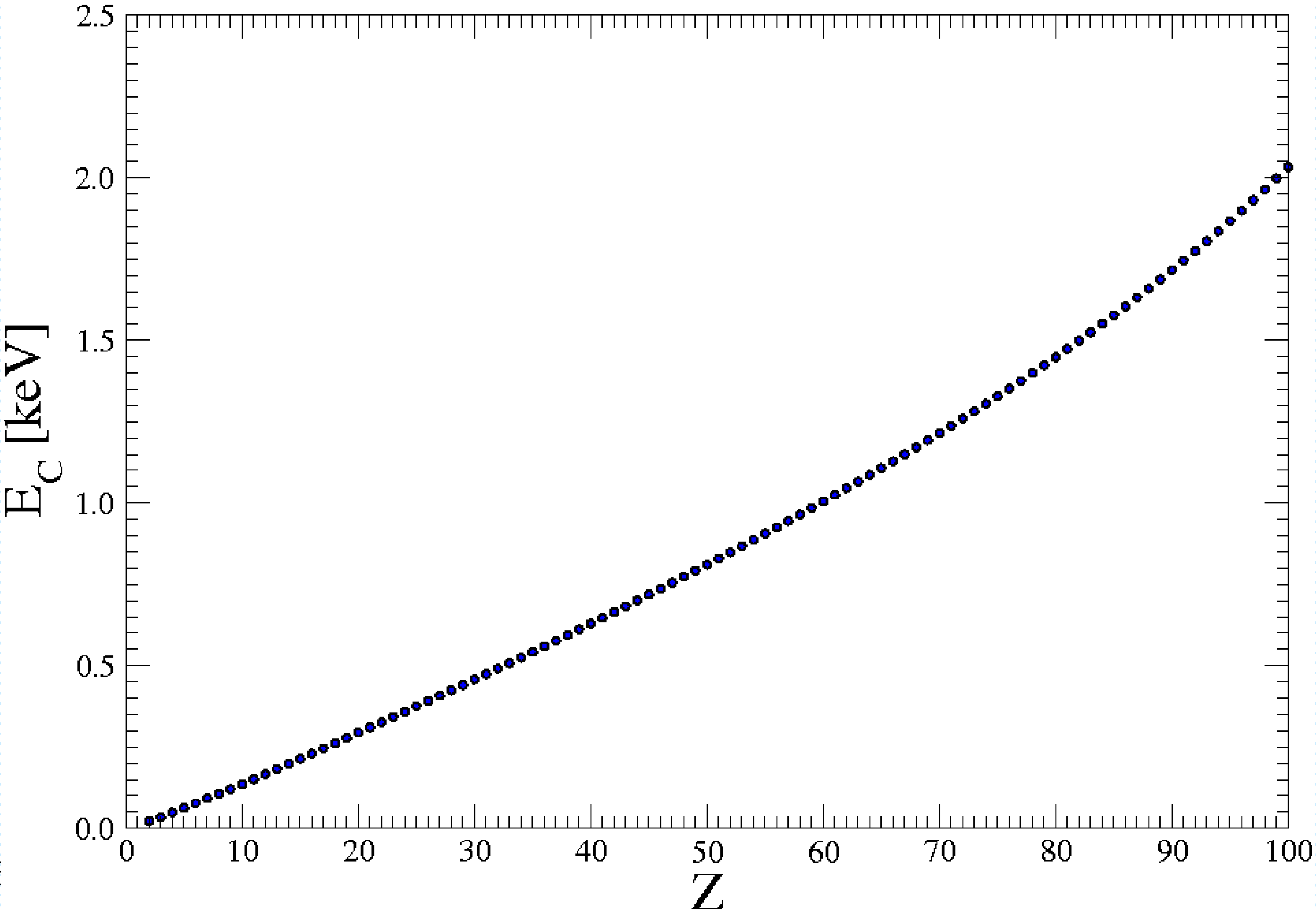
$$d_{\beta\alpha} = \int \frac{1}{a^{3/2}}R_{nl}(\frac{r}{a})\frac{1}{a^{3/2}}R_{n'l'}(\frac{r}{a})r^3dr$$



Repulsion coulombic energy of two holes

$$\varepsilon = m - \frac{\alpha^2 Z_*^2}{2n^2} m,$$

$$\epsilon_C = \int d\mathbf{x}_1 d\mathbf{x}_2 \Psi_{\beta\delta}^{JM+}(\mathbf{x}_1, \mathbf{x}_2) \frac{e^2}{|\mathbf{x}_1 - \mathbf{x}_2|} \Psi_{\beta\delta}^{JM}(\mathbf{x}_1, \mathbf{x}_2)$$



Normalized 0νECEC half-lives

For comparison 0νββ-half-life

$$\tilde{T}_{1/2} = T_{1/2}\left|\frac{m_{\beta\beta}}{1\ \mathrm{eV}}\right|^2 \left|\frac{M^{0\nu}(J_f^\pi)}{M^{0\nu}(0_f^+)}\right|^2 .$$

= 3

$$\begin{aligned}\tilde{T}^{0\nu\beta\beta}_{1/2} &= (1.4, 9.7)\times 10^{24}\ y \quad (^{76}Ge)\\ &= (0.14, 1.8)\times 10^{24}\ y \quad (^{100}Mo)\\ &= (1.8, 15.6)\times 10^{24}\ y \quad (^{130}Te)\end{aligned}$$

$T^{min}_{1/2}$: $M_i = M_f$ (full degeneracy)

$$\begin{aligned}\Delta M^2 &= (M^{**}_{A,Z-2} - M_{A,Z})^2 + \Delta M^2_{\mathrm{expt}}\\ \Delta M^2_{\mathrm{expt}} &= \delta M^2_{A,Z-2} + \delta M^2_{A,Z} + \delta R^2_{A,Z-2}.\end{aligned}$$

$^{184}_{76}Os \rightarrow {}^{184}_{74}W^*$ (0.02%)

J_f^π	$M_f^* - M_f$	$M_f^{**} - M_i$	$(n2jl)_\alpha$	$(n2jl)_\beta$	ϵ_α^*	ϵ_β^*	ϵ_C	$\Gamma_{\alpha\beta}$	$\tilde{T}^{min}_{1/2}$	$\tilde{T}^{max}_{1/2}$
(0^+)	1322.152±0.022	-11.3± 1.3± 0.9	110	110	69.53	69.53	1.31	6.7×10^{-2}	2×10^{22}	3×10^{27}

$^{180}_{74}W \rightarrow {}^{184}_{72}Hf$ (0.13%)

Half-lives in years

J_f^π	$M_f^* - M_f$	$M_f^{**} - M_i$	$(n2jl)_\alpha$	$(n2jl)_\beta$	ϵ_α^*	ϵ_β^*	ϵ_C	$\Gamma_{\alpha\beta}$	$\tilde{T}^{min}_{1/2}$	$\tilde{T}^{max}_{1/2}$
0^+	0	12.0± 3.9± 2.1	110	110	65.35	65.35	1.26	5.9×10^{-2}	3×10^{22}	5×10^{27}

$^{106}_{48}Cd \rightarrow {}^{106}_{46}Pd$ (1.25%)

All masses/energies in keV

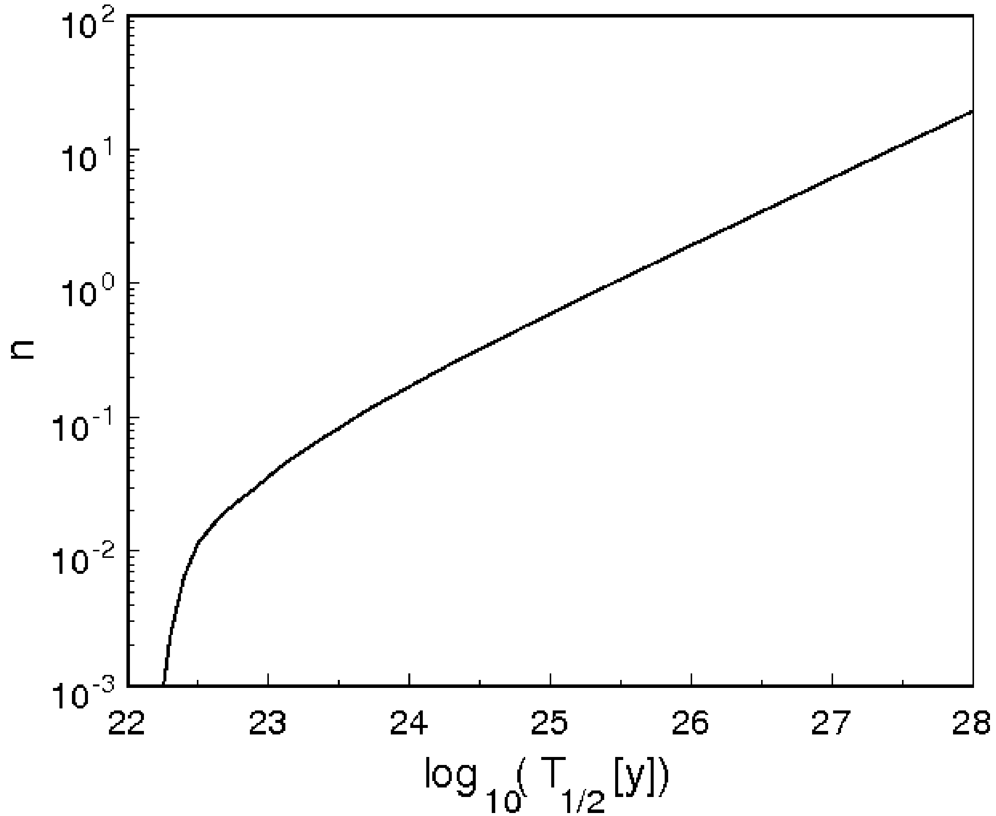
J_f^π	$M_f^* - M_f$	$M_f^{**} - M_i$	$(n2jl)_\alpha$	$(n2jl)_\beta$	ϵ_α^*	ϵ_β^*	ϵ_C	$\Gamma_{\alpha\beta}$	$\tilde{T}^{min}_{1/2}$	$\tilde{T}^{max}_{1/2}$
	2717.59±0.21	3.0± 5.9± 4.1	110	110	24.35	24.35	0.74	$7.1\ 10^{-3}$	$2\ 10^{23}$	$8\ 10^{29}$
	2737±1	-16.5± 5.9± 4.1	110	110	24.35	24.35	0.74	$7.1\ 10^{-3}$	$2\ 10^{23}$	$4\ 10^{30}$
		4.8± 5.9± 4.1	110	210	24.35	3.60	0.23	$3.6\ 10^{-3}$	$3\ 10^{23}$	$7\ 10^{30}$
		5.1± 5.9± 4.1	110	211	24.35	3.33	0.21	$3.1\ 10^{-3}$	$5\ 10^{25}$	$2\ 10^{33}$
		7.9± 5.9± 4.1	110	310	24.35	0.67	0.07	$3.6\ 10^{-3}$	$1\ 10^{24}$	$4\ 10^{31}$
		8.0± 5.9± 4.1	110	311	24.35	0.56	0.06	$3.5\ 10^{-3}$	$1\ 10^{26}$	$6\ 10^{33}$

$^{130}_{56}Ba \rightarrow ^{130}_{54}Xe$ (0.11%)

J_f^π	$M_f^* - M_f$	$M_f^{**} - M_i$	$(n2jl)_\alpha$	$(n2jl)_\beta$	ϵ^*_α	ϵ^*_β	ϵ_C	$\Gamma_{\alpha\beta}$	$\tilde{T}^{min}_{1/2}$	$\tilde{T}^{max}_{1/2}$
	2544.43±0.08	5.7± 2.8± 0.7	110	110	34.56	34.56	0.89	$1.5\ 10^{-2}$	**$1\ 10^{23}$**	**$7\ 10^{28}$**
	2608.426±0.019	0.6± 2.8± 0.7	210	210	5.45	5.45	0.17	$2.3\ 10^{-4}$	**$7\ 10^{22}$**	$5\ 10^{31}$
		1.0± 2.8± 0.7	210	211	5.45	5.10	0.17	$2.6\ 10^{-4}$	$5\ 10^{24}$	$3\ 10^{33}$
		5.0± 2.8± 0.7	210	310	5.45	1.15	0.08	$1.3\ 10^{-4}$	**$7\ 10^{22}$**	$6\ 10^{32}$
		5.2± 2.8± 0.7	210	311	5.45	1.00	0.07	$1.2\ 10^{-4}$	$7\ 10^{24}$	$7\ 10^{34}$
		6.0± 2.8± 0.7	210	410	5.45	0.21	0.03	$1.1\ 10^{-4}$	$1\ 10^{23}$	$2\ 10^{33}$
	2622.32±0.09	-13.3± 2.8± 0.7	210	210	5.45	5.45	0.17	$2.3\ 10^{-4}$	**$7\ 10^{22}$**	$1\ 10^{33}$
		-12.9± 2.8± 0.7	210	211	5.45	5.10	0.17	$2.6\ 10^{-4}$	$5\ 10^{24}$	$6\ 10^{34}$
		-8.9± 2.8± 0.7	210	310	5.45	1.15	0.08	$1.3\ 10^{-4}$	**$7\ 10^{22}$**	$1\ 10^{33}$
		-8.7± 2.8± 0.7	210	311	5.45	1.00	0.07	$1.2\ 10^{-4}$	$7\ 10^{24}$	$2\ 10^{35}$
		-7.9± 2.8± 0.7	210	410	5.45	0.21	0.03	$1.1\ 10^{-4}$	$1\ 10^{23}$	$3\ 10^{33}$
	2628.36±0.10	-14.9± 2.8± 0.7	210	310	5.45	1.15	0.08	$1.3\ 10^{-4}$	**$7\ 10^{22}$**	$4\ 10^{33}$
		-14.7± 2.8± 0.7	210	311	5.45	1.00	0.07	$1.2\ 10^{-4}$	$7\ 10^{24}$	$4\ 10^{35}$
		-13.9± 2.8± 0.7	210	410	5.45	0.21	0.03	$1.1\ 10^{-4}$	$1\ 10^{23}$	$9\ 10^{33}$
		-13.8± 2.8± 0.7	210	411	5.45	0.15	0.02	$9.7\ 10^{-5}$	$1\ 10^{25}$	$1\ 10^{36}$
		-13.7± 2.8± 0.7	210	510	5.45	0.02	0.02	$1.1\ 10^{-4}$	$3\ 10^{23}$	$2\ 10^{34}$
	2629.389±0.023	-15.0± 2.8± 0.7	210	410	5.45	0.21	0.04	$1.1\ 10^{-4}$	$5\ 10^{23}$	$4\ 10^{34}$
		-14.9± 2.8± 0.7	210	411	5.45	0.15	0.02	$9.7\ 10^{-5}$	$1\ 10^{25}$	$1\ 10^{36}$
		-14.7± 2.8± 0.7	210	510	5.45	0.02	0.02	$1.1\ 10^{-4}$	$3\ 10^{23}$	$2\ 10^{34}$
		-14.7± 2.8± 0.7	210	511	5.45	0.01	0.01	$1.1\ 10^{-4}$	$3\ 10^{25}$	$2\ 10^{36}$
		-14.6± 2.8± 0.7	211	410	5.10	0.21	0.02	$1.5\ 10^{-4}$	$2\ 10^{25}$	$1\ 10^{36}$
	2633.2 ±0.4	-14.4± 2.8± 0.7	310	410	1.15	0.21	0.03	$1.9\ 10^{-5}$	**$8\ 10^{22}$**	$2\ 10^{35}$
		-14.4± 2.8± 0.7	310	411	1.15	0.15	0.02	$1.6\ 10^{-5}$	$7\ 10^{24}$	$2\ 10^{37}$
		-14.2± 2.8± 0.7	310	510	1.15	0.02	0.01	$1.9\ 10^{-5}$	$2\ 10^{23}$	$4\ 10^{35}$
		-14.2± 2.8± 0.7	310	511	1.15	0.01	0.01	$1.8\ 10^{-5}$	$2\ 10^{25}$	$4\ 10^{37}$
		-14.3± 2.8± 0.7	311	410	1.00	0.21	0.02	$1.3\ 10^{-5}$	$6\ 10^{24}$	$3\ 10^{37}$

Data analysis of most likely resonant transitions

10 chance of 100 for $T_{1/2} < 10^{24}$ y
100 100 $< 10^{25}$ y



Half-life of a particular isotope

$$\tilde{T}_{1/2} = \tilde{T}^{\min}_{1/2} \frac{\Delta M^2 + \Gamma_\gamma^2/4}{\Gamma_\gamma^2/4}$$

$$\Delta M(\tilde{T}_{1/2}) = \frac{\Gamma_\gamma}{2} \left(\frac{\tilde{T}_{1/2}}{\tilde{T}^{\min}_{1/2}} - 1 \right)^{1/2} .$$

Number of transitions n with half –life $T^a_{1/2} < T_{1/2}$

$$n = \sum_{T^a_{1/2} < T_{1/2}} \frac{\Delta M(\tilde{T}^a_{1/2})}{\Delta M_{\mathrm{expt}}}$$

Q-value measurements

Klaus Blaum "LAUNCH09 (Nov. 09)"

Accuracy below 300 eV is not a problem

Decay	ββ Q-value	Precision
^{76}Ge – ^{76}Se	2039.006(50) G. Douysset et al., PRL 86, 4259 (2001)	6E-10
^{130}Te – ^{130}Xe	2527.518(13) M. Redshaw et al., PRL 102, 212502 (2009)	1E-10
^{136}Xe – ^{136}Ba	2457.83(37) M. Redshaw et al., PRL 98, 053003 (2007)	3E-09
	ECEC	
^{112}Sm – ^{112}Cd	1919.82(16) S. Rahaman et al., PRL 103, 042501 (2009)	1E-09
^{120}Te – ^{120}Sm	1714.81(1.25) N. Scielzo et al., PRC 80, 025501 (2009)	1E-08

Is it possible to manipulate atomic mass difference?

Magnetic field of 10 T would be not enough …

Statistical properties of ν and 2νββ-decay

$2\nu\beta\beta$-decay: fermionic (f) or bosonic (b) ν

$$|\nu_1\ \nu_2> \ = \ \hat{a}_1^\dagger\ \hat{a}_2^\dagger|0>$$

Dolgov, Smirnov, PLB 621, 1 (2005)

$$\{\hat{a}_i, \hat{a}_j^\dagger\}_+ \ = \ \delta_{i,j} \quad (fermionic\ \nu)$$
$$\left[\hat{a}_i, \hat{a}_j^\dagger\right]_- \ = \ \delta_{i,j} \quad (bosonic\ \nu)$$

$$dW^{f,b}(0^+ \to 0^+) \ \sim \ \left(3\left|\mathcal{M}^{f,b}{}_K + \mathcal{M}^{f,b}{}_L\right|^2 + \left|\mathcal{M}^{f,b}{}_K - \mathcal{M}^{f,b}{}_L\right|^2\right) \ d\vec{p_{e_1}}\, d\vec{p_{e_2}}\, d\vec{p_{\nu_1}}\, d\vec{p_{\nu_2}}$$

$$dW^{f,b}(0^+ \to 2^+) \ \sim \ \left|\mathcal{M}^{f,b}{}_K - \mathcal{M}^{f,b}{}_L\right|^2 \ d\vec{p_{e_1}}\, d\vec{p_{e_2}}\, d\vec{p_{\nu_1}}\, d\vec{p_{\nu_2}}$$

$$\mathcal{M}^{f,b}{}_K \ = \ \sum_m \left(\frac{M_m^I(1^+)M_m^F(1^+)}{E_m - E_i + e_1 + \nu_1} \ \pm \ \frac{M_m^I(1^+)M_m^F(1^+)}{E_m - E_i + e_2 + \nu_2}\right)$$

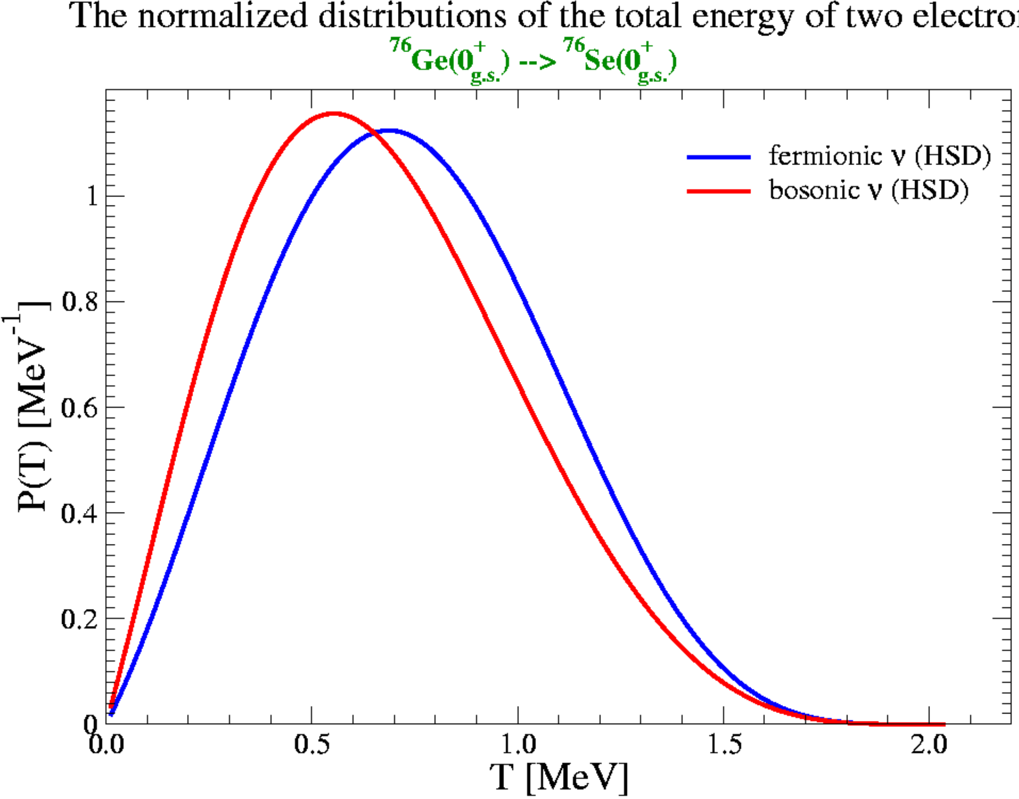
$$\mathcal{M}^{f,b}{}_K = \mathcal{M}^{f,b}{}_L(\nu_1 \leftrightarrow \nu_2)$$

Sign difference!!!
Lepton energies!!!

$$\frac{T_{1/2}^{2\nu-SSD}(2_f^+)}{T_{1/2}^{2\nu-SSD}(0_f^+)} \ = \ 2.41 \times 10^4 \quad \text{fermionic } \nu \qquad T_{1/2}^{2\nu}(2^+) \ = \ 1.73 \times 10^{23} years$$

$$= \ 403 \quad \text{bosonic } \nu \qquad = \ 2.74 \times 10^{21} years$$

$$T_{1/2}^{2\nu-exp}(2^+) \ > \ 1.6 \times 10^{21} years$$

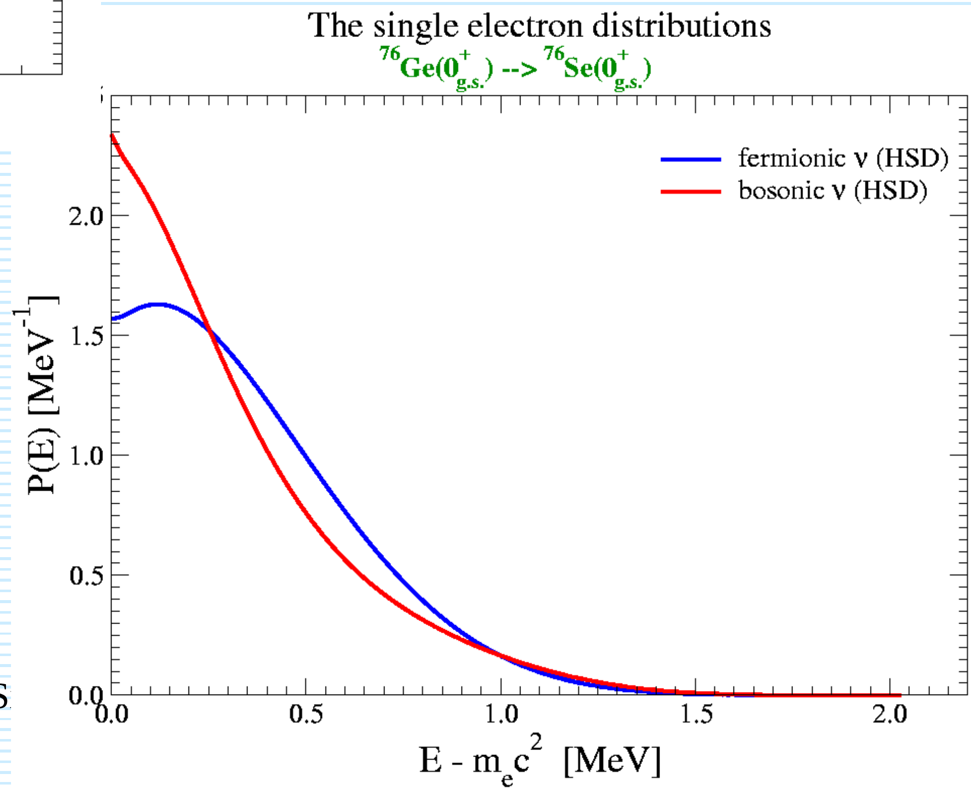


$2\nu\beta\beta$-decay of ^{76}Ge

$0^+_{g.s.} \longrightarrow 0^+_{g.s.}$

HSD

Barabash, Dolgov, Smirnov, F.Š, R. Dvornický, NPB 783, 90 (2007)



bosonic **fermionic**

$$P^{f,b}(T) = \frac{1}{W^{f,b}} \frac{dW^{f,b}}{dT}$$

$$P^{f,b}(E) = \frac{1}{W^{f,b}} \frac{dW^{f,b}}{dE}$$

Mixed statistics for neutrinos

Definnition of mixed state

$$
\begin{aligned}
|\nu> &= \hat{a}^\dagger|0> \\
&\equiv \cos\delta \ \ \hat{f}^\dagger|0> \ + \ \sin\delta \ \ \hat{b}^\dagger|0> \\
&= \cos\delta \ \ |f> \ + \ \sin\delta \ \ |b>
\end{aligned}
$$

with commutation Relations

$$
\begin{aligned}
\hat{f}\hat{b} &= e^{i\phi}\hat{b}\hat{f} \qquad \hat{f}^\dagger\hat{b}^\dagger = e^{i\phi}\hat{b}^\dagger\hat{f}^\dagger \\
\hat{f}\hat{b}^\dagger &= e^{-i\phi}\hat{b}^\dagger\hat{f} \qquad \hat{f}^\dagger\hat{b} = e^{-i\phi}\hat{b}\hat{f}^\dagger
\end{aligned}
$$

Amplitude for 2νββ

$$
\begin{aligned}
A^{2\nu} &= [\cos\delta^4 + \cos\delta^2\sin\delta^2(1-\cos\phi)]A^f \ + \ [\cos\delta^4 + \cos\delta^2\sin\delta^2(1+\cos\phi)]A^b \\
&= \cos\chi^2 A^f \ + \ \sin\chi^2 A^b
\end{aligned}
$$

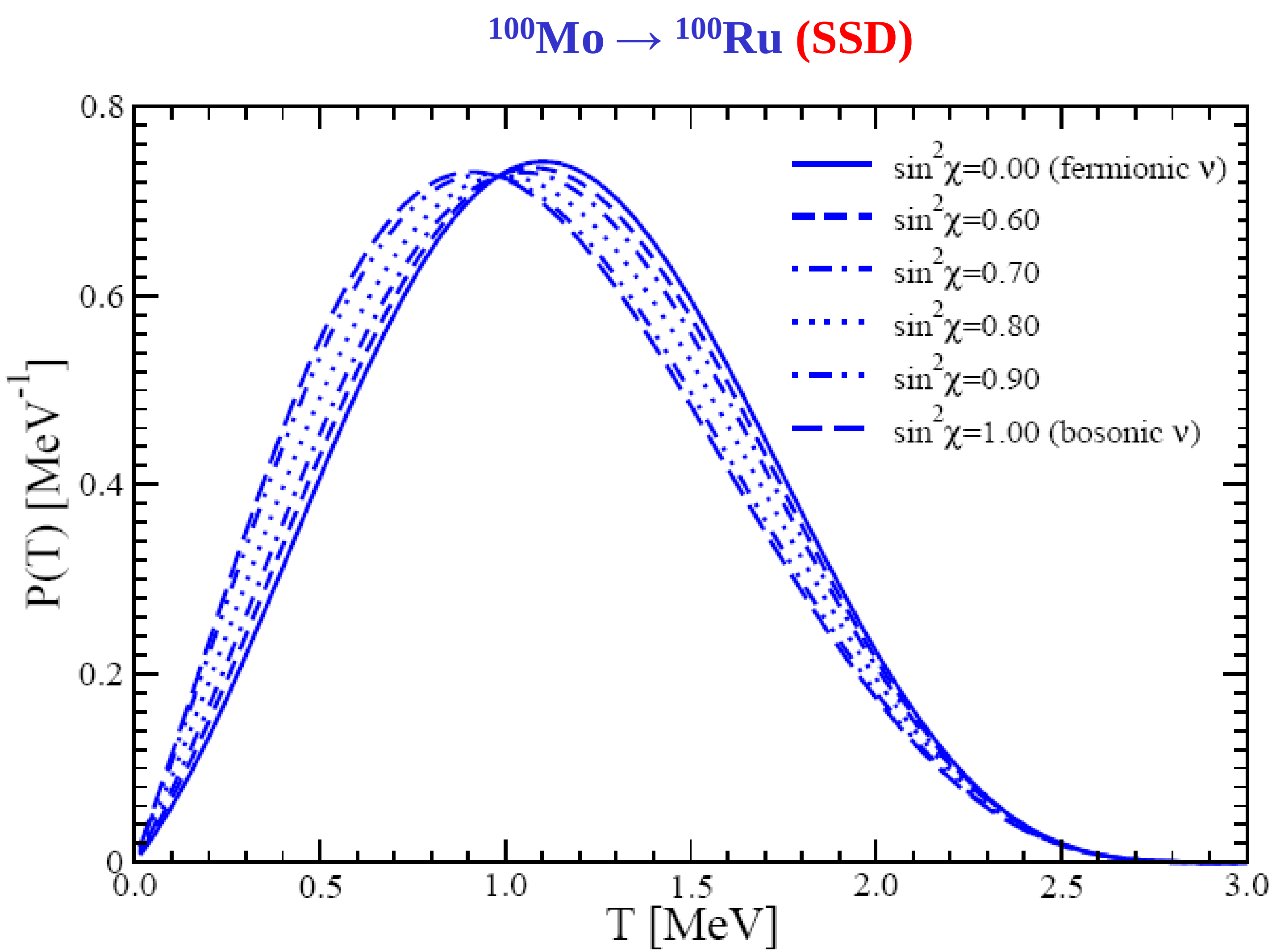
Decay rate

$$
\begin{aligned}
W^{2\nu} &= \cos\chi^4 \ W^f \ + \ \sin\chi^4 \ W^b \\
&= (1-b^2) \ W^f \ + \ b^2 \ W^b
\end{aligned}
$$

Partly bosonic neutrino requires knowing NME or log ft values for HSD or SSD

(calculations coming up soon)

Mixed ν excluded for $\sin^2\chi < 0.6$



Analogues of neutrinoless double beta decay

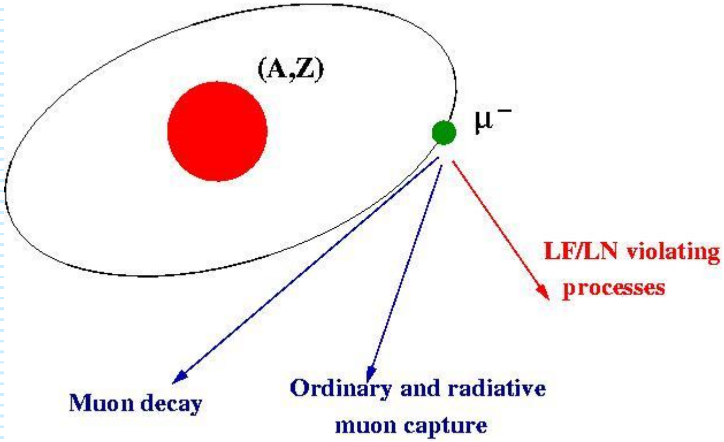
$$\mu^- + (A,Z) \rightarrow (A,Z\text{-}2) + e^+$$
$$\mu^- + (A,Z) \rightarrow (A,Z\text{-}2) + \mu^+$$
$$e^- + e^- \rightarrow W^- + W^-$$
$$K^+ \rightarrow \pi^- + \mu^+ + \mu^+$$

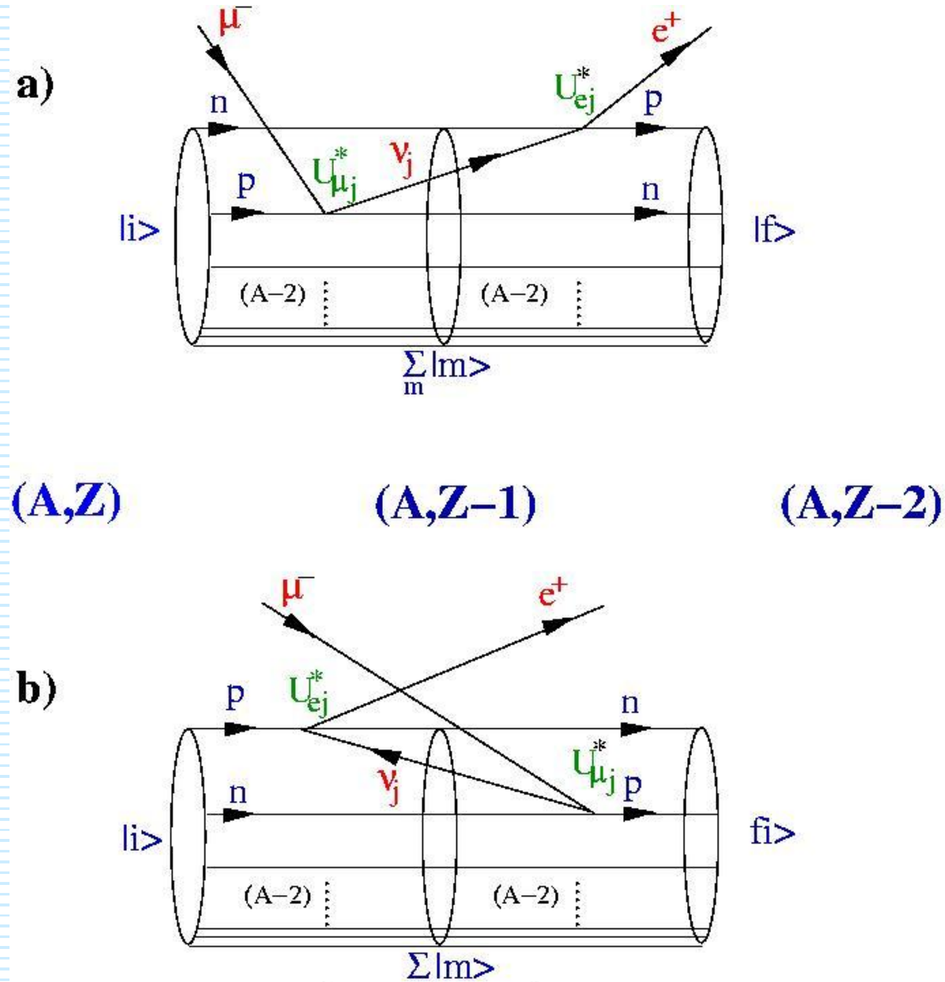
$$m_{\beta\beta} \longrightarrow \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{\mu e} & M_{\mu\mu} & M_{\mu\tau} \\ M_{\tau e} & M_{\tau\mu} & M_{\tau\tau} \end{pmatrix}$$

Muon-positron conversion

$$\mu_b^- + (A,Z) \rightarrow (A, Z-2) + e^+$$

$$\sum_m \frac{|m><m|}{q - E_{\mu^-} + E_m - E_i + i\epsilon_m}$$





$$\frac{\Gamma_{\mu e^+}}{\Gamma_{\beta\beta}} = 176 \left| \frac{m_{\mu e}}{m_{\beta\beta}} \right|^2$$

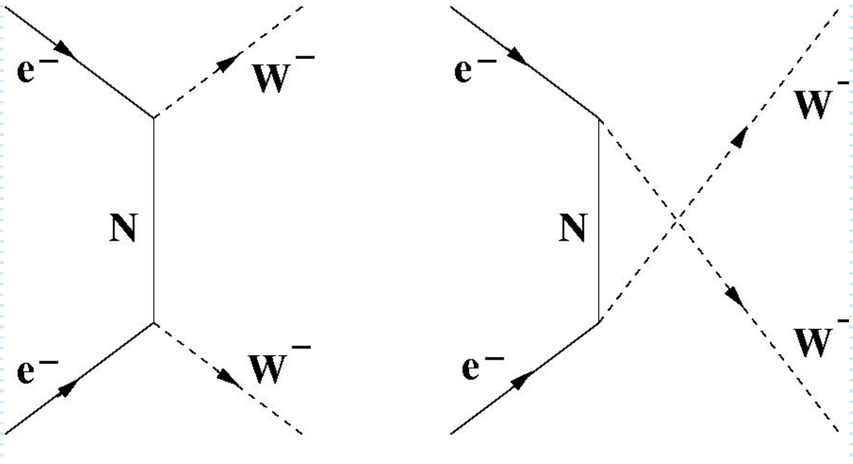
$$\frac{\Gamma_{\mu e^+}}{\Gamma_{\mu}} = 1.3\ 10^{-25} \left| \frac{m_{\mu e}}{m_{e}} \right|^2$$

Domin, Kovalenko, Faessler, Šimkovic, PRD 70, 065501 (2004)

$$\sum_m \frac{|m><m|}{q + E_{e^+} + E_m - E_i + i\epsilon_m}$$

Inverse 0νββ-decay: $e^- e^- \to W^- W^-$

$$\frac{d\sigma}{d\cos\theta} = \frac{g^4}{10024\pi M_W^4}\left[\sum_i M_i |U_{ei}|^2 \left(\frac{t}{(t-M_i^2)} + \frac{u}{(u-M_i^2)}\right)\right]^2$$



Belanger et al. PRD 53 (1996) 6292

The same LNV parameters as in 0νββ–decay:

$|m_{\beta\beta}| < 0.55$ eV

$|\eta_N| < 10^{-7}$

Small neutrino masses

$$\frac{d\sigma}{d\cos\theta} = \frac{g^4}{256\pi M_W^4}|m_{\beta\beta}|^2 \leq 1.3 \times 10^{-17}\ fb$$

Not observable at any future collider

Heavy neutrino masses

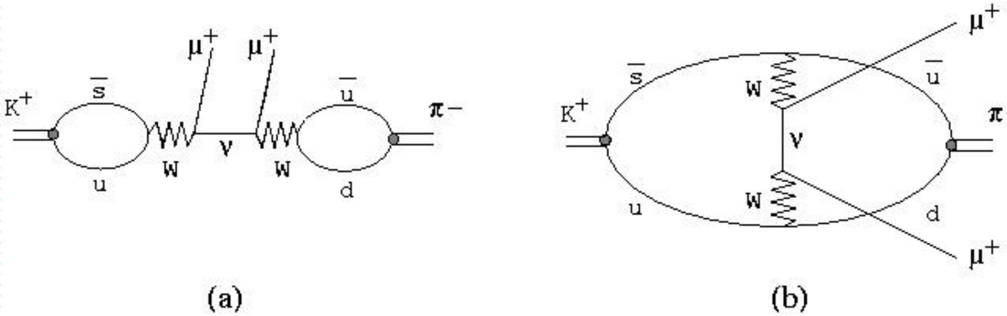
$$\frac{d\sigma}{d\cos\theta} = \frac{g^4}{1024\pi M_W^4}\frac{s^2}{m_p^2}|\eta_N|^2 \leq 4.9 \times 10^{-3}\ fb$$

The hoped-for luminosity at a $\sqrt{s}$=1 TeV NLC is 80 fb^{-1}

K-meson neutrinoless double muon decay

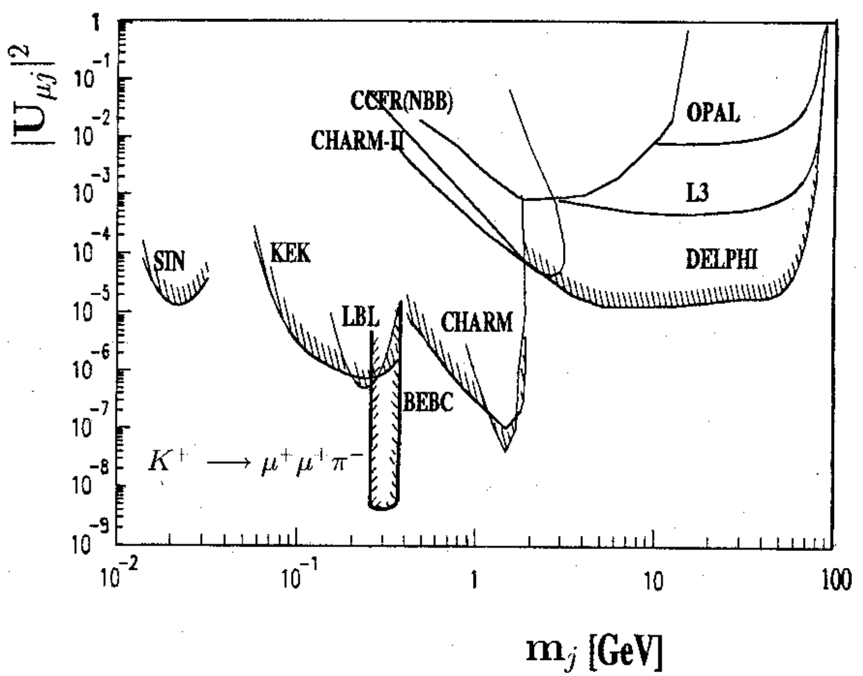
$K^+ \rightarrow \pi^- \mu^+ \mu^+$

Dib, Gribanov, Kovalenko, Schmidt,
PLB 493 (2000) 82



E865 experiment at BNL:

R < 2.0 10^{-9}



The decay width of sterile neutrino play important role

$(m_j \rightarrow m_j + i\Gamma_{\nu\kappa}/2)$

$\nu_k \rightarrow e^+\pi^-, \mu^+\pi^- \ldots$

$$245\ MeV \leq m_{\nu_j} \leq 398\ MeV \Rightarrow |U_{\mu j}|^2 \leq (5.6 \pm 1) \times 10^{-9}$$

Dirac and Majorana ν Magnetic Moment

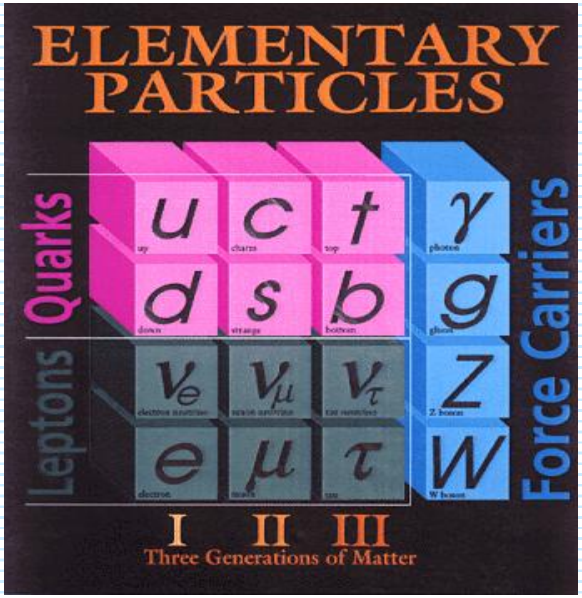
$$H_{\rm eff}^{D} = \frac{1}{2}\mu_{\nu_{ij}}^{D}\bar{\nu}_{iL}\sigma^{\alpha\beta}\nu_{jR}F_{\beta\alpha} + {\rm h.c.}$$

$$H_{\rm eff}^{M} = \frac{1}{2}\mu_{ij}^{M}\bar{\nu}_{iL}\sigma^{\alpha\beta}\nu_{jL}^{c}F_{\beta\alpha} + {\rm h.c.}$$

	Flavor changing MM	Flavor unchanging MM
Dirac	☺	☺
ΔL= 2 Majorana	☺	×

- **Dirac:** can have both diagonal and non-diagonal element
- **Majorana:** cannot have diagonal elememts, means spin flip causes flavor changing.

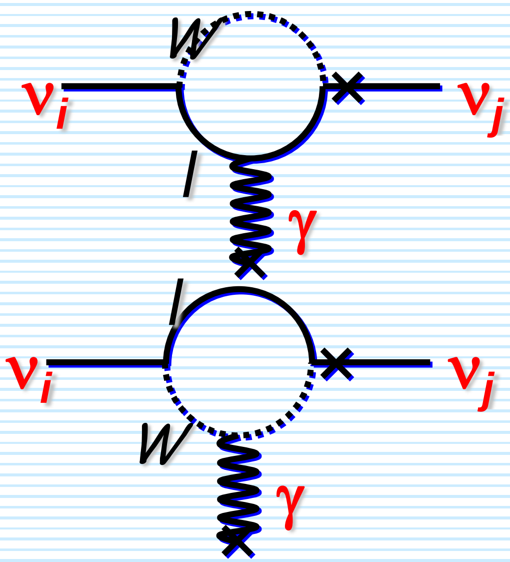
ν magnetic moment in non-minimal SM (+ RH ν)



$SU(3)_{strong}$ x $SU(2)_{weak}$ x U_{em}

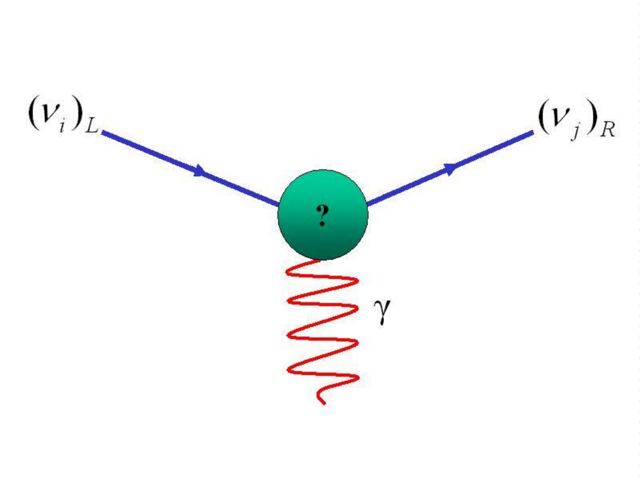
Left-handed	Right-handed
$\begin{pmatrix}\nu_{eL} & u_L\\ e_L & d_L\end{pmatrix}$	e_R u_R d_R ν_{eR}
$\begin{pmatrix}\nu_{eL} & u_L\\ e_L & d_L\end{pmatrix}$	μ_R c_R s_R $\nu_{\mu R}$
$\begin{pmatrix}\nu_{eL} & u_L\\ e_L & d_L\end{pmatrix}$	τ_R t_R b_R $\nu_{\tau R}$

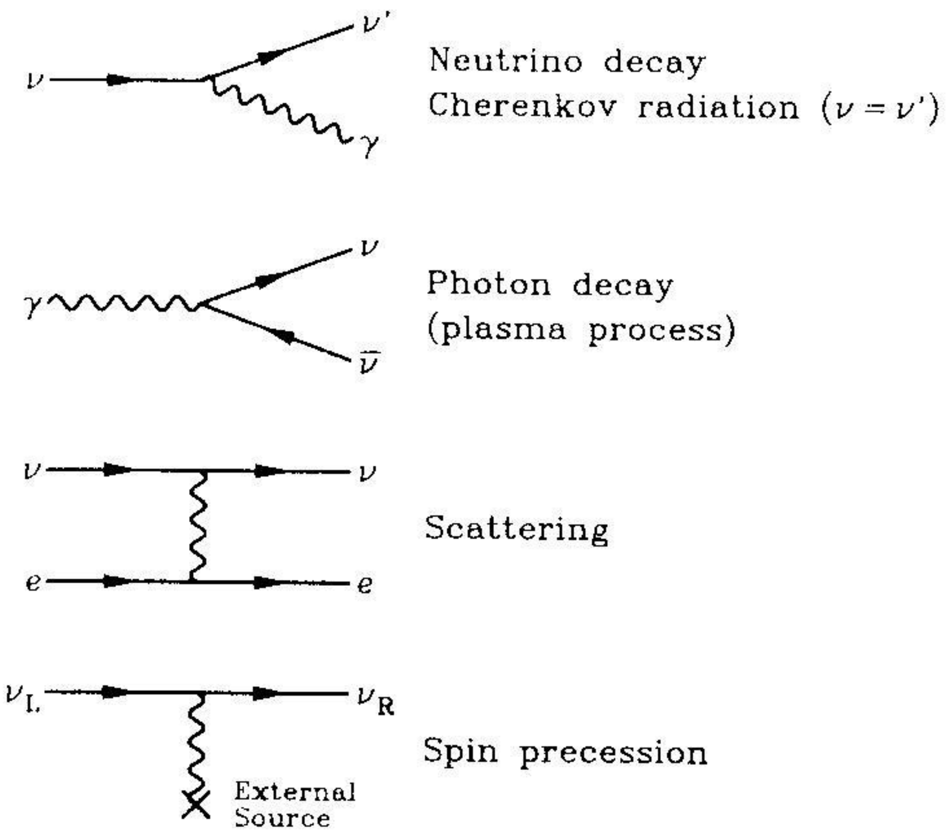
added



$$\mu_\nu = \frac{3}{4\pi^2}\frac{G_F m_e m_\nu}{\sqrt{2}}\mu_B \geq 4\times 10^{-20}\ \mu_B$$

Measuring ν magnetic moment



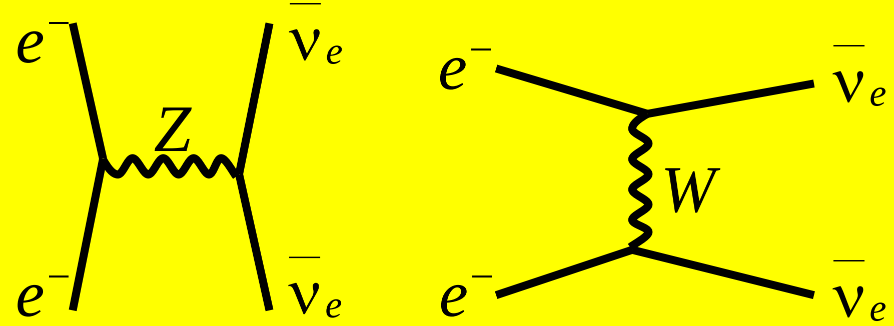


The most restrictive constraint come from astrophysics:

From cooling rate of globular cluster stars ⇒

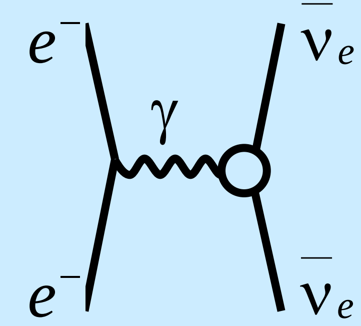
$$\mu^{D}_{\nu_{ij}} \leq 3 \times 10^{-12}\ \mu_B$$

ν scattering experiments



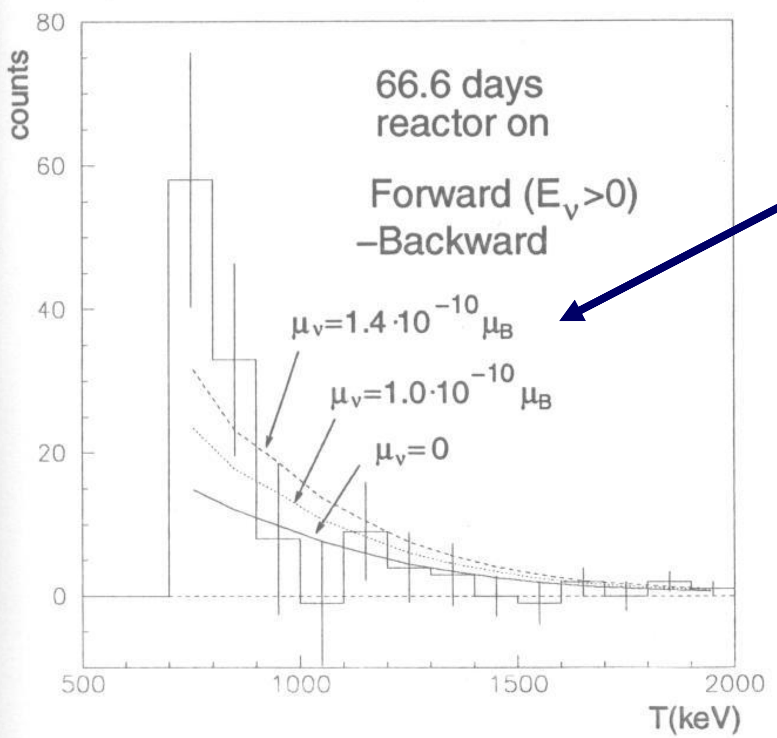
Weak Interactions

+



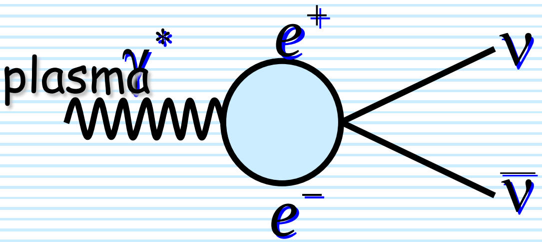
Magnetic Moment

Sensitivity to electron recoil energy



Reactor ν (Rovno, Bugey, Irvine, Krasnoyarsk), solar ν

$$\mu^D_{\nu_{ej}} \leq 0.9 \times 10^{-10}\ \mu_B,$$
$$\mu^D_{\nu_{\mu j}} \leq 6.8 \times 10^{-10}\ \mu_B,$$
$$\mu^D_{\nu_{\tau j}} \leq 3.9 \times 10^{-7}\ \mu_B$$



Neutrino Magnetic Moment (status)

$$\mu^D_{\nu_{ij}} \leq 3 \times 10^{-12} \ \mu_B$$

Astrophysics

$$\mu_\nu = \frac{3}{4\pi^2}\frac{G_F m_e m_\nu}{\sqrt{2}}\mu_B \geq 4 \times 10^{-20} \ \mu_B$$

Standard model

- Fundamental property of the neutrino
- Sizable magnetic moment (near current limits) ⇒ indication of new physics
- Different beyond the SM theories predict different sizes for neutrino magnetic moment

Majorana magnetic moment in R-parity breaking MSSM?

R-parity breaking terms in superpotential

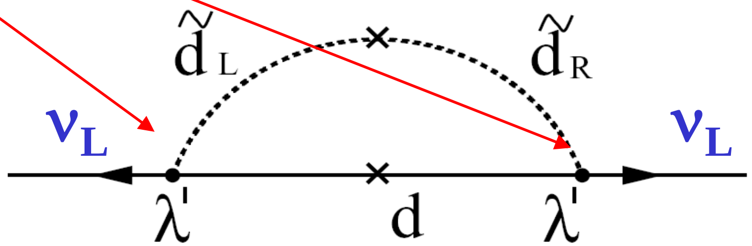
R-parity Breaking MSSM

$$\lambda_{ij<k}\, LLE + \lambda'_{ijk}\, LQD + \lambda''_{ij<k}\, UDD$$

$$9 \quad + \quad 27 \quad + \quad 9 \quad = \quad 45 \text{ coupling constants}$$

Radiative corrections to neutrino mass

$$M_\nu = M^{tree} + M^{l} + M^{q}$$



$$\mathcal{M}^{q}_{ii'} = \frac{3}{16\pi^2}\sum_{jkl}\left\{\left(\lambda'_{ijk}\lambda'_{i'kl}\sum_a V_{ja}V_{la}v^{q}_{ak}m_{d^a}\right)\right.$$
$$\left. + \left(\lambda'_{ijk}\lambda'_{i'lj}\sum_a V_{ka}V_{la}v^{q}_{aj}m_{d^a}\right)\right\}.$$

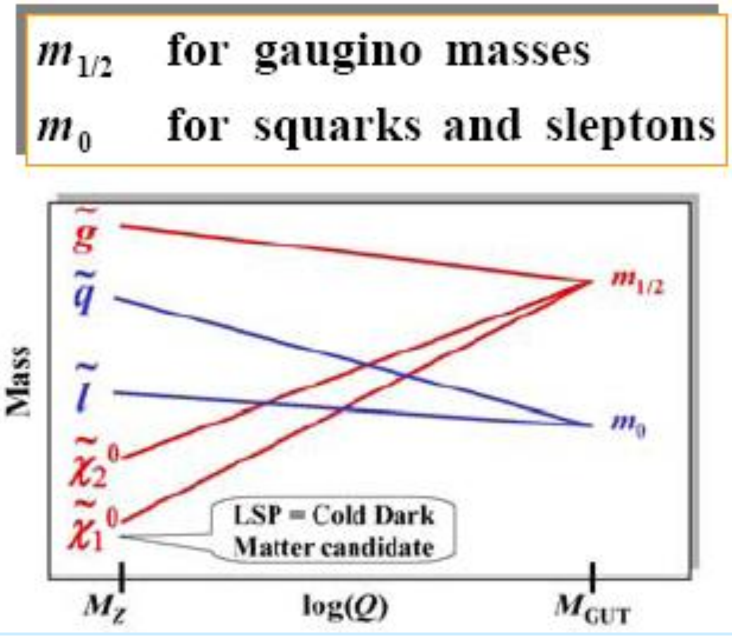
Transitional magnetic moment of Majorana neutrinos

$$\mu^{q}_{\nu_{ii'}} = (1-\delta_{ii'})\frac{12Q_d m_e}{16\pi^2}\sum_{jkl}\left\{\lambda'_{ijk}\lambda'_{i'kl}\sum_a V_{ja}V_{la}\frac{w^{q}_{ak}}{m_{d^a}}\right.$$
$$\left. - \lambda'_{ijk}\lambda'_{i'lj}\sum_a V_{ka}V_{la}\frac{w^{q}_{aj}}{m_{d^a}}\right\}\mu_B$$

Minimal Supergravity Model (mSUGRA)

SUSY model with two Higgs fields in the framework of unification

All SUSY masses are unified at the grand unified scale



$m_{1/2}$ = gaugino mass parameter
$m_0(M_Z)$ = scalar mass parameter for squarks and sleptons
A_0 = Common Yukawa coupling (A_b-bottom sector, A_t-top sector)
$\tan\beta = \langle H_1\rangle/\langle H_2\rangle$
μ = Higgsino mass parameter

GUT constrained low energy spectrum found by solving RGE

- **finite Yukawa coupling at GUT scale**
- **requirements for masses at low energies**
- **FCNC phenomenology ($b\rightarrow s\gamma$ processes)**

Magnetic moment and ν mass

SUSY input				The SUSY conversion coefficient		
A_0 $[GeV]$	m_0 $[GeV]$	$m_{1/2}$ $[GeV]$	$\tan\beta$	$f^q_{\rm SUSY}$ $[eV^{-1}]$	$f^{q-CKM}_{\rm SUSY}$ $[eV^{-1}]$	$f^\ell_{\rm SUSY}$ $[eV^{-1}]$
100	150	150	5	$(0.3, 1.0)\times 10^{-16}$	$(2.8, 8.8)\times 10^{-17}$	$(0.5, 1.5)\times 10^{-15}$
			19	$(0.3, 1.2)\times 10^{-16}$	$(2.8, 9.8)\times 10^{-17}$	$(0.5, 1.6)\times 10^{-15}$
500	1000	1000	5	$(1.1, 2.8)\times 10^{-18}$	$(1.0, 2.4)\times 10^{-18}$	$(1.7, 4.1)\times 10^{-17}$
			19	$(1.1, 3.1)\times 10^{-18}$	$(1.0, 2.7)\times 10^{-18}$	$(1.7, 4.3)\times 10^{-17}$

Loop integrals

$$w^\ell_{jk} = \frac{\sin 2\phi^k}{2}\left(\frac{y_2^{jk}\ln y_2^{jk} - y_2^{jk} + 1}{(1-y_2^{jk})^2} - (y_2 \to y_1)\right)$$

$$v^q_{jk} = \frac{\sin 2\theta^k}{2}\left(\frac{\ln x_2^{jk}}{1-x_2^{jk}} - \frac{\ln x_1^{jk}}{1-x_1^{jk}}\right)$$

$$x_1^{jk} \equiv m^2_{dj}/m^2_{\tilde d^k_1}, \qquad x_2^{jk} \equiv m^2_{dj}/m^2_{\tilde d^k_2}$$

Magneticmagnetic moment expressed with elements of ν mass matrix

$$\mu^q_{\nu_{ii'}} \simeq (1-\delta_{ii'})\frac{4}{3}\mu_B m_{e^1}\mathcal{M}^q_{ii'}\left[\frac{\sum_a V_{ja}V_{la}w^q_{ak}/m_{q^a}}{\sum_a V_{ja}V_{la}v^q_{ak}m_{q^a}}\right]_{\rm max}$$

$$\equiv (1-\delta_{ii'})\mathcal{M}^q_{ii'} f^q_{\rm SUSY}$$

Phenomenological ν mass matrix

Best fit values

$$
\begin{aligned}
|\Delta m^2_{23}| &= 2.1\times 10^{-3}\ ^2 & \sin^2 2\theta_{23} &= 1.00\\
\Delta m^2_{12} &= 7.1\times 10^{-5}\ ^2 & \tan^2\theta_{12} &= 0.40\\
\sin^2\theta_{13} &\leq 5\times 10^{-2} & (90\%\ \text{c.l.})
\end{aligned}
$$

$$
U_{PNMS} = \begin{pmatrix} 1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}
\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta_{13}} & 0 & c_{13}\end{pmatrix}
\begin{pmatrix} c_{12} & s_{12} & 0\\ s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}
\begin{pmatrix} 1 & 0 & 0\\ 0 & e^{i\lambda_{21}} & 0\\ 0 & 0 & e^{i\lambda_{31}}\end{pmatrix}
$$

$$
= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}}\\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23} & s_{23}c_{13}\\ s_{12}s_{23} - c_{12}c_{23} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{23}c_{13}\end{pmatrix}
\begin{pmatrix} 1 & 0 & 0\\ 0 & e^{i\lambda_{21}} & 0\\ 0 & 0 & e^{i\lambda_{31}}\end{pmatrix}
$$

Majorana magnetic moment

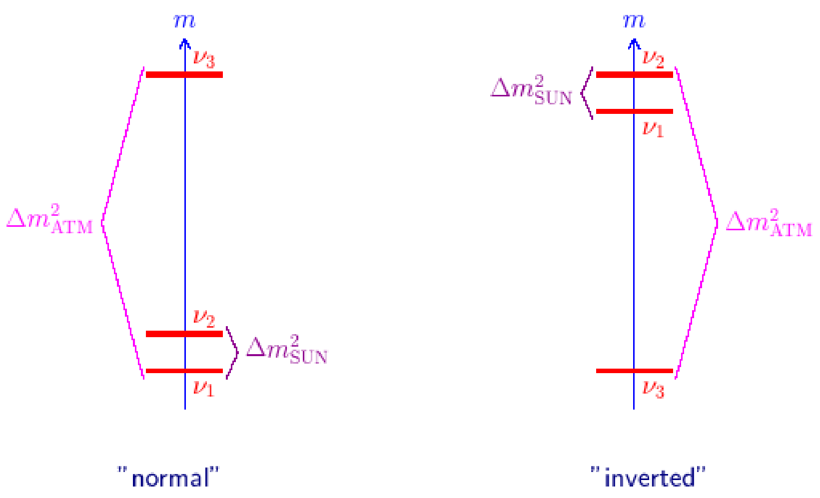
$m_{\beta\beta}$

$$
\begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau}\\ M_{\mu e} & M_{\mu\mu} & M_{\mu\tau}\\ M_{\tau e} & M_{\tau\mu} & M_{\tau\tau}\end{pmatrix}
$$

Assuming arbitrary phases and best fit ν oscaillation data

Upper limits on $M_{\alpha\beta}$ from the measured 0νββ-decay of ^{76}Ge

$$|\mathcal{M}^{ph-HM}| \leq \begin{pmatrix} 0.55 & 1.29 & 1.29 \\ 1.29 & 1.35 & 1.04 \\ 1.29 & 1.04 & 1.35 \end{pmatrix} \; eV$$



Inverted hierarchy

$$|\mathcal{M}^{ph-IH}| = 10^{-2} \; eV \times \begin{pmatrix} (1.80, 4.52) & (0.025, 3.12) & (0.025, 3.12) \\ (0.025, 3.12) & (0.028, 2.40) & (0.945, 2.39) \\ (0.025, 3.12) & (0.945, 2.39) & (0.028, 2.40) \end{pmatrix}$$

Normal hierarchy

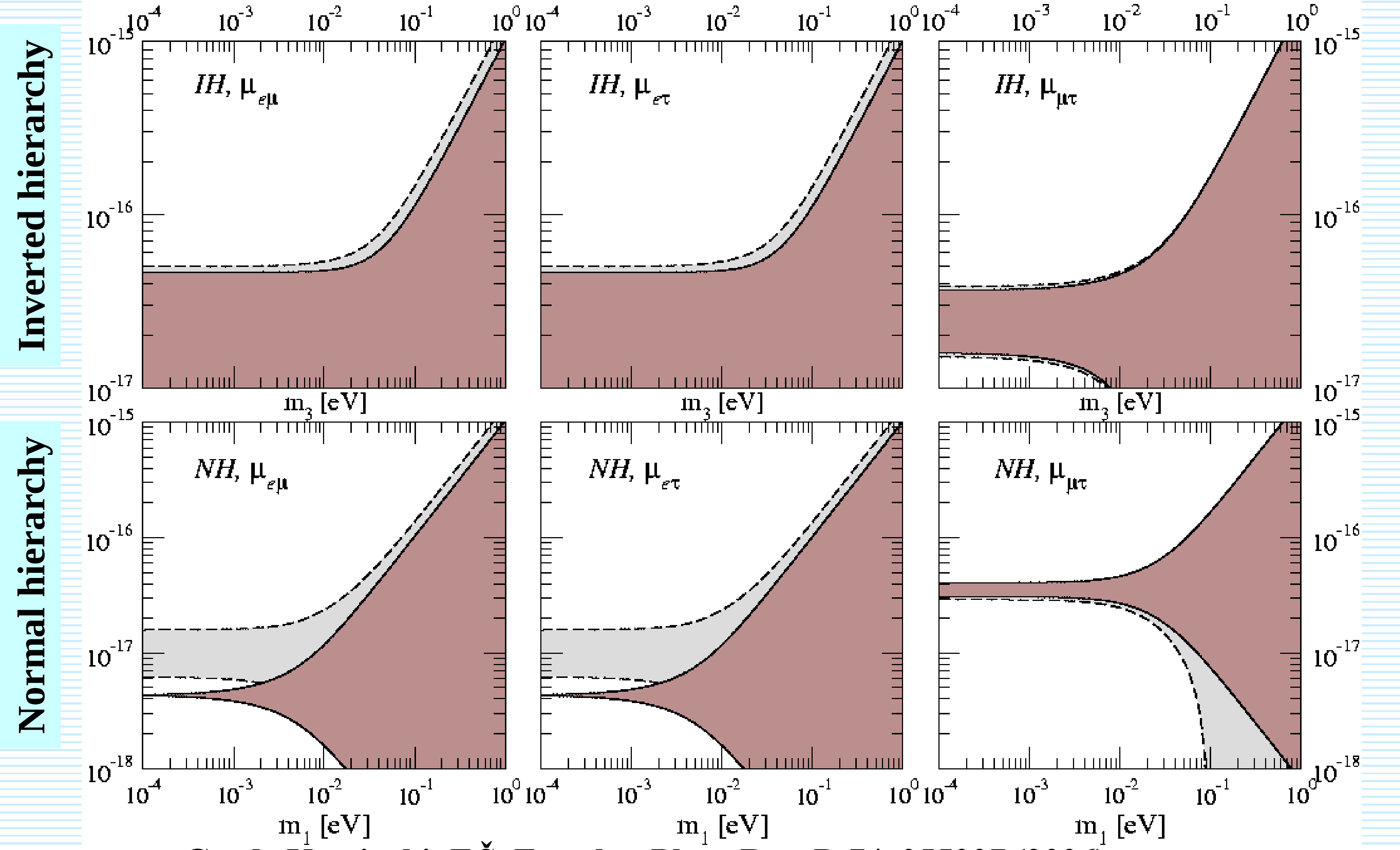
$$|\mathcal{M}^{ph-NH}| = 10^{-4} \; eV \times \begin{pmatrix} (0.53, 45.2) & (26.9, 98.9) & (26.9, 98.9) \\ (26.9, 98.9) & (173, 254) & (182, 254) \\ (26.9, 98.9) & (182, 254) & (173, 254) \end{pmatrix}$$

Assuming mass of the lightest ν is negligible

Transitional magnetic moment of Majorana ν

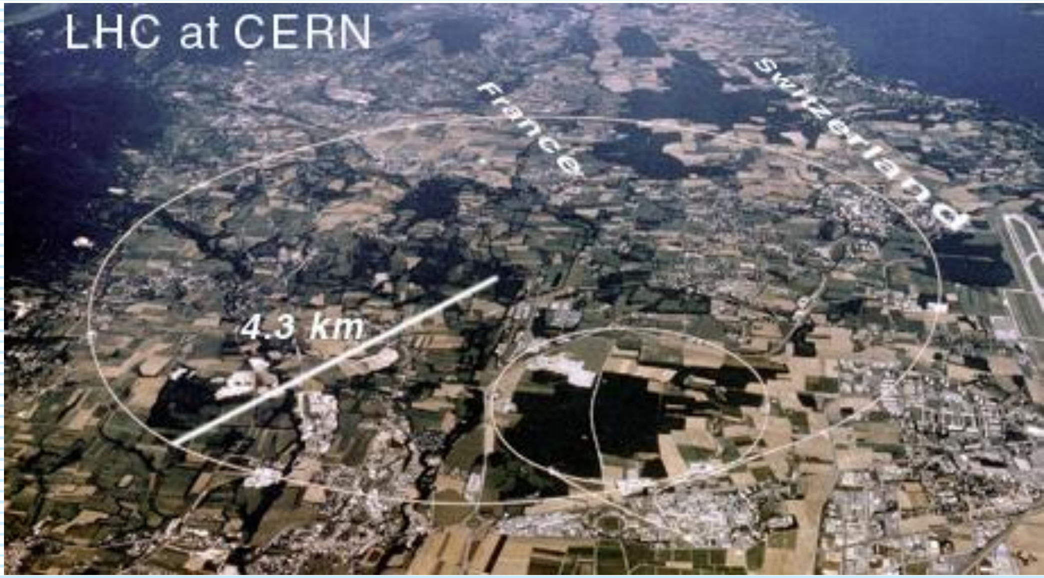
R-parity violation SUSY, masses generated via lepton-slepton loops

$m_0 = m_{1/2} = 150$ GeV, $A_0 = 100$ GeV
$\tan\beta = 19$



Gozdz,Kaminski, F.Š, Faessler, Phys. Rev. D 74, 055007 (2006)

SUSY parameter space will be fixed at colliders?!



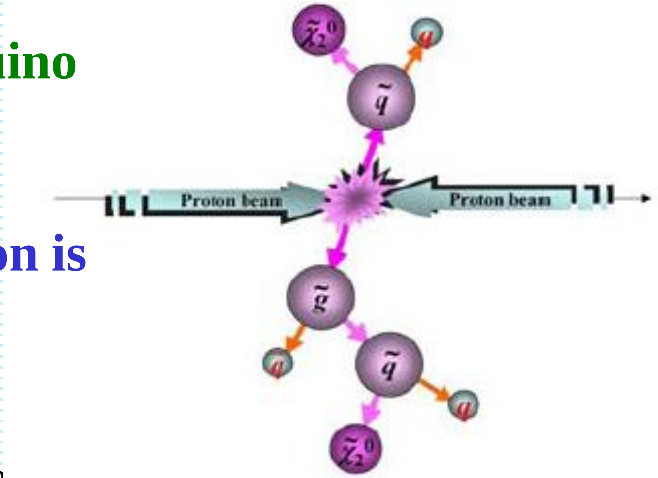
Tevatron (2 TeV, p anti-p)
LHC (14 TeV, pp)

LHC experiments will provide a crucial test for SUSY. The LHC will be powerfull enough to produce many SUSY particles. Mass reach of squark and gluino search is 2 TeV.

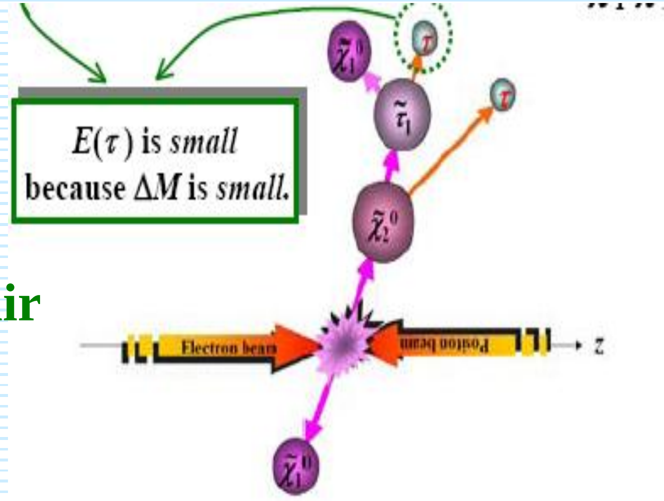
LC (500-800 GeV, e^-,e^+)

Squark-gluino production

cross-section is large



neutralino-pair production



We have no end of fun with neutrino physics.

Mathematics is Egyptian

Neutrino physics is Babylonian

We have to communicate more with neutrinos.