Mössbauer Antineutrinos: Recoilless Resonant Emission and Absorption of Electron Antineutrinos

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Outline

I) Conventional Mössbauer spectroscopy (photons)

II) β-decay; bound-state β-decay: resonant character

^{3}H – ^{3}He system

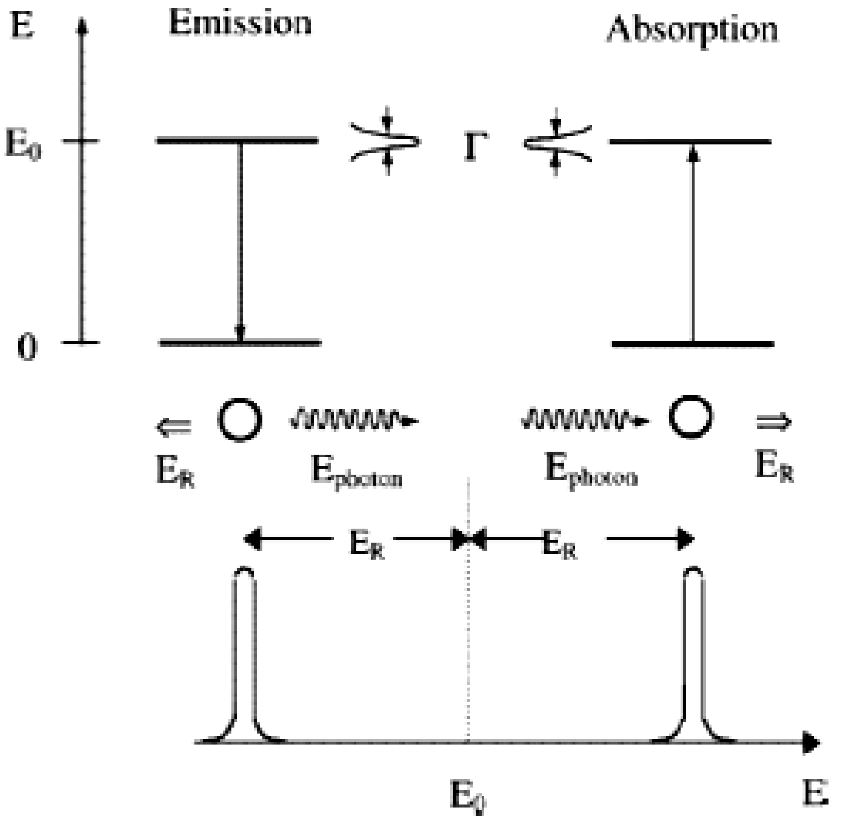
III) Mössbauer $\bar{\nu}_e$: Basic questions – Principal problems

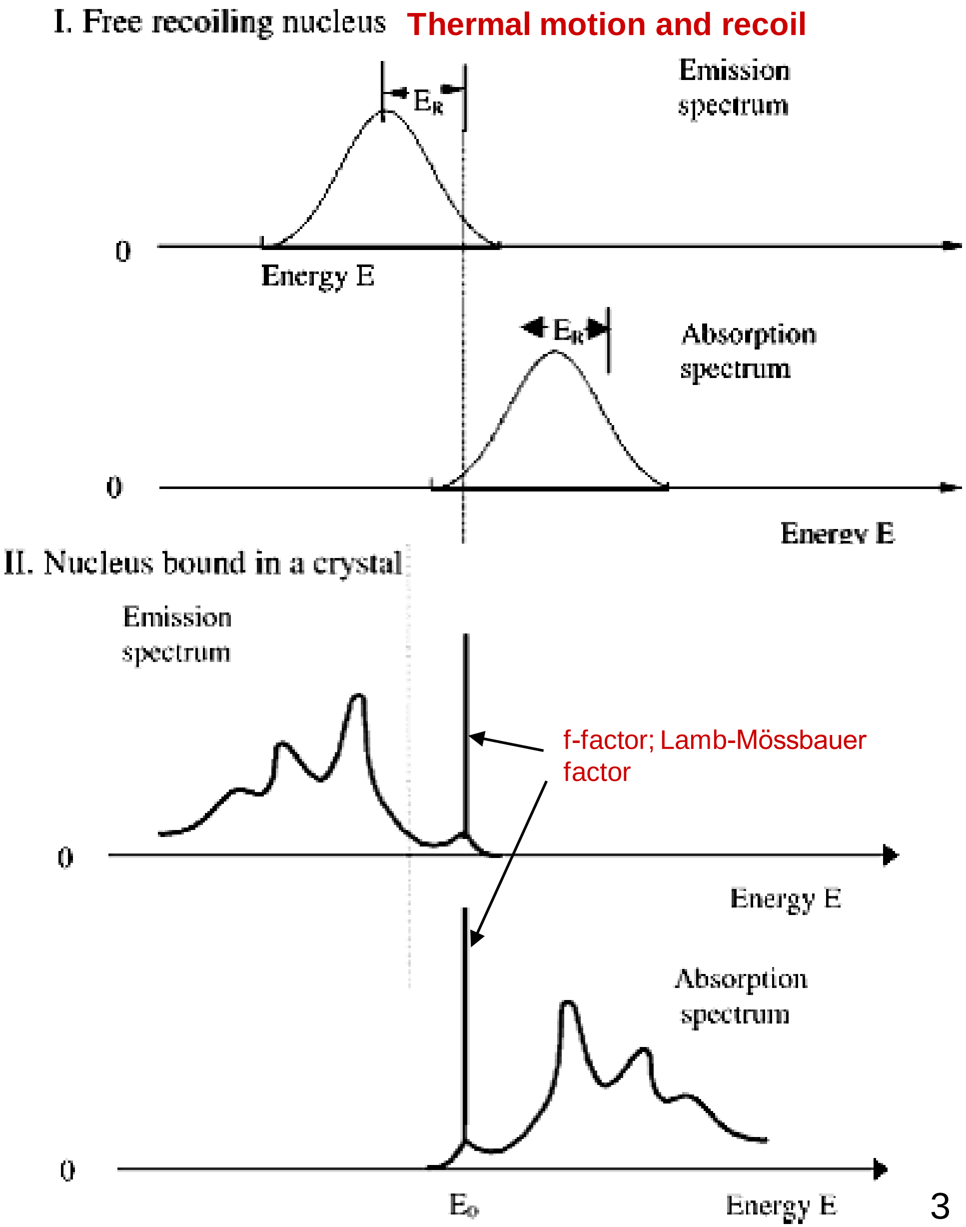
1) Phononless transition: Recoilfree fraction; lattice expansion and contraction
2) Linewidth: homogeneous and inhomogeneous broadening
3) Relativistic effects: Second-order Doppler shift
   a) temperature
   b) zero-point motion
4) Answers to basic questions
5) Rare-earth systems

IV) Interesting experiments

V) Conclusions

I) Conventional Mössbauer Spectroscopy





Thermal motion and recoil

f-factor; Lamb-Mössbauer factor

R. L. Mössbauer, Hyperfine Interact. **126**, 1 (2000)

I) Conventional Mössbauer Spectroscopy

Mössbauer Isotope	Transition energy E_0 [eV]	Recoil energy E_R [eV] $E_R = \frac{E_0^2}{2Mc^2}$	Natural linewidth Γ [mm/s] [eV]	Maximal resonance cross section σ_R [cm^2]
^{57}Fe	14 400	1.9×10^{-3}	0.09 $\approx 4.3 \times 10^{-9}$ eV	1.2×10^{-17}
^{181}Ta	6 300	0.1×10^{-3}	3.2×10^{-3} $\approx 6.7 \times 10^{-11}$eV	6.2×10^{-17}
^{67}Zn	93 300	69.2×10^{-3}	1.6×10^{-4} $\approx 4.8 \times 10^{-11}$eV	2.8×10^{-19}
^{109}Ag	87 700	37.6×10^{-3}	4.1×10^{-11} $\approx 1.2 \times 10^{-17}$eV	3.2×10^{-19}

Recoil energy:

$$E_R = \frac{E_0^2}{2Mc^2}$$

Typical: meV

$$\Gamma << E_R$$

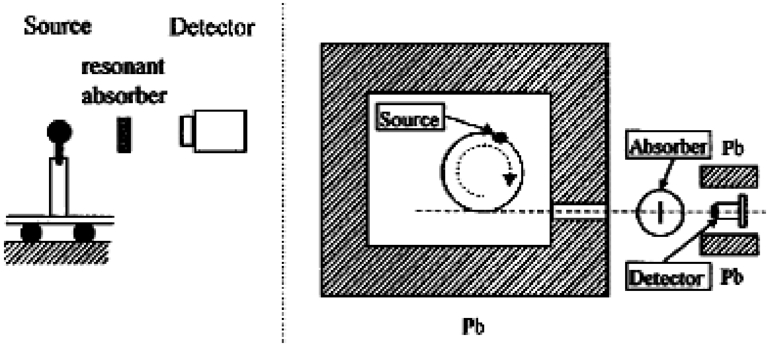
$$\bar{\lambda} = \frac{\lambda}{2\pi}$$

Wavelength of emitted photon with energy E_0

Max. resonance cross section:

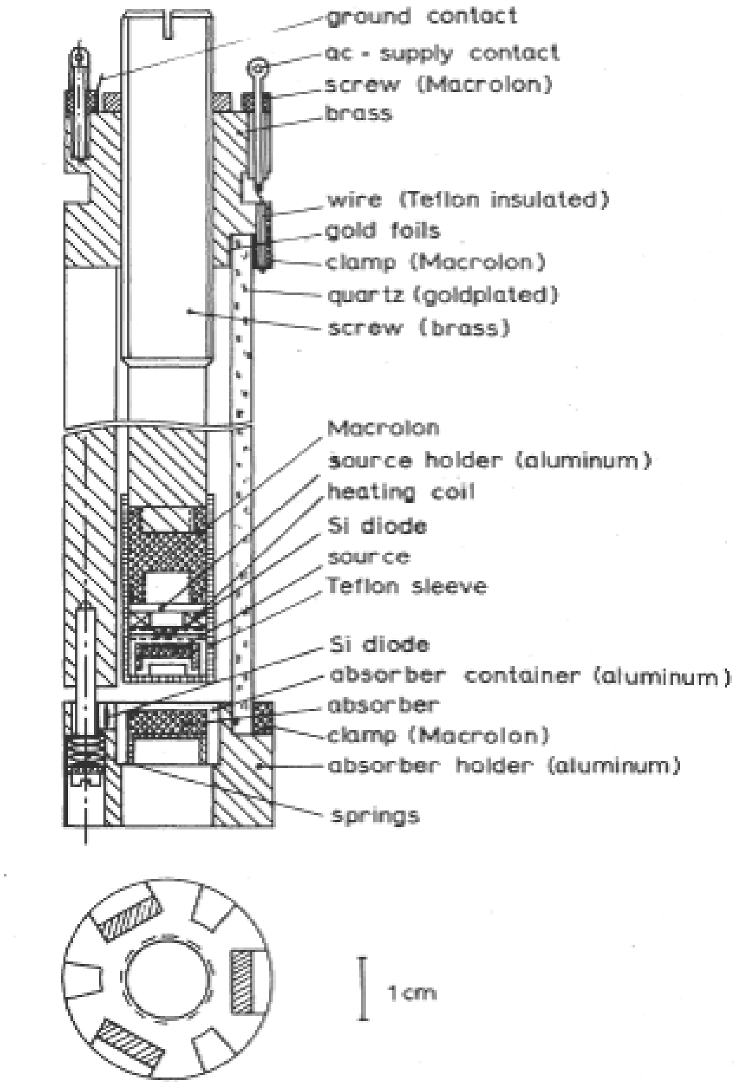
$$\boxed{\sigma_R = 2\pi\bar{\lambda}^2 (s^2 f^2) \frac{\Gamma}{\Gamma_{\exp}}}$$

Many interesting applications:
Solid-State Physics, Chemistry,
Special and General Relativity



R. L. Mössbauer, Hyperfine Interact. **126**, 1 (2000)

I) Conventional Mössbauer Spectroscopy



Piezoelectric drive used for high-resolution ^{67}Zn Mössbauer spectroscopy

accuracy: $(\Delta E/E) \leq 1\text{x}10^{-18}$

i.e., 3Å/s ≈ 1cm/y

W. Potzel et al.,
Phys. Rev. B**30**, 4980 (1984)

Gravitational Redshift Experiment

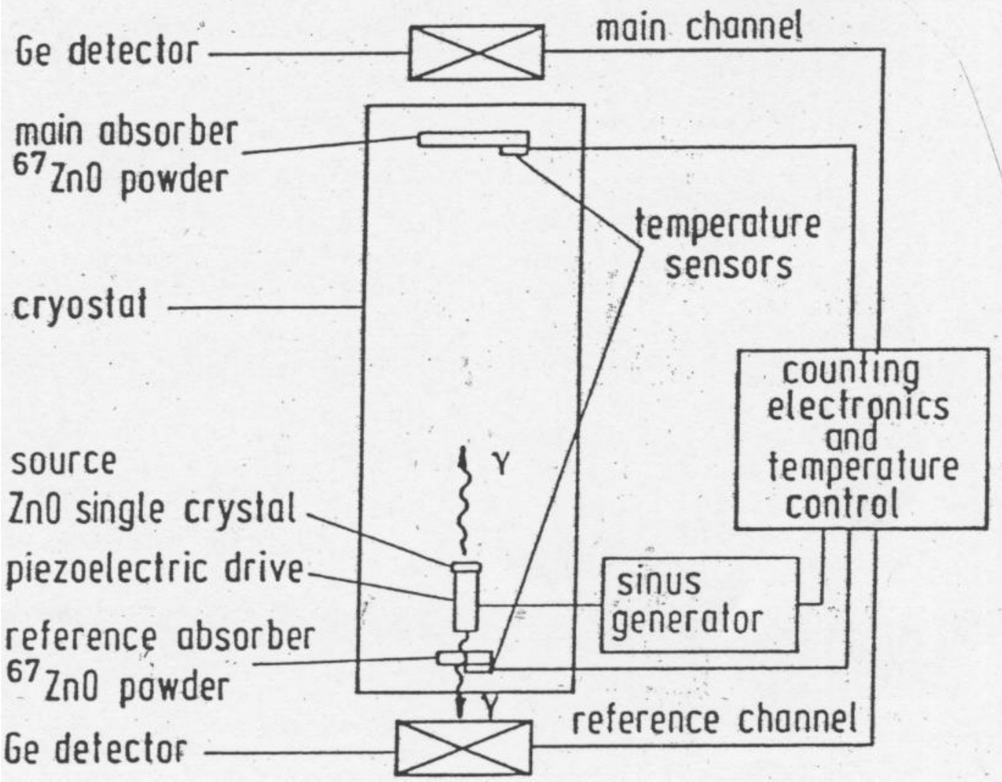
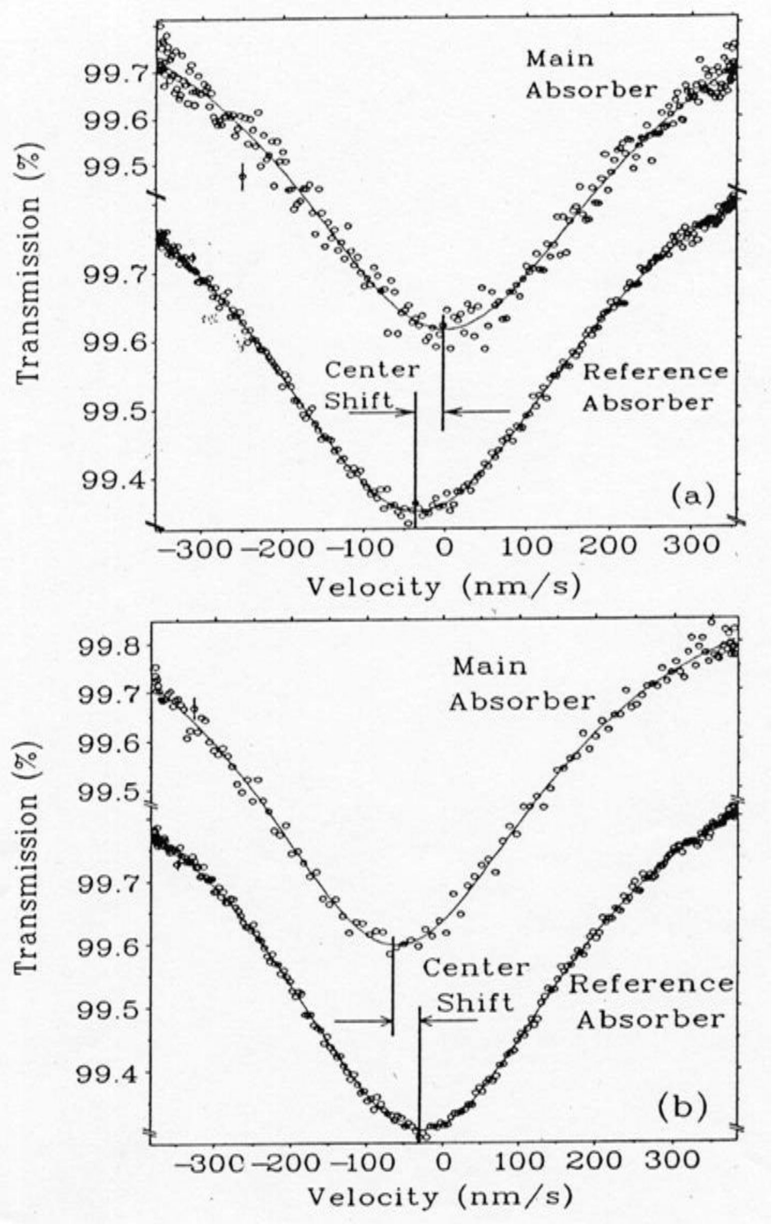


Fig. 3. Basic set-up of the Mössbauer gravitational redshift experiment. Two transmission experiments are carried out simultaneously: through the reference absorber and through the main absorber. A piezoelectric drive moves the source sinusoidally with respect to both absorbers.

W. Potzel et al., Hyp. Interact. 72, 197 (1992)

Red(blue)shift ^{67}ZnO-Mössbauer exp.



gravitational redshift

difference in height: 1m
in gravitational field of Earth

gravitational blueshift

accuracy: $(\Delta E/E) \leq 1\mathrm{x}10^{-18}$
i.e., 3Å/s ≈ 1cm/y

W. Potzel et al., Hyp. Interact.
72, 197 (1992)

Gravitational shift ^{109m}Ag

^{109m}Ag: gravitational spectrometer

$$\Gamma \approx 1.2 \cdot 10^{-17} eV \qquad \tau \approx 40s$$

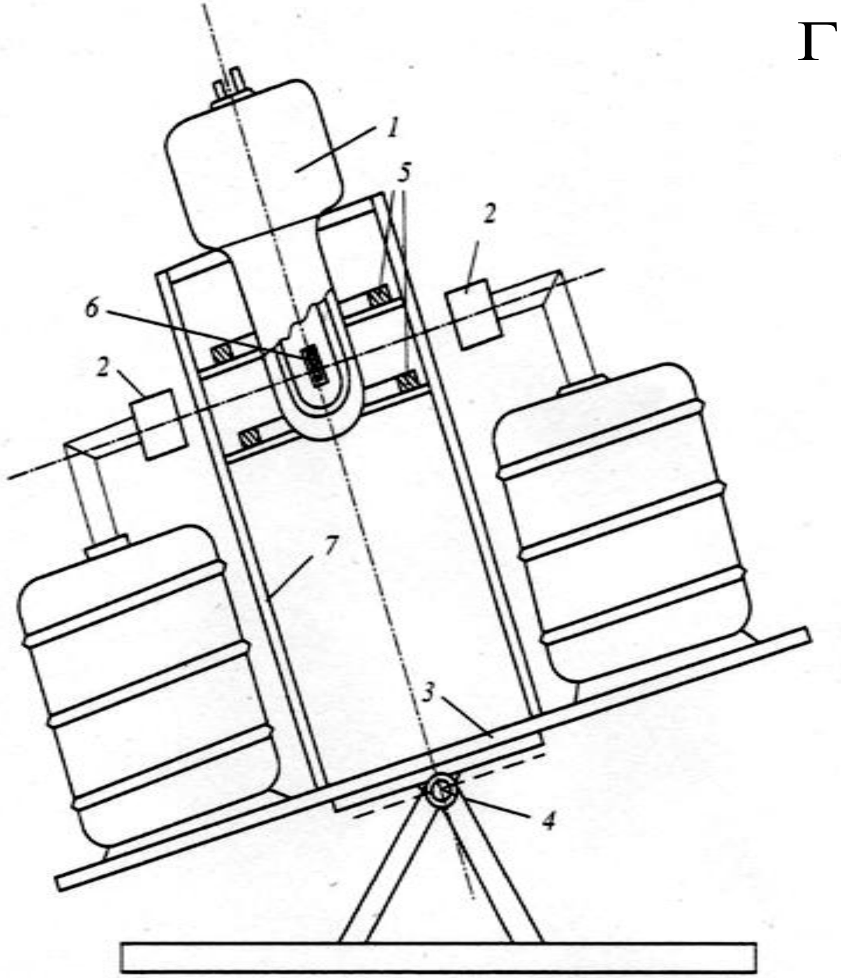
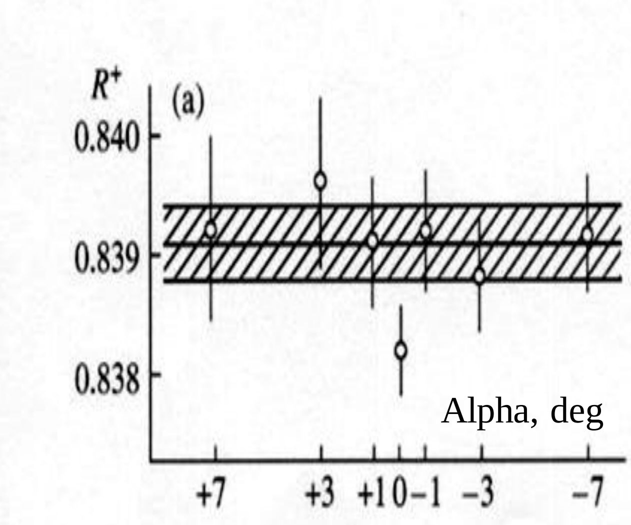


Fig. 1. Scheme of the gravitational gamma spectrometer: (*1*) cryostat, (*2*) germanium gamma detectors, (*3*) rotating platform, (*4*) support of cryostat and Helmholtz coils, (*5*) Helmholtz coils, (*6*) gamma sources, and (*7*) rotation axic of the platform.



V.G. Alpatov et al., Laser Physics 17 (2007) 1067
Yu.D. Bayukov et al., JETP Letters 90 (2009) 499

e^-

II) β-decay

1) Usual β-decay

$$A(Z-1) \rightarrow A(Z) + \underbrace{e^- + \bar{\nu}_e}_{\text{occupy states in continuum}}$$

neutron transforms into a proton

3-body process: $e^-, \bar{\nu}_e$ show (broad) energy spectra

Maximum $\bar{\nu}_e$ energy: $E_{\bar{\nu}_e}^{\max} = Q$

where $Q = (M_{Z-1} - M_Z)c^2$

is the end-point energy

II) Bound-state β-decay

2) Bound-state β-decay

J. N. Bahcall, Phys. Rev. 124, 495 (1961)

$$A(Z-1) \rightarrow \boxed{A(Z) + e^-} + \bar{\nu}_e$$

$\bar{\nu}_e - source$

mono-energetic

Bound-state atomic orbit.
Not a capture of e$^-$ initially created in a continuum state (less probable).

2-body process, $\bar{\nu}_e$ has a fixed energy:

$\boxed{E_{\bar{\nu}_e} = Q + B_z - E_R}$ where

$Q = (M_{Z-1} - M_Z)c^2$ end-point energy

B_z binding energy of electron

E_R recoil energy $\boxed{{}^3He + e^-}$ recoils

Example:

$${}^3H \rightarrow \boxed{{}^3He + e^-} + \bar{\nu}_e$$

Atomic orbit in ^{3}He

II) Bound-state β-decay

Reverse process (absorption):

$$\bar{\nu}_e + \boxed{A(Z) + e^-} \rightarrow A(Z-1)$$

in atomic orbit

target for $\bar{\nu}_e$

Example:

$$\bar{\nu}_e + \boxed{{}^3He + e^-} \rightarrow {}^3H$$

in atomic orbit of ^{3}He

energy required for $\bar{\nu}_e$:

$$\boxed{E_{\bar{\nu}_e}{}' = Q + B_z + E_R'}$$

^3H recoils

Bound-state β-decay has a resonant character which is (partially) destroyed by the recoil in source and target.

II) Bound-state β-decay

Resonance cross section

$$\sigma = 4.18 \cdot 10^{-41} g_0^2 \cdot \frac{\rho(E_{\bar{\nu}_e}^{res})}{ft_{1/2}} \left[cm^2\right]$$

L.A. Mikaélyan, et al.: Sov.
J. Nucl. Phys. 6, 254 (1968)

$$g_0 = 4\pi \left(\frac{\hbar}{mc}\right)^3 |\Psi|^2 \approx 4\left(\frac{Z}{137}\right)^3$$

for low Z, hydrogen-like ψ
m: electron mass
ψ^2: probability density of e in A(Z)

$\rho(E_{\bar{\nu}_e}^{res})$: resonant spectral density, i.e., number of $\bar{\nu}_e$ in an energy interval of 1MeV around $E_{\bar{\nu}_e}^{res}$

$ft_{1/2}$ value: reduced half-life of decay

$ft_{1/2} \approx 1000$: super-allowed transition

II) ^{3}H-^{3}He system

$Decay$	$E_{\bar{\nu}_e}^{res}$	$ft_{1/2}$	$B\beta / C\beta$
$^3H \rightarrow {}^3He$ $\tau = 17.81y$	18.60 keV	1132 sec	α=6.9x10^{-3} (80% ground state, 20% excited states)

Resonance cross section (without Mössbauer effect): $\sigma \approx 1\text{x}10^{-42}$ cm^2

To observe bound-state β-decay: 100-MCi sources (^{3}H) and kg-targets (^{3}He) would be necessary

Thermal motion:

Doppler energy profile, width: 0.16 eV

Recoil energy:

$$E_R = \frac{(E_{\bar{\nu}_e}^{res})^2}{2Mc^2} \approx 0.06eV$$

→ Source and target imbedded in solid-state lattices

Mössbauer effect: $$\sigma_R = \underbrace{2\pi\lambdabar^2}_{7.1\text{x}10^{-18}\text{ cm}^2}(s^2\alpha^2 f^2)\frac{\Gamma}{\Gamma_{\exp}}$$

For ^{3}H / ^{3}He: $\sigma_R \approx 1.8\text{x}10^{-22}$ cm^2 (maximal)

W. M. Visscher, Phys. Rev. 116, 1581 (1959)
W. P. Kells and J. P. Schiffer, Phys. Rev. C 28, 2162 (1983)

R. S. Raghavan, hep-ph/0601079 v3, 2006

Mössbauer Resonance Cross Section

For Mössbauer neutrinos:

W. M. Visscher, Phys. Rev. 116, 1581 (1959)
W. P. Kells and J. P. Schiffer, Phys. Rev. C 28, 2162 (1983)

$$\boxed{\sigma_R = 2\pi\lambdabar^2 (s^2\alpha^2 f^2)\frac{\Gamma}{\Gamma_{\exp}}}$$

Breit-Wigner formula

^{3}H – ^{3}He system, bound-state ß-decay
Resonant, transition energy: 18.6 keV

σ_R=1.8x10^{-22} cm^2 (as compared to 1.1x10^{-42} cm^2)

Although weak interaction !

II) ^{3}H-^{3}He system

Preparation of source and target

Source:

^{3}H chemically loaded into metals to form hydrides (tritides), e.g., Nb: in tetrahedral interstitial sites (IS).

Target:

^{3}He accumulates with time due to the tritium trick:

$$\mathrm{Nb^3H_x} \xrightarrow{\text{time=200d}} \mathrm{Nb^3H_{x\text{-}y}{}^3He_y} \xrightarrow{\text{remove}} \mathrm{Nb^3He_y}$$

Remove ^{3}H by isotopic exchange ^{3}H→D

R. S. Raghavan, hep-ph/0601079 v3, 2006

II) ^3H-^3He system

How much metal for source and target?

Source:

1 kCi of ^3H (~100mg ^3H): ~3g of Nb^3H

for NMR studies: 0.5 kCi ^3H in 2.4g $PdH_{0.6}$

Target:

100mg of ^3He implies ~100g of Nb^3H aged for 200 d

III) Mössbauer Nu‘s: Basic Questions

1) Phononless transition:

a) Recoilfree fraction:

Stop thermal motion!
Make E_R negligibly small!

recoil energy:

$$E_R = \frac{(E_{\bar{\nu}_e}^{res})^2}{2Mc^2}$$

^{3}H as well as ^{3}He in metallic lattices:
freeze their motion →no Doppler broadening.
M→$M_{lattice}$>>M
Leave lattice unchanged, leave phonons unchanged.

zero-point energy

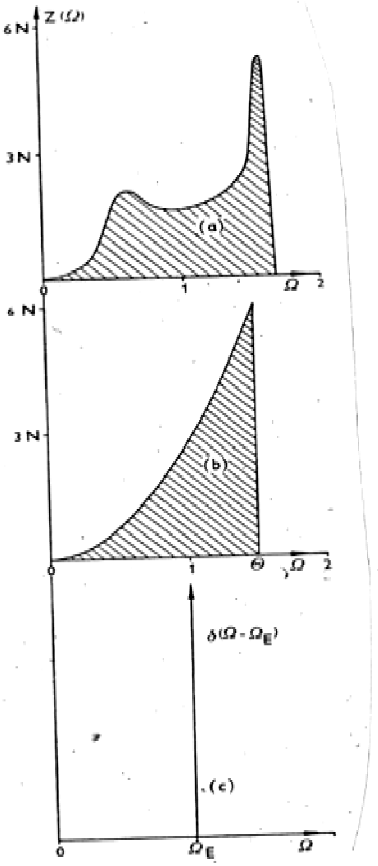
Energy of lattice with N particles: $E_L = \sum_{s=1}^{3N} (n_s + 1/2)\hbar\omega_s$ $\quad (n_s = 0,1,2,\dots)$

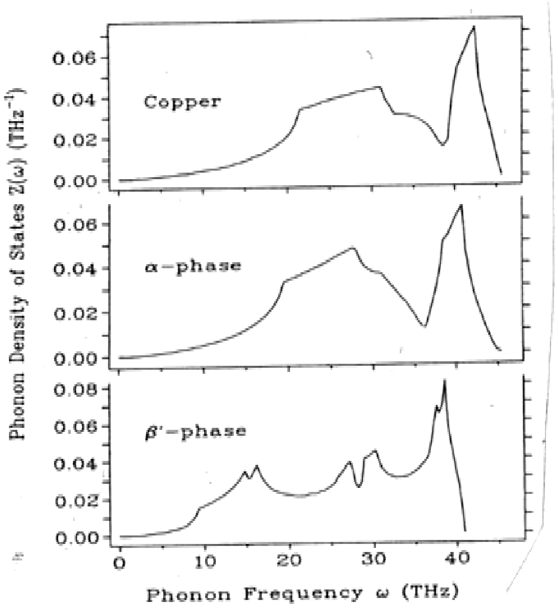
3N normal modes

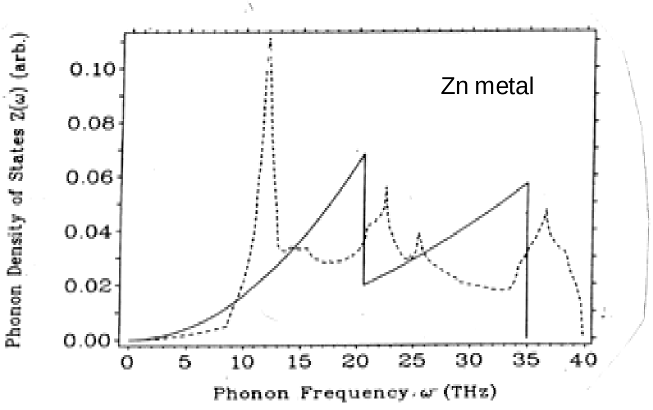
$$E_L = \int_0^{\omega_{\max}} \left(\overline{n(\omega)} + 1/2\right)\omega \cdot Z(\omega)\, d\omega$$ with $\overline{n(\omega)} = 1/\left(\exp(\hbar\omega/k_B T) - 1\right)$

$Z(\omega)\cdot d\omega$: number of oscillators with frequency *ω* between *ω* and *ω* + *dω*

Phonon density of states







III) Mössbauer Nu‘s: Basic Questions

1) Phononless transition:

a) Recoilfree fraction:

$$f = e^{-\left(\frac{E}{\hbar c}\right)^2 \cdot \langle x^2 \rangle} \longrightarrow f < 1$$

Conventional Mössbauer effect (with photons):

Source and absorber (target) involve the <u>same</u> type of atoms, e.g., the isotope ^{57}Fe.

recoil energy:

$$E_R = \frac{(E^{res}_{\bar{\nu}_e})^2}{2Mc^2}$$

Debye model:

$T \rightarrow 0$:

$$f(T \rightarrow 0) = \exp\left\{ - \underbrace{\frac{E^2}{2Mc^2}}_{\text{recoil energy}} \cdot \frac{3}{2k_B\Theta} \right\}$$

f depends on: transition energy E
mass M of the atom
Debye temperature Θ

Example: ^{3}H – ^{3}He

typically: $f(0) \approx 0.27$ for Θ≈800K

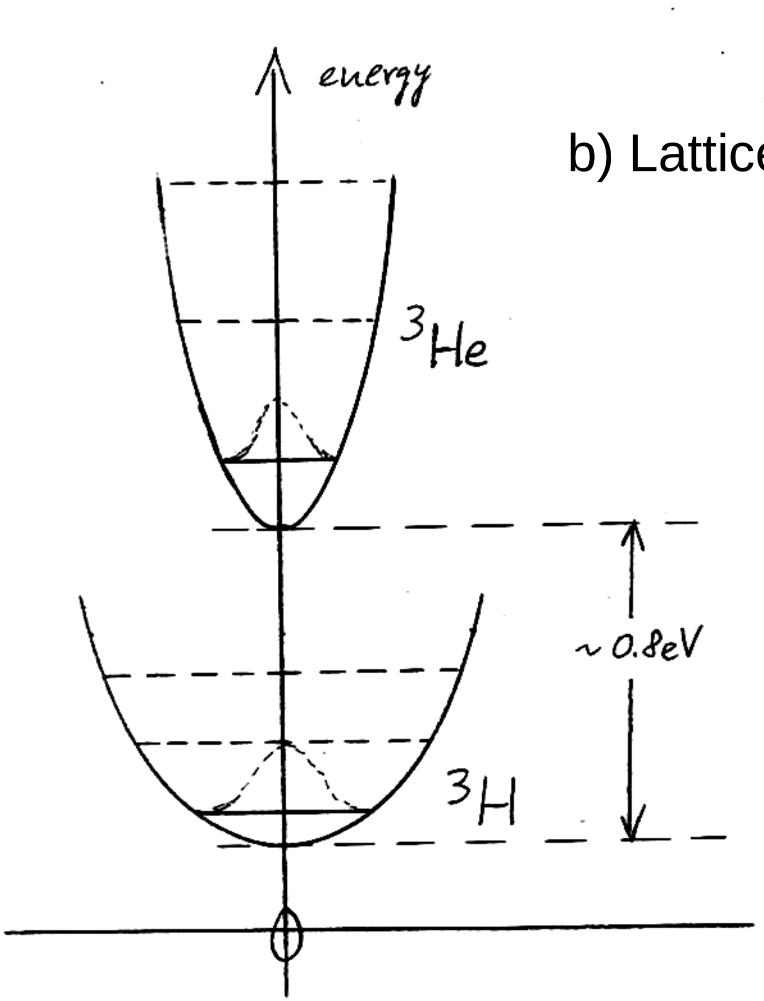
Emission and absorption:

$f^{\,^3H} \cdot f^{\,^3He} \approx 0.07$ for T→0

III) Mössbauer Nu‘s: Basic Questions

^{3}H as well as ^{3}He in metallic lattices:

Nb metal, tetrahedral interstitial sites



M. J. Puska et al., PRB 29, 5382 (1984)

b) Lattice expansion and contraction: in addition to recoil

Nuclear transformations occur when $\overline{\nu}_e$ is emitted or captured. ^{3}He and ^{3}H use different amounts of lattice space. Will this cause lattice excitations (phonons)?

Lattice-deformation energies of ^{3}H and ^{3}He in Nb metal:

$$E_L(^3H) = 0.099eV; E_L(^3He) = 0.551eV$$

$$f^L(T \rightarrow 0) \leq \exp\left\{ -\frac{E_L(^3He) - E_L(^3H)}{k_B \Theta} \right\} \approx 1 \cdot 10^{-3}$$

$f^L(0)^2 \approx 1 \cdot 10^{-6}$ and $f(0)^2 \cdot f^L(0)^2 \approx 7 \cdot 10^{-8}$

→ Theoretical calculations:

David Ceperley, Univ. of Illinois at Urbana-Champaign USA

III) Mössbauer Nu‘s: Basic Questions

2) Linewidth

minimal width (natural width): $\Delta E^{nat} = \Gamma = \hbar / \tau$ τ: lifetime

^{3}H: τ = 17.81 y ⟶ $\Delta E^{nat} = \Gamma = 1.17 \cdot 10^{-24} eV$ (extremely narrow)

R.S. Raghavan: possible to reach nat. width
Phys. Rev. Lett. **102**, 091804 (2009)

Unfortunately: not possible
W. Potzel and F.E. Wagner
Phys. Rev. Lett. **103**, 099101 (2009)
and arXiv: 0908.3985 [hep-ph];
J.P. Schiffer
Phys. Rev. Lett. **103**, 099102 (2009)

Two types of line broadening:

a) homogeneous broadening

↓

due to fluctuations, e. g. of magnetic fields, **stochastic processes**

b) inhomogeneous broadening

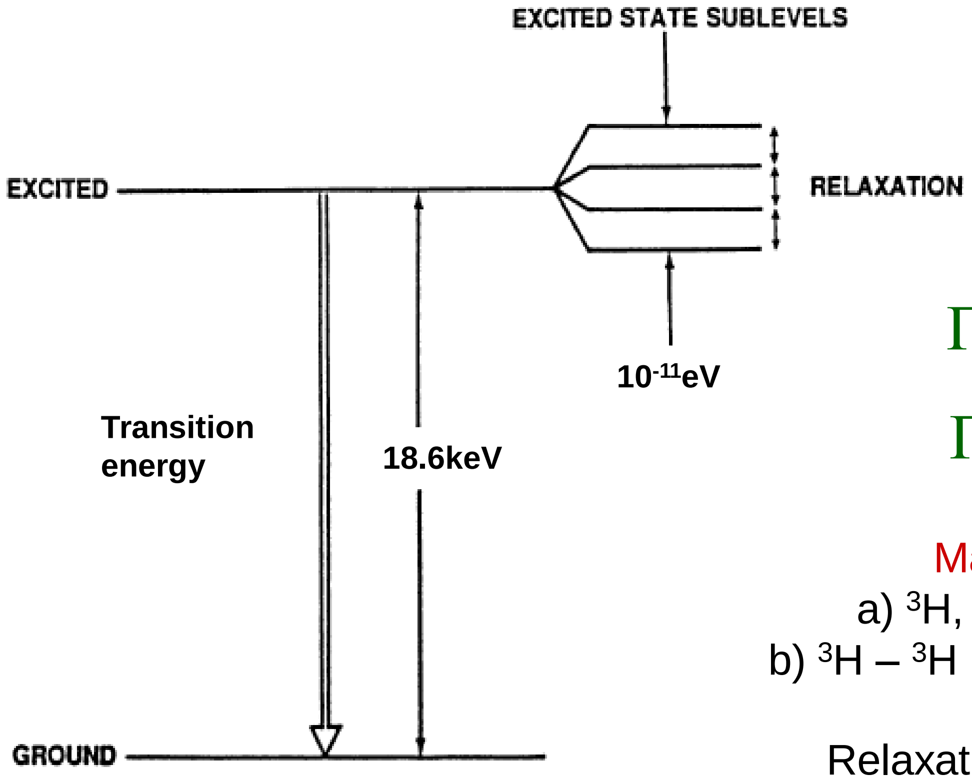
↓

due to stationary effects, e.g. impurities, **lattice defects which cause variations of line shifts**

How big are these broadening effects?

III) Mössbauer Nu's: Basic Questions

a) homogeneous broadening: **stochastic processes**, average rate is **not** connected to frequency of lattice vibrations; present at all temperatures



Measurements: ^{3}H (Pd), ^{3}H (Ti-H), NbH

Typical relaxation times: T_2~2ms, 79μs

$$\Gamma_{exp} \sim 5x10^{-11}eV \sim 4x10^{13}\,\Gamma$$

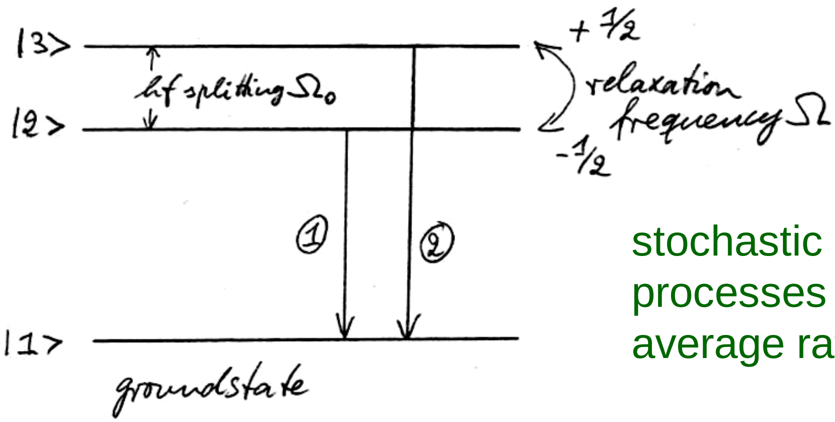
$$\Gamma_{exp} \sim \Gamma\ (^{67}Zn)$$

Magnetic interactions:
a) ^{3}H, ^{3}He with nuclei of metallic lattice
b) ^{3}H – ^{3}H magnetic dipolar spin-spin interaction

Relaxation between the sublevels affects the lineshape and the total linewidth.

The linewidth is determined by the relaxation rate.

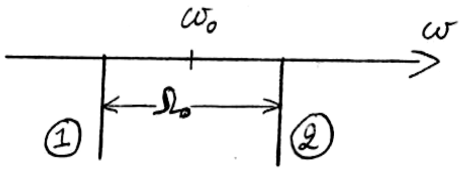
Homogeneous Broadening: Magnetic Relaxation



stochastic processes average rate:Ω

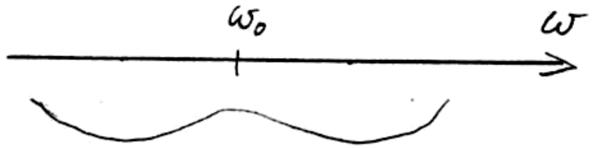
Simplest magnetic relaxation model

Sudden, irregular transitions (relaxation) between hyperfine-split states
→irregular (random) phase changes of transitions to ground state, no correlation to original phase
→it might take a long time to come back to the original frequency



$\Omega << \Omega_0$

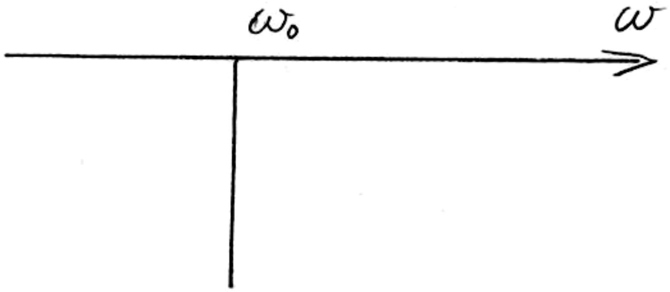
Two lines of (almost) natural width:
With increasing Ω, the lines broaden
→effective lifetime (time-energy uncertainty princ.)



$\boxed{\Omega \approx \Omega_0}$

Intensity is distributed over a broad pattern, which extends over the total hf splitting Ω_0 as suggested by the time-energy uncertainty principle. **Broad lines can not be decomposed into multiple sharp lines.**

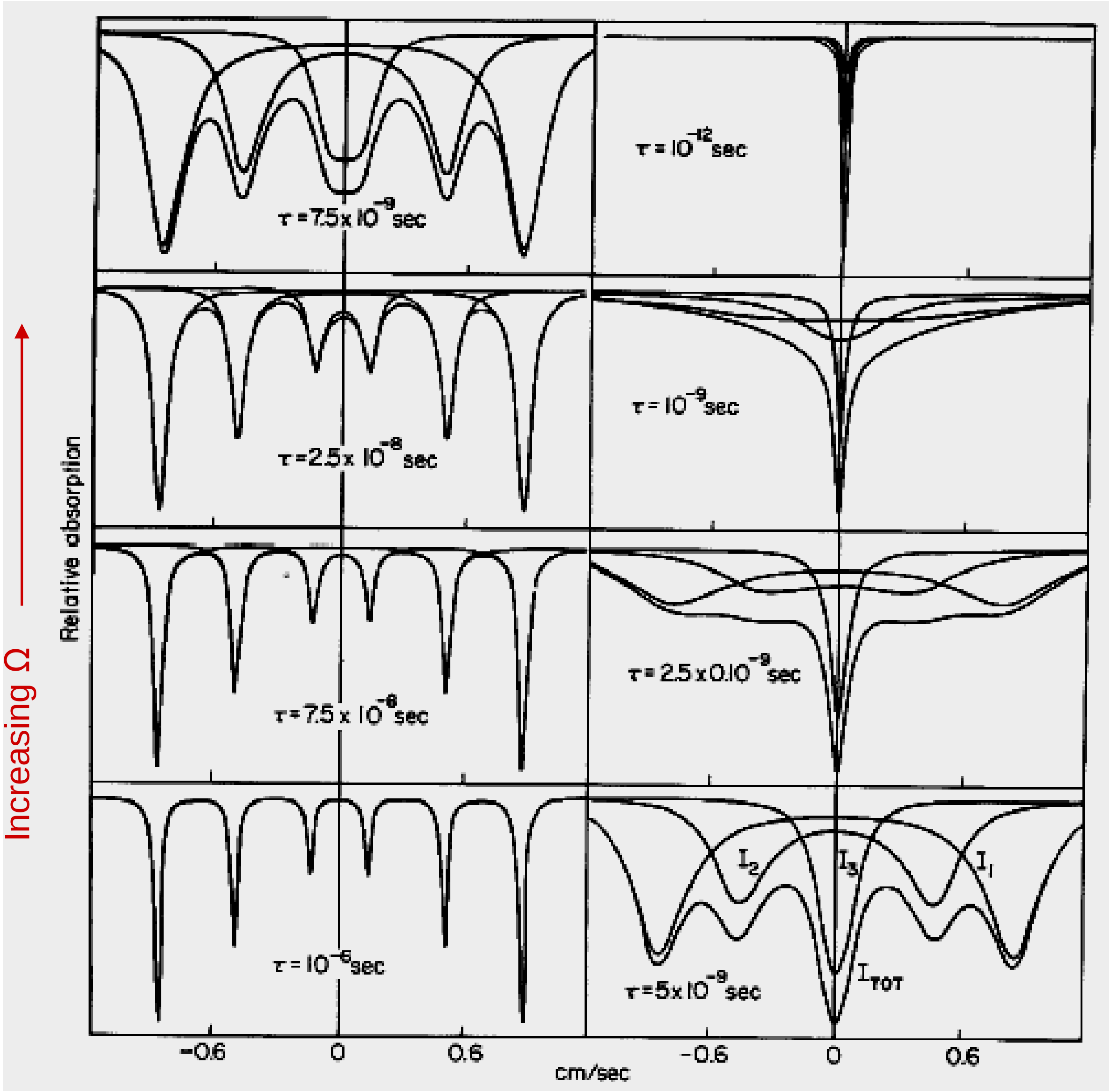
^{3}H/^{3}He system in Nb metal: Ω_0~10^5 s^{-1} and Ω~8x10^4 s^{-1}. → Γ_{exp}~5x10^{-11}eV ~4x10^{13} Γ.



$\Omega >> \Omega_0$

Motional narrowing: one line at the center of the hf splitting of practically natural width. Stochastic frequency changes: between lines 1 and 2. Averaging process over many short parts of the lifetime. **Not the case for ^{3}H/^{3}He**.

III) Mössbauer Nu‘s: Basic Questions



Increasing Ω

Relaxation spectra for ^{57}Fe
Superposition of 3 doublets

Average relaxation rate: $\Omega = 2\pi / \tau$

H.H. Wickman and G.K. Wertheim in: *Chemical Applications of Mössbauer Spectroscopy,* V.I. Goldanskii and R. Herber, editors; pp. 548 (New York: Academic Press, 1968)

III) Mössbauer Nu‘s: Basic Questions

b) inhomogeneous broadening:

Stationary effects: lattice defects, impurities

Conventional Mössbauer spectroscopy: Different binding strengths due to inhomogeneities affect the energy of the photons in the **same type** of nucleus in source and target.

⟶ Shift of photon energy by typically 10^{-7} – 10^{-9} eV (**hyperfine interaction**)

In the best single crystals: inhomogeneities cause shifts of 10^{-13} to 10^{-12} eV.
For $\bar{\nu}_e$: corresp. to $10^{11}\,\Gamma$ or $10^{12}\,\Gamma$ or even larger.

$10^{12}\,\Gamma$ would still be fine; ⟶ $\sigma_R \approx 1.8\text{x}10^{-34}\ \text{cm}^2$.

Binding energies of ^{3}H and ^{3}He in an inhomogeneous metallic lattice **directly** influence the $\bar{\nu}_e$ energy.

Binding energies per atom: ~eV range (**Coulomb interaction**).

⟶ Variation of the $\bar{\nu}_e$ energy much larger than neV, **typically: meV range**.

⟶ Variation of the $\bar{\nu}_e$ energy by only 10^{-6} eV → $10^{18}\,\Gamma$.

III) Mössbauer Nu‘s: Basic Questions

3) Relativistic effects

Second-order Doppler shift due to mean-square atomic velocity <V^2>

moving system

Time-dilatation effect: $\Delta t = \dfrac{\Delta t'}{\sqrt{1-(V/c)^2}}$

stationary system

Frequencies: $\omega = \omega' \sqrt{1-(V/c)^2} \approx \omega' \left(1 - \dfrac{V^2}{2c^2}\right)$

Second-order Doppler shift: $\Delta\omega = \omega - \omega' = -\omega' \dfrac{V^2}{2c^2}$

Reduction of frequency (energy)

III) Mössbauer Nu‘s: Basic Questions

Within the Debye model:

$$\frac{\Delta E}{E} = \frac{9k_B}{16Mc^2}(\Theta_s - \Theta_t) + \frac{3k_B}{2Mc^2}\left[T_s \cdot f\left(\frac{T_s}{\Theta_s}\right) - T_t \cdot f\left(\frac{T_t}{\Theta_t}\right)\right]$$

Zero-point energy

where

$$f\left(\frac{T}{\Theta}\right) = 3\left(\frac{T}{\Theta}\right)^3 \cdot \int_0^{\Theta/T} \frac{x^3}{\exp x - 1}\,dx$$

If $|T_s - T_t| = 1$ degree $\longrightarrow$ $\Delta E = 1.9 \cdot 10^{-9}\,eV \approx 1.6 \cdot 10^{15}\,\Gamma$

Low temperatures: $T_s \approx T_t \approx 0 \longrightarrow [.....] \approx 0$

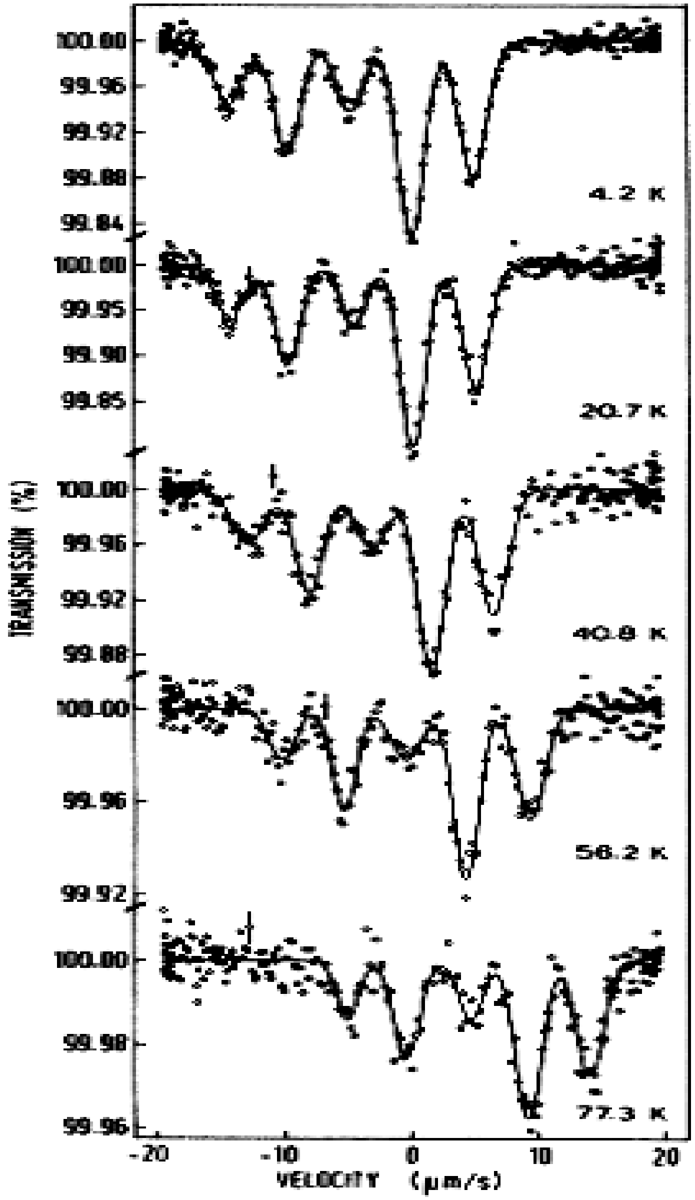
However, zero-point energy remains!

If $|\Theta_s - \Theta_t| = 1$ degree (0.08meV) $\rightarrow \Delta E / E \approx 2 \cdot 10^{-14} \longrightarrow \Delta E \approx 4 \cdot 10^{-10}\,eV$

$\approx 3 \cdot 10^{14}\,\Gamma$

Similar conclusion as reached from different binding energies.

III) Mössbauer Nu‘s: Basic Questions



^{67}Zn Mössbauer absorption spectra

Source: ^{67}GaZnO single crystal at various temperatures.

Absorber: ^{67}ZnO powder at 4.2K.

C. Schäfer et al., Phys. Rev. B37, 7247 (1988)

^{3}H-^{3}He system

Event rates for ^{3}H – ^{3}He recoilless resonant capture of antineutrinos

Base line	^{3}H	^{3}He	Antineutrino capture per day	Rβ(Δt=65d) per day
5 cm	1 kCi	100 mg	~40x10^{3}	~40
10 m	1 MCi	1 g	~10^{3}	~10

1) <u>only</u> homogeneous broadening, assuming
$\rightarrow\Gamma_{exp}=9$◌̇$x10^{-12}eV\approx 8x10^{12}$
$\sigma_{res}\approx 3x10^{-33}$ cm^{2}

2) **no** lattice expansion and contraction

Rβ(Δt)/day: Reverse β-activity rate after growth period Δt=65d=0.01τ

R. S. Raghavan, hep-ph/0601079 v3, 2006

4) Answers to Basic Questions

A) Principal difficulties

1) Probability for phononless emission and detection: ~ 7×10^{-8} ?
Due to lattice expansion and contraction (not present with conventional Mössbauer effect) and due to recoil.

Show Stopper

2) Homogeneous line broadening: $\Gamma_{exp} > 4 \times 10^{13}\, \Gamma \approx 5 \times 10^{-11}$ eV.

Acceptable!

3) Inhomogeneous line broadening due to random distribution of ^3H and ^3He in metal lattice (entropy) → **direct** influence of binding energies.

$\Gamma_{exp} \gg 10^{12}\, \Gamma$, maybe that even $\Gamma_{exp} \approx 10^{18}\, \Gamma \approx 10^{-6}$ eV
(200 times broader than the ^{57}Fe Mössbauer resonance).

Critical!

4) Relativistic effects (zero point energy) due to different binding energies:

$$\Gamma_{exp} > 3 \times 10^{14}\, \Gamma \approx 3 \times 10^{-10} \text{ eV}.$$

Acceptable!

4) Answers to Basic Questions

B) Technological difficulties (to mention only two)

1) Heat production in source of 1kCi is 0.1 W, → temperature gradients ΔT. For natural width Γ, $\Delta T \ll 10^{-11}$K (relativistic effect).

2) Stability of apparatus for continuous measurement:
e.g., mechanical and temperature variations must be negligible for time comparable to lifetime, i.e. for ~ 20 years.

4) Answers to Basic Questions

C) Age of the source

To use ^{3}H sources produced at different times: The age itself of the ^{3}H source does not influence the linewidth.

For an exponential decay (constant Γ):

Fourier transform from t=0 to t→∞: $F(\Omega)$
Fourier transform from $t=t_0>0$ to t→∞: $F(\Omega)\cdot e^{-(\Gamma/2)t_0}$
Only amplitude is reduced, width is the same.

In an $\bar{\nu}_e$ experiment, the clock is started together with the measurement when source and target are arranged in their fixed positions.

Candidates for recoilless neutrino emission and absorption

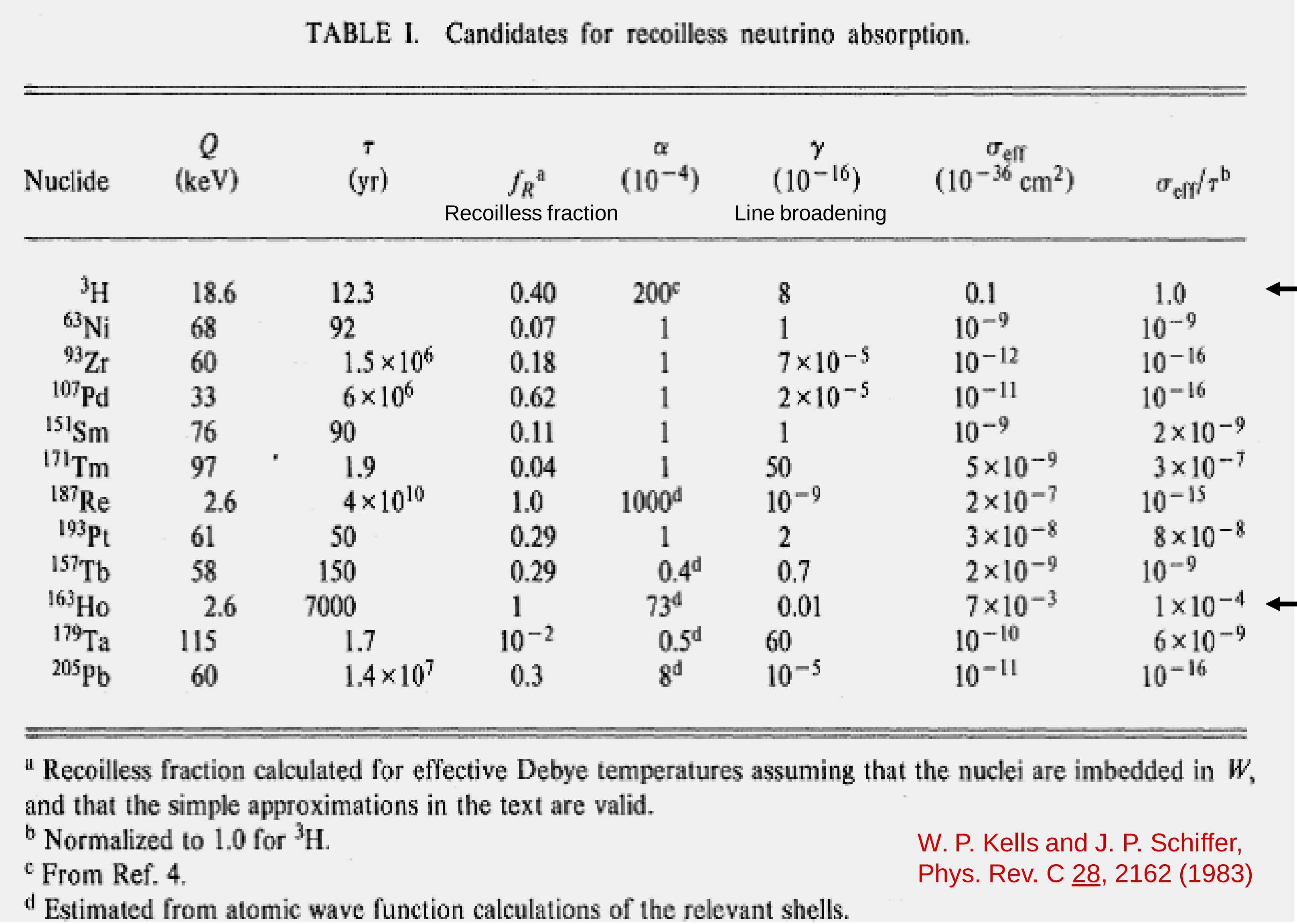


TABLE I. Candidates for recoilless neutrino absorption.

Nuclide	Q (keV)	τ (yr)	f_R[a]	α (10^{-4})	γ (10^{-16})	σ_{eff} (10^{-36} cm^2)	$\sigma_{\mathrm{eff}}/\tau$[b]
^{3}H	18.6	12.3	0.40	200[c]	8	0.1	1.0
^{63}Ni	68	92	0.07	1	1	10^{-9}	10^{-9}
^{93}Zr	60	1.5×10^{6}	0.18	1	7×10^{-5}	10^{-12}	10^{-16}
^{107}Pd	33	6×10^{6}	0.62	1	2×10^{-5}	10^{-11}	10^{-16}
^{151}Sm	76	90	0.11	1	1	10^{-9}	2×10^{-9}
^{171}Tm	97	1.9	0.04	1	50	5×10^{-9}	3×10^{-7}
^{187}Re	2.6	4×10^{10}	1.0	1000[d]	10^{-9}	2×10^{-7}	10^{-15}
^{193}Pt	61	50	0.29	1	2	3×10^{-8}	8×10^{-8}
^{157}Tb	58	150	0.29	0.4[d]	0.7	2×10^{-9}	10^{-9}
^{163}Ho	2.6	7000	1	73[d]	0.01	7×10^{-3}	1×10^{-4}
^{179}Ta	115	1.7	10^{-2}	0.5[d]	60	10^{-10}	6×10^{-9}
^{205}Pb	60	1.4×10^{7}	0.3	8[d]	10^{-5}	10^{-11}	10^{-16}

Recoilless fraction

Line broadening

[a] Recoilless fraction calculated for effective Debye temperatures assuming that the nuclei are imbedded in *W*, and that the simple approximations in the text are valid.

[b] Normalized to 1.0 for ^{3}H.

[c] From Ref. 4.

[d] Estimated from atomic wave function calculations of the relevant shells.

W. P. Kells and J. P. Schiffer,
Phys. Rev. C 28, 2162 (1983)

5) Rare Earth Systems?

W. P. Kells and J. P. Schiffer,
Phys. Rev. C 28, 2162 (1983)

Next-best case: ^{163}Ho/^{163}Dy

Advantages: a) large mass → relativistic effects are smaller comp. to ^{3}H/^{3}He

b) low Q value, 2.6 keV, → high recoilfree fraction

c) More similar chemical behaviour of Rare Earth
→ lattice expansion and contraction more favourable
→ larger probability of phononless transitions in source and target.

Disadvantages: a) Rare Earth atoms have large magnetic moments (4f electrons)
→ magnetic relaxation phenomena are decisive

b) less technological knowledge concerning fabrication of high-purity materials and of single crystals.

c) large amounts of ^{163}Ho (kg) and ^{163}Dy (100kg) necessary.

IV) Interesting experiments

1) Do Mössbauer (anti)neutrinos oscillate?

2) Oscillating Mössbauer neutrinos

    2.1) Ultra-short base lines

    2.2) Determination of mass hierarchy and
    oscillation parameters
    Δm_{32} and Δm^2_{12}: 0.6% and $\sin^2 2\theta_{13}$: 0.002

    2.3) Search for sterile neutrinos

    2.4) Gravitational redshift experiments (Earth).

IV) Interesting experiments

1) Question: Do Mössbauer neutrinos oscillate?

A) Evolution of neutrino state in time

Schrödinger equation for evolution of any quantum state:

$$i\frac{\partial|\psi(t)\rangle}{\partial t} = H|\psi(t)\rangle \longrightarrow |\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$

$$P(\nu_l \to \nu_{l'}) = \left|\sum_{k=1}^{3} U_{l'k} e^{-i(E_k - E_i)t} U_{lk}^*\right|^2$$

No matter what the neutrino momenta are !

If $E_k = E_i$, there will be no neutrino oscillations:
The neutrino state is stationary

$$P(\nu_l \to \nu_{l'}) = \delta_{l'l}$$

If E_k are different, neutrino state is non-stationary.
→time-energy uncertainty relation holds:

$$\Delta E \cdot \Delta t \geq 1$$

Δt is the time interval during which the state of the system is significantly changed

If $E_k \neq E_i$, the time-energy uncertainty relation takes the form:

$$(E_k - E_1) \cdot t \approx \frac{\Delta m_{1k}^2}{2E} t$$

IV) Interesting experiments

B) Evolution of neutrino state in time and space

Mixed neutrino state at space-time point $x = (t, \vec{x})$:

$$\left|\nu_l\right\rangle_x = \sum_{k=1}^{3} e^{-ip_k x} U_{lk}^{*} \left|\nu_k\right\rangle \longrightarrow P(\nu_l \to \nu_{l'}) = \left| \sum_{k=1}^{3} U_{l'k} e^{-i(p_k - p_1)x} U_{lk}^{*} \right|^2$$

with $(p_k - p_1)x = \frac{E_k^2 - E_1^2}{E_k + E_1} t - (p_k - p_1)L$ and $E_i^2 = p_i^2 + m_i^2$

ultra-relativistic neutrinos: $t \approx L \longrightarrow (p_k - p_1)x \approx \frac{\Delta m_{1k}^2}{2E} L$ oscillatory phase

Mössbauer neutrinos: different masses have the *same* energy

$\longrightarrow$ neutrino wave function is stationary in time, but oscillatory in space

$$\longrightarrow p_k \neq p_i : \quad (p_k - p_i)x = \frac{\Delta m_{1k}^2}{2E} L \qquad P(\nu_l \to \nu_{l'}) = \left| \sum_{k=1}^{3} U_{l'k} e^{-i\Delta m_{1k}^2 \frac{L}{2E}} U_{lk}^{*} \right|^2$$

IV) Interesting experiments

1) Do Mössbauer neutrinos oscillate?
Different approaches to neutrino oscillations

A) Evolution of the neutrino state Ψ(t) in time:

Neutrino oscillations occur if Ψ(t) is a superposition of states of neutrinos with different energies
→ non-stationary phenomenon
→ No oscillations for Mössbauer neutrinos, since very narrow energy distribution

B) Evolution of the neutrino state in space and time:

→ Oscillations are possible in both the non-stationary and also in the stationary case (Mössbauer neutrinos: wave function oscillates in space)

IV) Interesting experiments

2) Oscillating Mössbauer neutrinos:

2.1) Ultra-short base lines for neutrino-oscillation experiments

For only two flavors: $P(\nu_a \to \nu_b) = \sin^2 2\Theta \cdot \sin^2(\pi L / L_0)$

Oscillation length: $L_0 = 4\pi\hbar c \dfrac{E}{|\Delta m^2|} \approx 2.480 \dfrac{E / MeV}{|\Delta m^2| / eV}$ [m]

A) Determination of Θ_{13}: *E=18.6 keV* instead of *3 MeV.*

Δm_{23}^2 observed with *atmospheric* neutrinos

Chooz experiment: $\sin^2 2\Theta_{13} \leq 2 \cdot 10^{-1}$ Oscillation base line: $L_0/2$~9.3 m

→ Base line *L* of *9.3 m* instead of *1500 m*

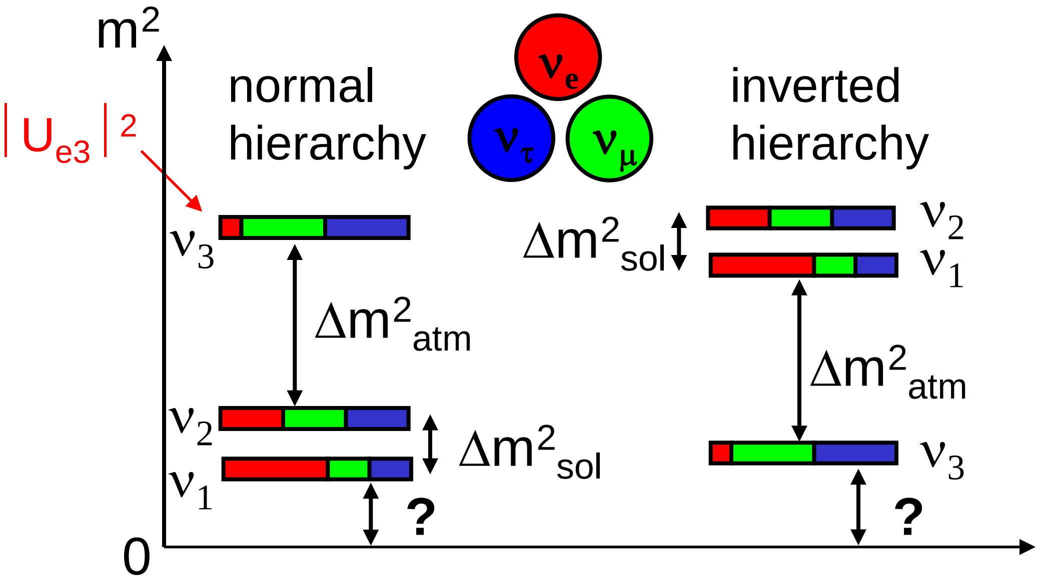
B) Determination of oscillation parameters of solar neutrinos

Δm_{12}^2 observed with *solar* neutrinos

Amplitude: $\sin^2 2\Theta_{12} \approx 0.82$ Oscillation base line: $L_0/2$~300 m

IV) Interesting experiments

2.2) Mass hierarchy and oscillation parameters



H. Nunokawa et al., hep-ph/0503283

To determine mass hierarchy: Measure Δm^2 in reactor-neutrino and muon-neutrino (accelerator long-baseline) disappearance channels to better than a fraction of 1%

H. Minakata et al., hep-ph/0602046

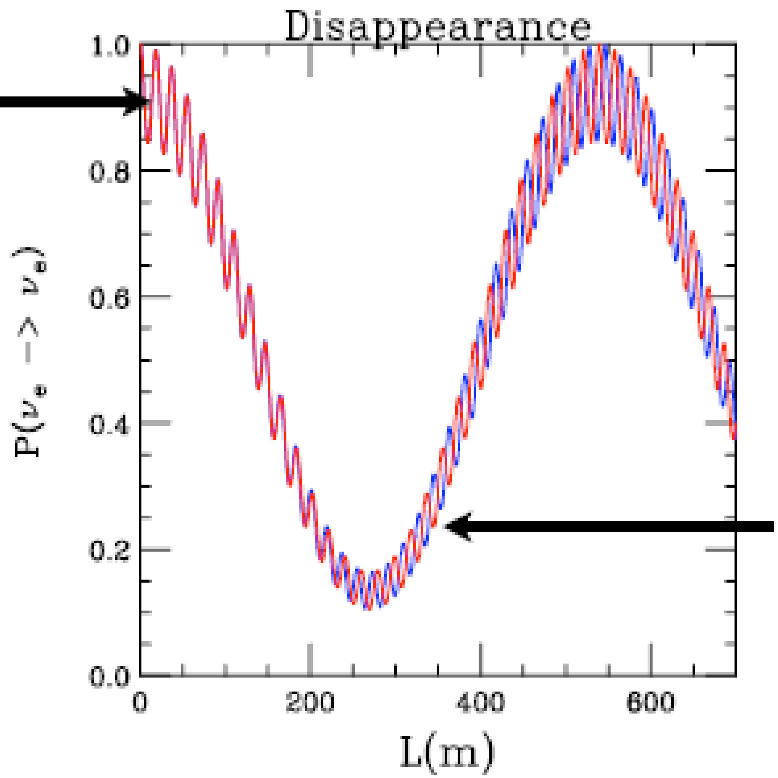
For $\sin^2 2\theta_{13}=0.05$ and 10 different detector locations one can reach uncertainties:
in Δm^2_{31} and Δm^2_{12}: 0.6%,
in $\sin^2 2\theta_{13}$: 0.002

H. Minakata et al.: hep-ph/0701151
S. Parke et al.: 0812.1879 (hep-ph)
Phase changes of atmosph. w.r. to solar oscill.

NH: $|\Delta_{31}| > |\Delta_{32}|$ — Phase of atm. osc. advances by $2\pi \sin^2 \Theta_{12}$ for every solar osc.

IH: $|\Delta_{31}| < |\Delta_{32}|$ — Phase of atm. osc. retarded by $2\pi \sin^2 \Theta_{12}$ for every solar osc.

IV) Interesting experiments



in phase

out of phase by pi/2

$\delta m^2_{31} > 0$

$\delta m^2_{31} < 0$

Long period: solar oscillation Δm^2_{12}
Short period: atm. oscillation Δm^2_{32}

$$L_O = 2.48 \frac{E / MeV}{\left|\Delta m_{ij}\right|^2 / eV^2} m$$

S. Parke et al.: Nucl. Phys. Proc. Suppl. 188, 115 (2008);
arXiv: 0812.1879 [hep-ph]

IV) Interesting experiments

2.3) Search for conversion of $\bar{\nu}_e \longrightarrow \nu_{sterile}$

LSND experiment: $\Delta m^2 \approx 1eV^2$ and sin^{2}2θ~0.1 to 0.001

(not completely excluded by MiniBooNE, experiment not conclusive)

Possibility: $\bar{\nu}_e \longrightarrow \nu_{sterile}$ V. Kopeikin et al. : hep-ph/0310246

Test: Disappearance experiment with 18.6 keV antineutrinos (2.6 keV, ^{163}Ho)

⟶ Oscillation length L_0 ~ 5cm! (~1 cm, ^{163}Ho)

⟶ Ultra-short base line, difficult to reach otherwise

IV) Interesting experiments

2.4) Gravitational redshift experiments (Earth)

Gravitational redshift: $\frac{\delta E}{E} = \frac{gh}{c^2}$

Experimental linewidth: $\Gamma_{\exp} = \Delta = 5 \cdot 10^{-11} eV$

$\Delta = \frac{\hbar\omega}{c^2} gh_\Delta$ where h_Δ is height corresponding to 1 experimental linewidth

$\longrightarrow$ $h_\Delta \approx 25m$ realistic experiment

For $\Gamma_{\exp} \approx 10^{-6} eV$ $\longrightarrow$ unrealistic

Can **not** be used to determine the neutrino mass

Gravitational spectrometer

V) Conclusions

A) Phononless resonant emission and detection of antineutrinos:

^{3}H – ^{3}He system.

B) Experiment is very difficult, if not impossible.

Not possible to reach natural width.

W. Potzel and F.E. Wagner
Phys. Rev. Lett. **103**, 099101 (2009)
and arXiv:0908.3985 [hep-ph]

J.P. Schiffer
Phys. Rev. Lett. **103**, 099102 (2009)

1) Principal difficulties:

a) Probability for phononless emission and detection might be smaller than expected due to lattice expansion and contraction after the transformation of the nucleus:

Additional reduction factor of $1\text{x}10^6$.

b) Homogeneous broadening (stochastic processes) and inhomogeneous broadening (variation of binding energies and of zero-point energies).

$\longrightarrow \quad \Gamma_{exp} \approx 10^{18}\,\Gamma \approx 10^{-6}$ eV

V) Conclusions

2) Many technological difficulties, for example:

a) Production of ^{3}H source and ^{3}He target in Nb metal
b) Temperature differences within the source and between source and target (temperature shift)
c) Stability (mechanical and temperature) of apparatus for continuous measuring times of several years.

C) Rare Earth Systems, e.g., ^{163}Ho/^{163}Dy ?

→ Larger probability of phononless transitions in source and target
→ Smaller relativistic effects, experiment at room temperature
→ However, magnetic relaxation phenomena are critical

D) Interesting experiments:

a) Do Mössbauer neutrinos oscillate?
b) Mass hierarchy and accurate determination of oscillation parameters
c) Search for sterile neutrinos (LSND experiment, MiniBooNE?)
d) Gravitational redshift experiments (Earth).

Papers

Earlier papers:

W. M. Visscher, Phys. Rev. 116, 1581 (1959)
W. P. Kells and J. P. Schiffer, Phys. Rev. C 28, 2162 (1983)

More recent papers:

R. S. Raghavan, hep-ph/0601079 v3, 2006
W. Potzel, Phys. Scrip. T127, 85 (2006)
S. M. Bilenky, F. von Feilitzsch, and W. Potzel, J. Phys. G: Nucl. Part. Phys. **34**, 987 (2007);
J. Phys. G: Nucl. Part. Phys. **35**, 095003 (2008); **36**, 078002 (2009); arXiv: 0903.5234 [hep-ph]
E. Kh. Akhmedov, J. Kopp, and M. Lindner, J. High Energy Phys. **0805**, 005 (2008); arXiv: 0802.2513 [hep-ph]
W. Potzel, J. Phys. Conf. Ser. **136**, 022010 (2008); arXiv: 0810.2170 [hep-ph]
R. S. Raghavan, Phys. Rev. Lett. **102**, 091804 (2009)
W. Potzel and F. E. Wagner, Phys. Rev. Lett. **103**, 099101 (2009); arXiv: 0908.3985 [hep-ph]
J. P. Schiffer, Phys. Rev. Lett. **103**, 099102 (2009)
R. S. Raghavan, Phys. Rev. Lett. **103**, 099103 (2009)
W. Potzel, Acta Physica Polonica **B40**, 3033 (2009); arXiv: 0912.2221 [hep-ph]
J. Kopp, J. High Energy Phys. **0906**, 049 (2009); arXiv: 0904.4346 [hep-ph]
D. V. Naumov and V. A. Naumov, J. Phys. G: Nucl. Part. Phys. **37**, 105014 (2010); arXiv: 1008.0306 [hep-ph]

Extra slides

Neutrinos

Example: Oscillations between two neutrino flavours (e.g., $\nu_e \leftrightarrow \nu_\mu$)

$$\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \underbrace{\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}}_{U}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

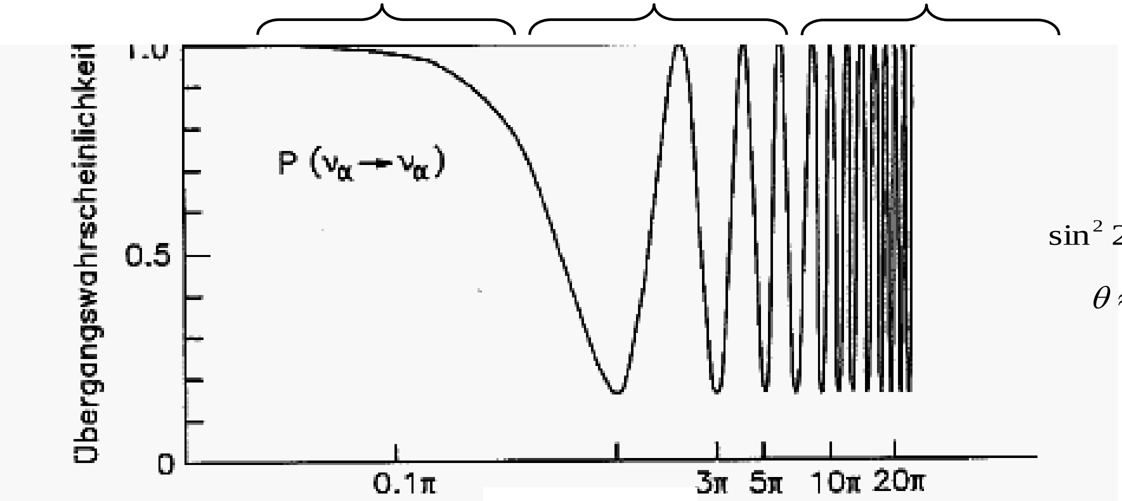
mixing angle: θ

Oscillations, if $\theta \neq 0$ and $\Delta m_{21}^2 = m_2^2 - m_1^2 \neq 0$

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \cdot \sin^2\left(\frac{\Delta m_{21}^2}{4} \cdot \frac{L}{E}\right) = \sin^2 2\theta \cdot \sin^2\left(\pi \frac{L}{L_O}\right)$$

where $L_O = 2.48 \frac{E / MeV}{\left|\Delta m_{12}\right|^2 / eV^2} m$

L << L$_0$ L ≈ L$_0$ L >> L$_0$



$$\Delta = \frac{\Delta m^2}{2} \cdot \frac{L}{E}$$

$\sin^2 2\theta = 0.83$

$\theta \approx 33^0$

L: distance between source and detector

$$P(\nu_\alpha \to \nu_\alpha) = 1 - P(\nu_\alpha \to \nu_\beta)$$

N. Schmitz: Neutrinophysik,
Teubner Studienbücher Physik,
B.G. Teubner, Stuttgart 1997, p. 252

Neutrinos: Mass Matrix

$\langle \nu_i | M | \nu_j \rangle = m\delta_{ij}$ and $m_i - m_j \neq 0$ for $i \neq j$ *M*: mass operator

diagonal in $|\nu_i\rangle$ – representation

In flavour representation:

$$m_{\alpha\beta} \equiv \langle \nu_\beta | M | \nu_\alpha \rangle = \sum_{i,j} \underbrace{\langle \nu_\beta | \nu_i \rangle}_{U^*_{\beta i}} \underbrace{\langle \nu_i | M | \nu_j \rangle}_{m_i \delta_{ij}} \underbrace{\langle \nu_j | \nu_\alpha \rangle}_{U_{\alpha j}} = \sum_i m_i U_{\alpha i} U^*_{\beta i}$$

In general not diagonal!

Masses of flavour eigenstates are the expectation values of the mass operator, i.e., weighted averages of the masses m_i:

$$m_\alpha \equiv m_{\alpha\alpha} = m(\nu_\alpha) = \langle \nu_\alpha | M | \nu_\alpha \rangle = \sum_i |U_{\alpha i}|^2 m_i$$

For *n*=2: $m_e = m_1 \cos^2\theta + m_2 \sin^2\theta$ $m_\mu = m_1 \sin^2\theta + m_2 \cos^2\theta$

$$m_{e\mu} = (m_2 - m_1) \sin\theta \cdot \cos\theta$$

$$\longrightarrow \quad m_{1,2} = \frac{1}{2}\left[m_e + m_\mu \mp \sqrt{(m_\mu - m_e)^2 + 4m_{e\mu}} \right] \qquad \tan 2\theta = \frac{2m_{e\mu}}{m_\mu - m_e}$$

Quark mixing

Charged weak current (CC) transitions between quark families can change the quark flavour

Unitary 3x3 mixing matrix U (Kobayashi-Maskawa matrix)

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = U \begin{pmatrix} d \\ s \\ b \end{pmatrix} \longrightarrow$$

three quark families

$$\underbrace{\begin{pmatrix} u \\ d' \end{pmatrix}\begin{pmatrix} c \\ s' \end{pmatrix}\begin{pmatrix} t \\ b' \end{pmatrix}}_{}$$

For weak interaction: weak CC transitions occur only within each of the three families

$$u \leftrightarrow d'$$
$$c \leftrightarrow s'$$
$$t \leftrightarrow b'$$

Neutral weak-currents: do not change the quark flavour.

Direct neutrino mass search

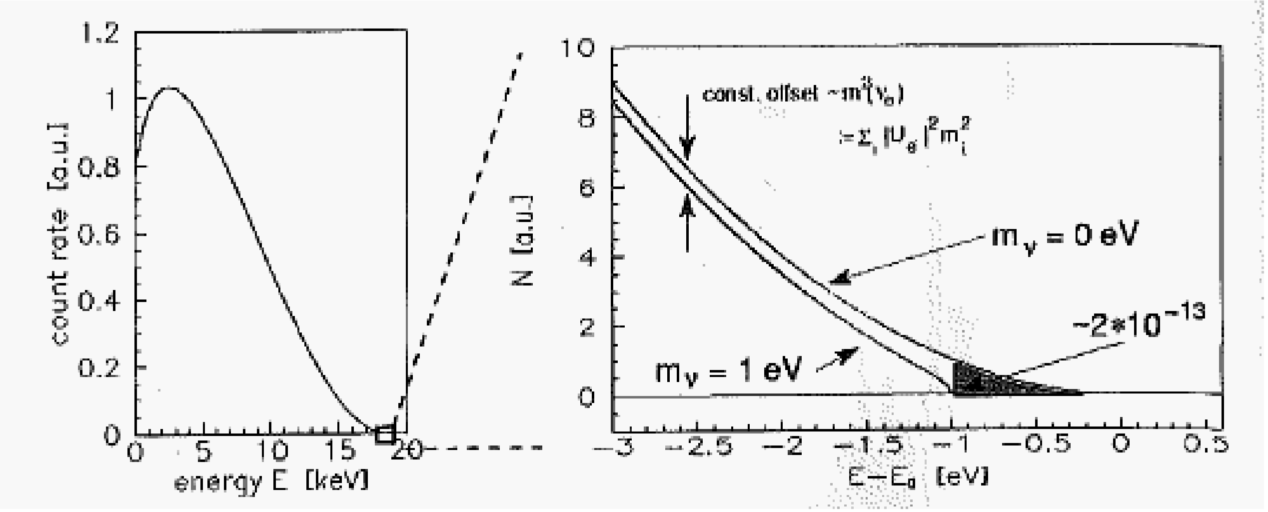
β-decay, e.g. KATRIN: $^3H \rightarrow {}^3He + e^- + \overline{\nu}_e$

β-spectrum: $N(E) = \frac{dN}{dE} = K \cdot F(E,Z) \cdot pE(E_0 - E) \cdot \sum_{i=1}^{3} U_{ei}^2 \cdot \sqrt{(E_0 - E)^2 - m_i^2}$

$E_0 = E_{\max}(m_\nu = 0)$ $m_\nu \geq 0$: end-point energy $E_{\max} = E_0 - m_\nu$

Kurie plot: $K(E) = \sqrt{\frac{N(E)}{K \cdot F(E,Z) \cdot pE}} = \left[(E_0 - E) \cdot \sum_{i=1}^{3} U_{ei}^2 \cdot \sqrt{(E_0 - E)^2 - m_i^2}\right]^{1/2}$

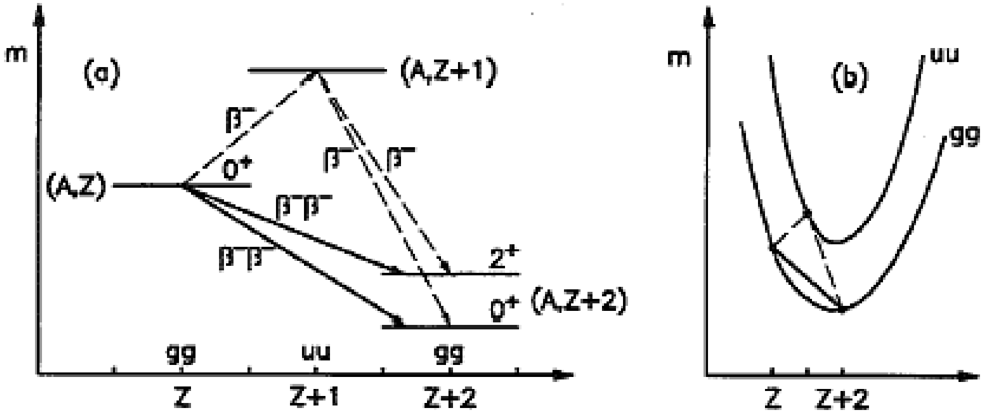
three spectra with end-point energies: $E_0 - m_i$ $(i = 1,2,3)$



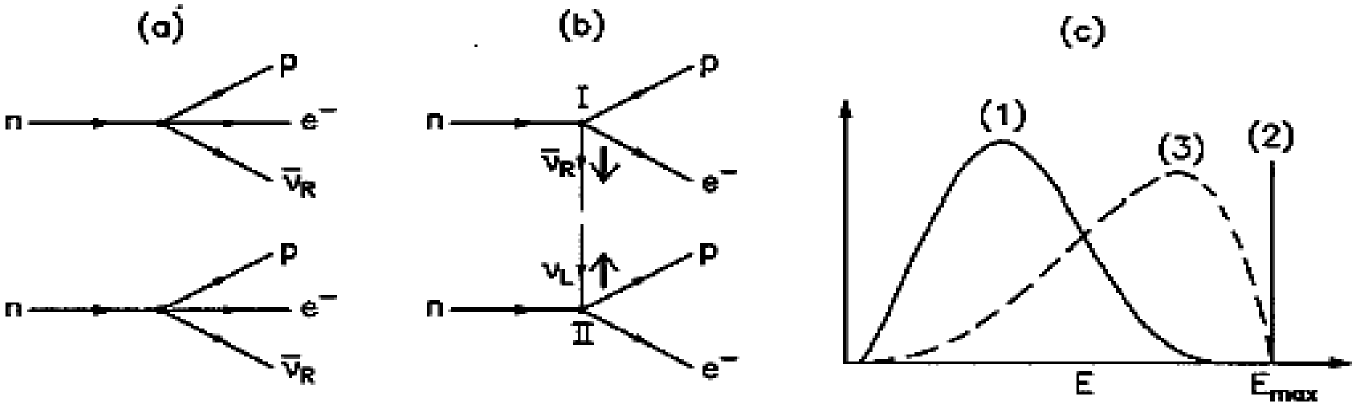
C. Weinheimer,
Nucl. Phys. B(Proc.Suppl.)
118, 279 (2003)

Neutrinoless double β decay (0νββ)

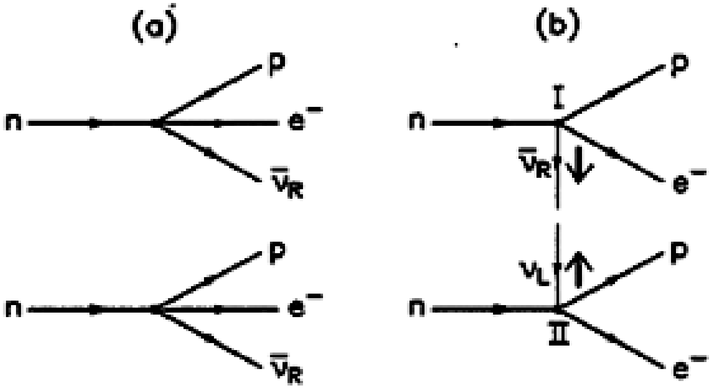
Are neutrinos Majorana or Dirac particles?



N. Schmitz: Neutrinophysik,
Teubner Studienbücher Physik,
B.G. Teubner, Stuttgart 1997, p. 302,303



Neutrinoless double β decay (0νββ)



Vertex I: $n \to p + e^- + \overline{\nu_R}$ H=+1, for m_ν=0

ΔL=2

Vertex II: $\nu_L + n \to p + e^-$ H=-1, for m_ν=0

Possible, if two conditions are fulfilled:

1) $\nu = \overline{\nu} = \nu^M$ Two helicity states: $\nu_L \equiv \overline{\nu}_L$ $\nu_R \equiv \overline{\nu}_R$

ΔL=2

2) a) Helicity has to be changed from H=+1 to H=-1

$m_\nu > 0 \longrightarrow$ Helicity is not a good quantum number.

b) In addition to left-handed charged current exists a (small) contribution of right-handed charged current.

a) and b) may contribute independently

Measured quantity: effective Majorana mass $< m_\nu >$

$$\left[T_{1/2}^{0\nu}(0^+ \to 0^+)\right]^{-1} = \left|\underbrace{M_{GT}^{0\nu} - \frac{g_\nu^2}{g_A^2} M_F^{0\nu}}_{\text{nuclear matrix elements}}\right|^2 \left(\frac{< m_\nu >}{m_e}\right)^2 \underbrace{G_1^{0\nu}}_{\text{phase-space integral}}$$

$$< m_\nu > = \left|\sum_j U_{ej}^2 m_j\right| = \left|\sum_j \left|U_{ej}^2\right|^2 e^{i2\alpha_j} m_j\right|$$

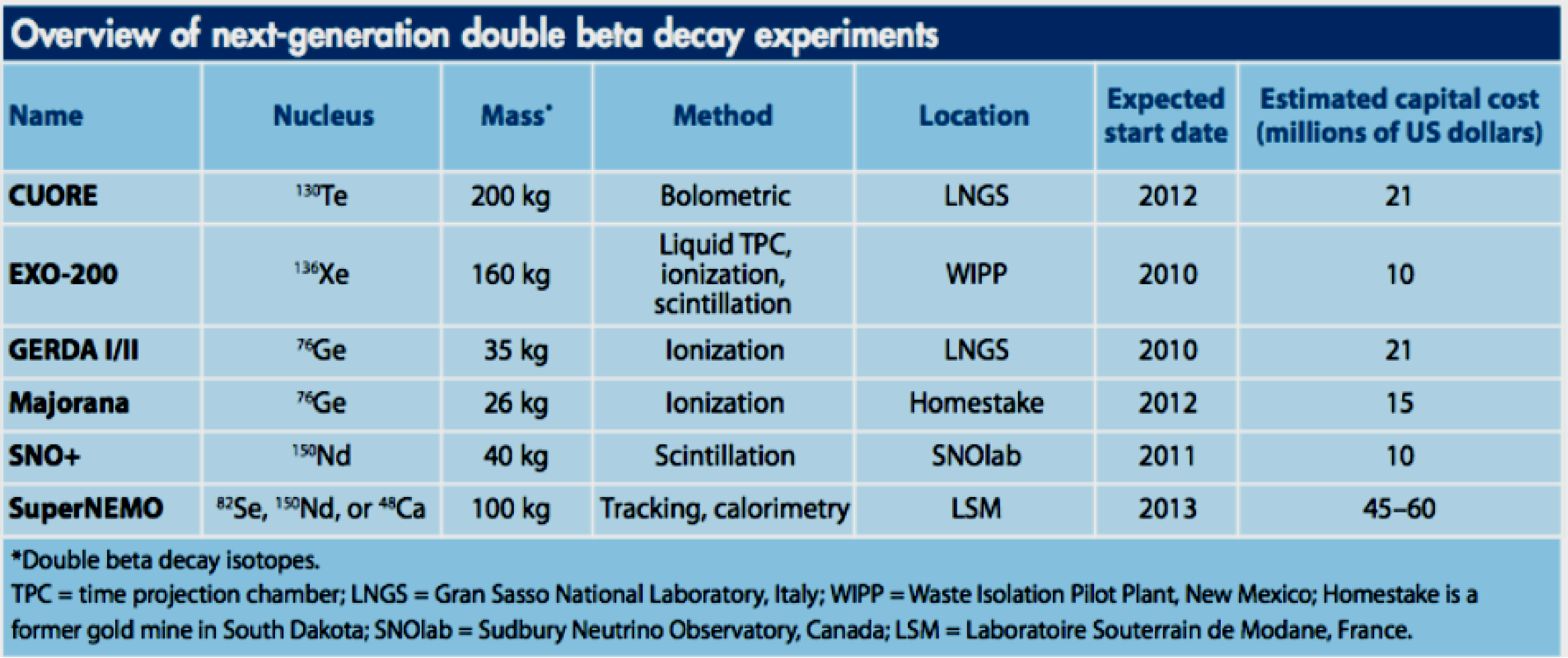
CP phases: α_j. If CP is conserved: αj=kπ

At least one $m_j \neq 0$ for 0νββ to happen.

Possible interference among mass terms in contrast to simple β decay.

⟶ Complementary methods and results.

Neutrinoless double β decay (0νββ)



Overview of next-generation double beta decay experiments

Name	Nucleus	Mass*	Method	Location	Expected start date	Estimated capital cost (millions of US dollars)
CUORE	^{130}Te	200 kg	Bolometric	LNGS	2012	21
EXO-200	^{136}Xe	160 kg	Liquid TPC, ionization, scintillation	WIPP	2010	10
GERDA I/II	^{76}Ge	35 kg	Ionization	LNGS	2010	21
Majorana	^{76}Ge	26 kg	Ionization	Homestake	2012	15
SNO+	^{150}Nd	40 kg	Scintillation	SNOlab	2011	10
SuperNEMO	^{82}Se, ^{150}Nd, or ^{48}Ca	100 kg	Tracking, calorimetry	LSM	2013	45–60

*Double beta decay isotopes.

TPC = time projection chamber; LNGS = Gran Sasso National Laboratory, Italy; WIPP = Waste Isolation Pilot Plant, New Mexico; Homestake is a former gold mine in South Dakota; SNOlab = Sudbury Neutrino Observatory, Canada; LSM = Laboratoire Souterrain de Modane, France.

Toni Feder: Physics Today, January 2010, p. 20

Recoilless antineutrino …

What does this mean for the effective values Θ_s and Θ_t ?

Source

$^3H \longrightarrow {}^3He$

$$f^{\,^3He}(M,{}^3H)$$

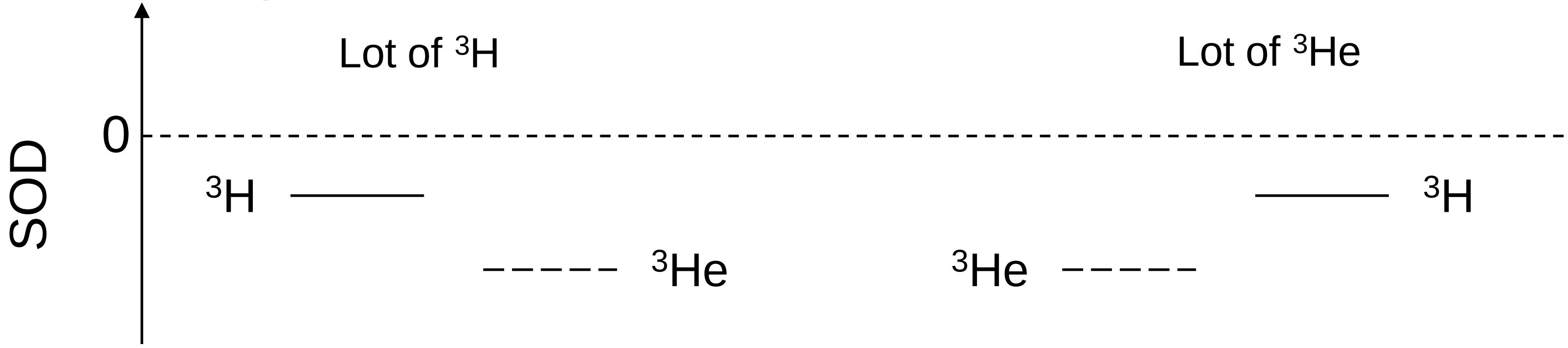
Lot of ^3H

Target

$^3He \longrightarrow {}^3H$

$$f^{\,^3H}(M,{}^3He)$$

Lot of ^3He

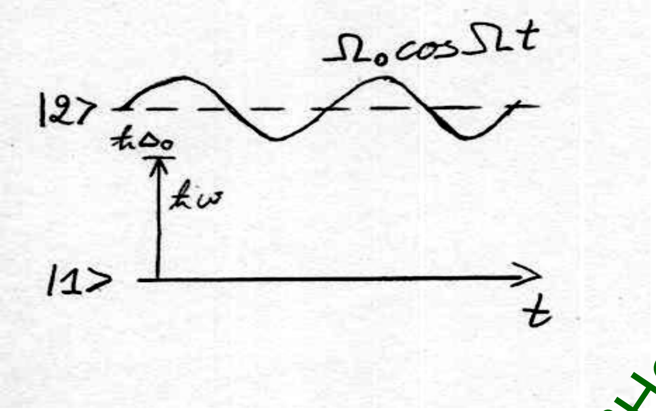


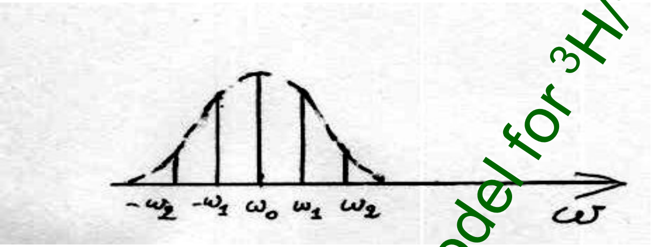
The differences of these SOD values in source and target have to be the same. In a practical experiment this means:

The Debye temperature for ^3H has to be the same in source and target. The same holds for ^3He. The Debye temperatures of ^3H and ^3He in the metal matrix do <u>not</u> have to be equal.

Homogeneous Broadening: Frequency Modulation

M. Salkola and S. Stenholm, Phys. Rev. A **41**, 3838 (1990)





Wrong model for ³H/³He system

$$A \propto \sum_{k=-\infty}^{k=+\infty} J_k^2(\eta) \frac{1}{\left[(\Delta_0/\Gamma) - k\xi\right]^2 + 1}$$

sum of Lorentzians, located at $\omega = \omega_0 \pm k\Omega$

Ω_0 : max. freq. deviation

Ω : relaxation frequency

$\eta = \dfrac{\Omega_0}{\Omega}$: modulation index

$\Delta_0 = \omega_0 - \omega$

Γ : linewidth

$\xi = \dfrac{\Omega}{\Gamma}$

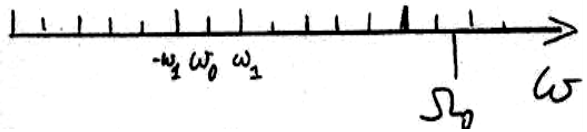
$\eta \approx 1 \Rightarrow \Omega \approx \Omega_0$

$\Gamma << \Omega$

motional narrowing: $\Omega >> \Omega_0 \Rightarrow \eta \approx 0$

only center line at ω_0 survives

For the ^{3}H/^{3}He system: $\Omega_0 \approx 10^5$ s^{-1}and $\Omega \sim 8\text{x}10^4$ s^{-1}
However, not continuous but stochastic oscillations;
→time-energy uncertainty relation determines width



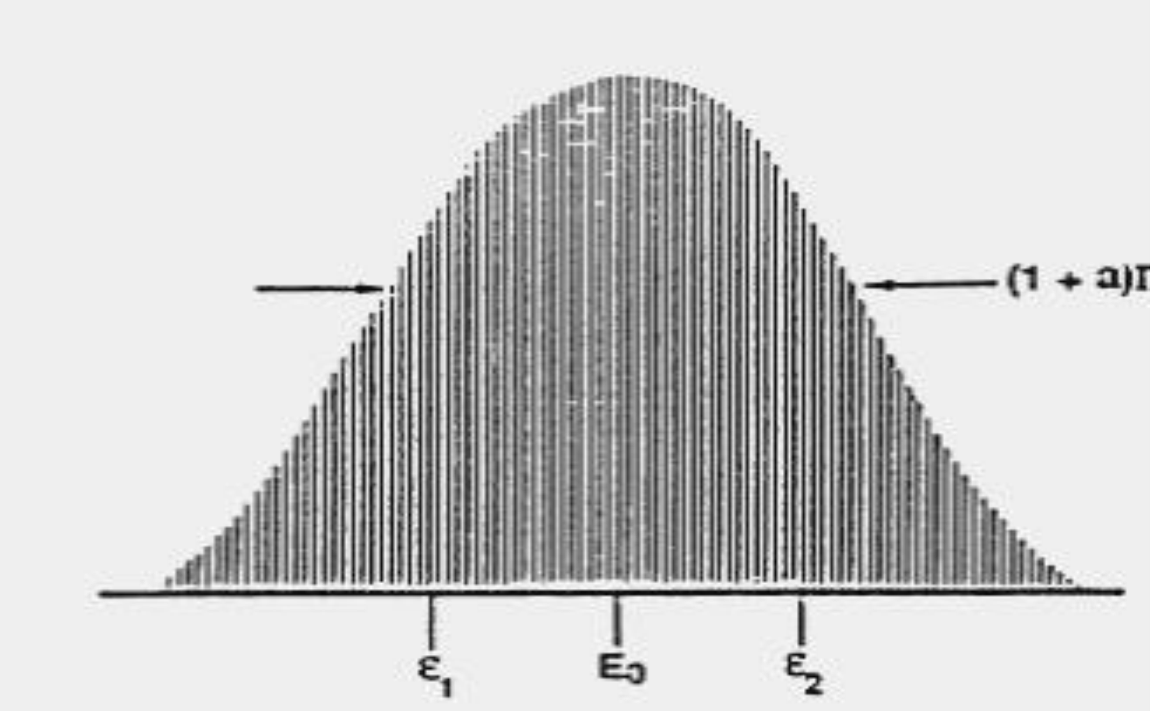
$\Omega_0 >> \Omega \Rightarrow \eta >> 1$

many sidebands→at ω_0
very little intensity

motional narrowing: not possible

Recoilless antineutrino …

b) inhomogeneous broadening

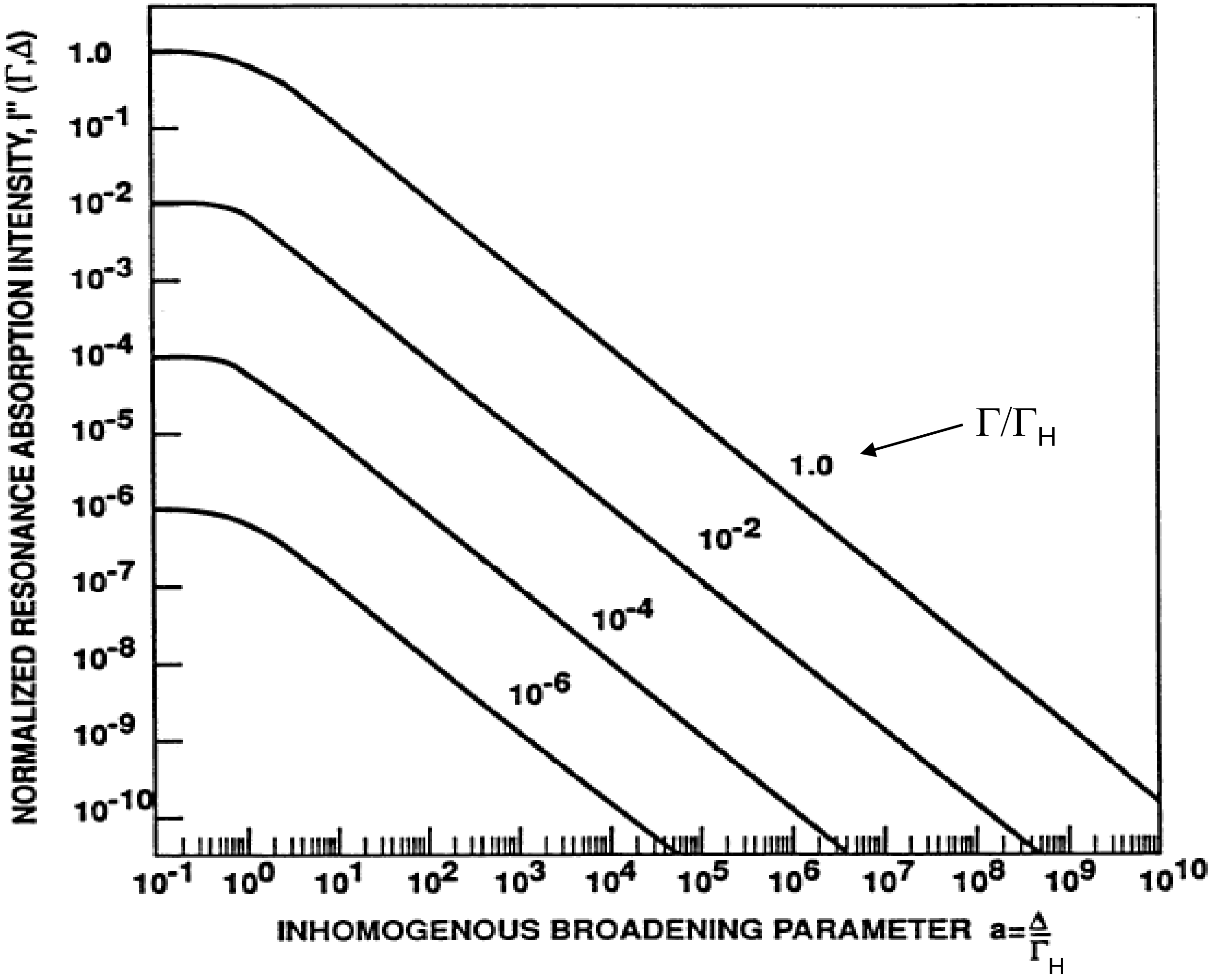


Many individual resonances displaced from the nonperturbed resonance energy E_0

In the best single crystals: $(1 + a)\Gamma \sim 10^{-13}$ eV corresp. to $10^{11}\,\Gamma$ or larger

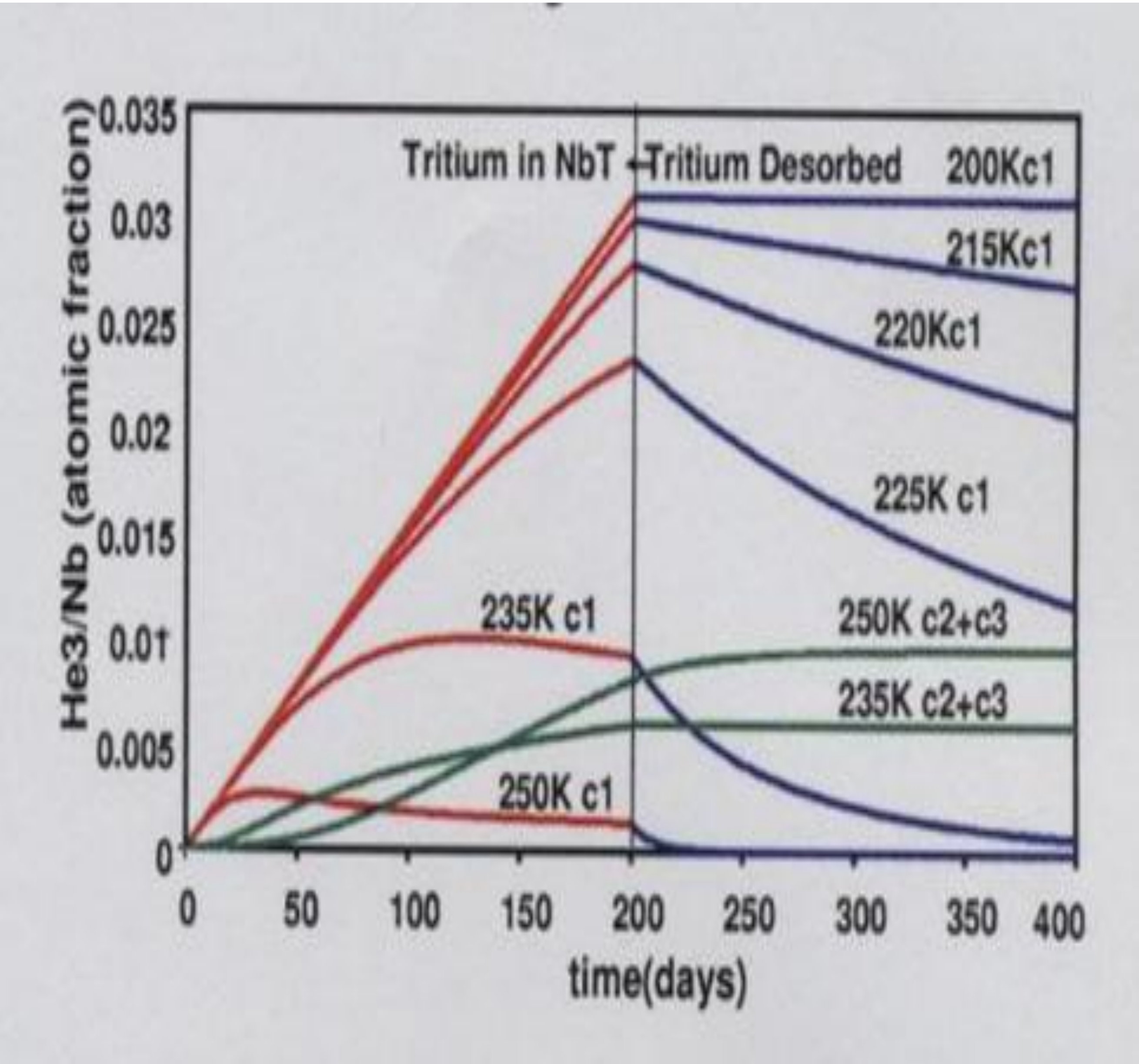
Both types of broadening reduce the resonant reaction intensity

Recoilless antineutrino …



B. Balko, I. W. Kay, J. Nicoll, J. D. Silk, and G. Herling, Hyperfine Interactions **107**, 283 (1997).

System NbT



^{3}He generated in Nb: c1: concentration in interstitial sites for different temperatures and times. The He in the T-free absorber below 200K is almost all interstitial.

R.S. Raghavan: hep-ph/0601079 revised v3; calculations: Sandia Natl. Lab., USA

System NbT

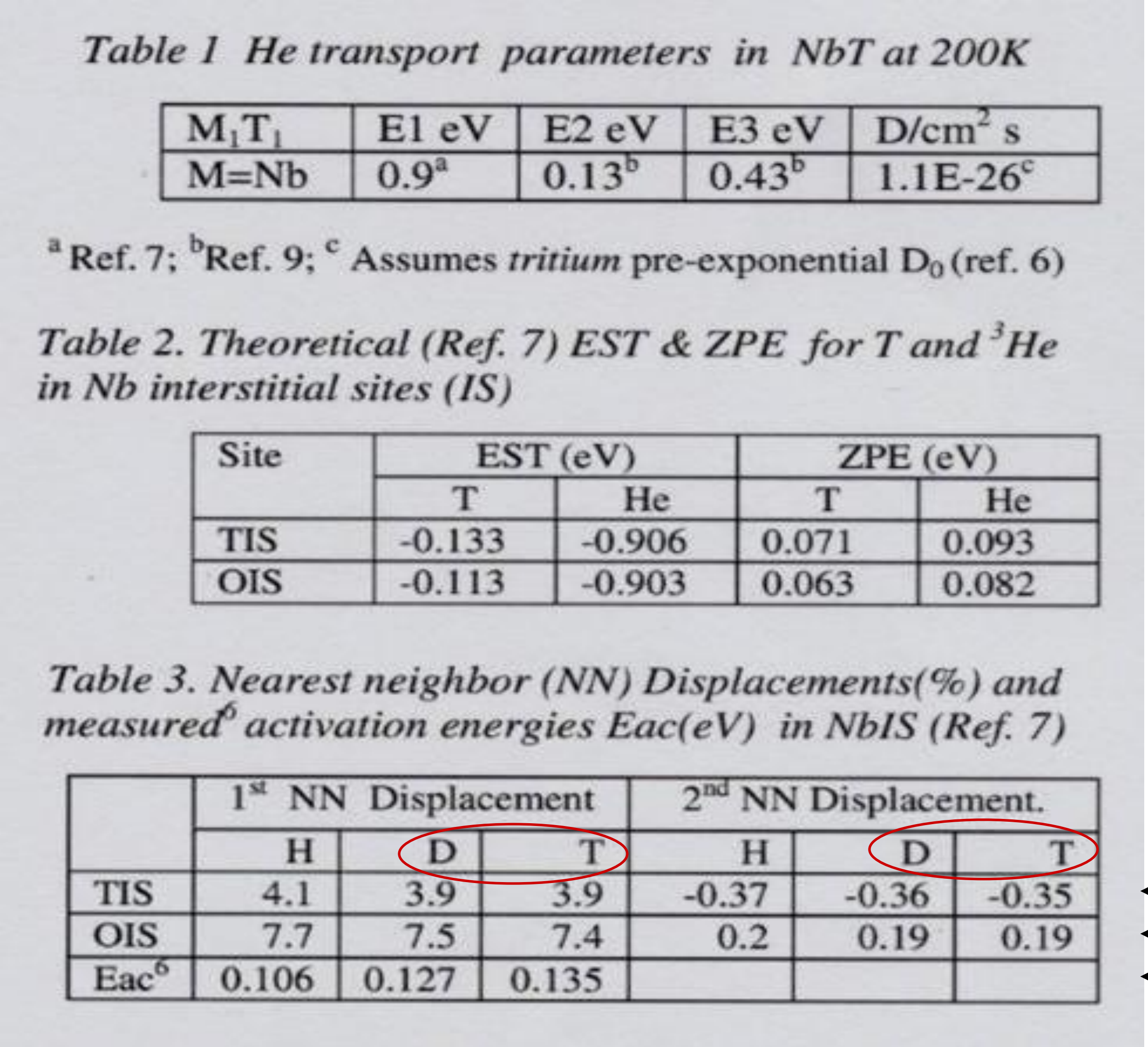


Table 1 He transport parameters in NbT at 200K

M_1T_1	E1 eV	E2 eV	E3 eV	D/cm^2 s
M=Nb	0.9[a]	0.13[b]	0.43[b]	1.1E-26[c]

[a] Ref. 7; [b] Ref. 9; [c] Assumes *tritium* pre-exponential D_0 (ref. 6)

Table 2. Theoretical (Ref. 7) EST & ZPE for T and ^{3}He in Nb interstitial sites (IS)

Site	EST (eV)		ZPE (eV)	
	T	He	T	He
TIS	-0.133	-0.906	0.071	0.093
OIS	-0.113	-0.903	0.063	0.082

Table 3. Nearest neighbor (NN) Displacements(%) and measured[6] activation energies Eac(eV) in NbIS (Ref. 7)

	1st NN Displacement			2nd NN Displacement.		
	H	D	T	H	D	T
TIS	4.1	3.9	3.9	-0.37	-0.36	-0.35
OIS	7.7	7.5	7.4	0.2	0.19	0.19
Eac[6]	0.106	0.127	0.135			

6 TIS
3 OIS

EST: self-trapping energy
ZPE: zero-point energy

Little difference between Deuterium and Tritium

theoretical

experimental activation energies

Interesting experiments

5) Real-time, ^{3}H-specific signal of $\bar{\nu}_e$ resonance

a) sudden change of the magnetic moment from
-2.1nm (^{3}He)→+2.79nm (^{3}H)

→ transient (~0.1ms) magnetic field which couples to electron moment of ^{3}H via hyperfine interaction

→ Read-out by SQUID

b) new electrons appear in the Nb bands when ^{3}H is formed.
These electrons cause additional specific heat that grows linearly with ^{3}H concentration.

→ detectable by ultra-sensitive (micro)-calorimeters ?