Phenomenology and theory of neutrino oscillations

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LTP

Three generations of leptons and quarks

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

NEUTRINOS ARE UNIQUE PARTICLES

FLAVOUR NEUTRINOS HAVE NO DEFINITE MASSES. THEY ARE “MIXED” PARTICLES

NEUTRINO MASSES ARE MANY ORDER OF MAGNITUDE SMALLER THAN MASSES OF QUARKS AND LEPTONSII

NEUTRINOS WITH DEFINITE MASSES CAN BE Majorana particles

NEUTRINOS HAVE ONLY WEAK INTERACTION

Neutrino interaction

STANDARD MODEL CC AND NC INTERACTION

$$\mathcal{L}_{\mathcal{I}}^{CC}(x) = -\frac{g}{2\sqrt{2}}\, j_{\alpha}^{CC}(x)\, W^{\alpha}(x) + \mathrm{h.c.} \quad \mathrm{j}_{\alpha}^{\mathrm{CC}}(\mathrm{x}) = 2 \sum_{\mathrm{l}=\mathrm{e},\mu,\tau} \bar{\nu}_{\mathrm{lL}}(\mathrm{x})\gamma_{\alpha}\mathrm{l}_{\mathrm{L}}(\mathrm{x})$$

$$\mathcal{L}_{\mathcal{I}}^{NC}(x) = -\frac{g}{2\cos\theta_W}\, j_{\alpha}^{NC}(x)\, Z^{\alpha}(x) \quad j_{\alpha}^{NC}(x) = \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x)\gamma_{\alpha}\nu_{lL}(x)$$

Neutrino mixing

NEUTRINO MASS TERM IS LORENZ INVARIANT PRODUCT OF L AND R COMPONENTS

LEFT-HANDED FIELDS ν_L AND $(\nu_R)^c$

RIGHT-HANDED FIELDS ν_R AND $(\nu_L)^c$

$(\nu_L)^c = C\bar{\nu}_L^T, \quad C\gamma_\alpha^T C^{-1} = -\gamma\alpha, \quad C^T = -C$ is conjugated field

THREE POSSIBILITIES FOR NEUTRINO MASS TERM

Dirac mass term

$$\mathcal{L}^D = -\sum_{l',l} \bar{\nu}_{l'L} M^D_{l'l} \nu_{l'R} + \text{h.c.}$$

CONSERVED TOTAL LEPTON NUMBER L

Majorana mass term

$$\mathcal{L}^{\mathcal{M}} = -\frac{1}{2}\sum_{l',l} \bar{\nu}_{l'L} M^{L}_{ll} (\nu_{l'L})^{c} + \text{h.c.}$$

NO CONSERVE LEPTON NUMBERS

The most general Dirac and Majorana mass term

$$\mathcal{L}^{\mathcal{D}+\mathcal{M}} = -\frac{1}{2}\sum_{l',l} \bar{\nu}_{l'L} M^{L}_{l'l} (\nu_{lL})^{c} - \sum_{l',l} \bar{\nu}_{l'L} M^{D}_{l'l} \nu_{l'R} - \frac{1}{2}\sum_{l',l} \overline{(\nu_{l'R})^{c}} M^{R}_{l'l} \nu_{lR} + \text{h.c.}$$

AFTER DIAGONALIZATION IN D AND M CASES

$$\nu_{lL} = \sum_{i=1}^{3} U_{li} \nu_{iL}$$

IN D CASE ν_i IS THE FIELD OF THE DIRAC NEUTRINO WITH MASS m_i

$$L(\nu_i) = 1. \quad L(\bar{\nu}_i) = -1$$

IN M CASE ν_i IS THE FIELD OF THE MJORANA NEUTRINO WITH MASS m_i; NEUTRINO$\equiv$ ANTINEUTRINO

$$\nu_i = \nu_i^c$$

IN D+M CASE FOR THE NEUTRINO MIXING

$$\nu_{lL} = \sum_{i=1}^{6} U_{li}\nu_{iL}, \quad (\nu_{lR})^c = \sum_{i=1}^{6} U_{\bar{l}i}\nu_{iL}, \quad \nu_i = (\nu_i)^c$$

SIX MASSIVE MAJORANA NEUTRINOS, POSSIBLE TRANSITIONS INTO STERILE STATES

State of flavor neutrino

NEUTRINOS ARE PRODUCED IN CC WEAK PROCESSES

$$\pi^+ \to \mu^+ + \nu_\mu, \quad (A,Z) \to (A, Z+1) + e^- + \bar{\nu}_e, \quad etc$$

AND ARE DETECTED IN WEAK RECTIO0NS

$$\nu_\mu + N \to \mu^- + X, \quad etc$$

ν_e, ν_μ, ν_τ PRODUCED TOGETHER WITH e^+, μ^+, τ^+

ARE CALLED FLAVOR NEUTRINOS

CONSIDER A DECAY

$$a \to b + l^+ + \nu_l$$

State of flavor neutrino

FINAL STATE

$$|f\rangle = \sum_i |l^+\rangle|\nu_i\rangle|b\ \rangle\langle l^+\nu_i\ b|\ S\ |a\rangle$$

$|\nu_i\rangle$ is the state of neutrino with mass m_i, momentum $\vec{p_i} = p_i\vec{k}$, energy $E_i = \sqrt{p_i^2 + m_i^2}$

Consider kinematics of the decay $\pi^+ \to \mu^+ + \nu_i.$

Neutrino energy $E_i = E + \frac{m_i^2}{2m_\pi}, \quad E = \frac{m_\pi^2 - m_\mu^2}{2m_\pi}$

NEUTRINO MOMENTUM $p_i = E + a_\pi \frac{m_i^2}{2E}, \qquad a_\pi = -\frac{1}{2}(1 + \frac{m_\mu^2}{m_\pi^2}) \simeq -0.75.$

$$p_i - p_k = a_\pi \frac{\Delta m_{ki}^2}{2E} \quad \Delta m_{ki}^2 = m_i^2 - m_k^2$$

State of flavor neutrino

IN THREE-BODY DECAY $p_i - p_k$: PROPORTIONAL TO Δm^2_{ki} :

$$\text{If } E \gg m_i \quad (p_i - p_k) \to 0$$

FROM DIMENSION $$p_i - p_k = a\,\frac{\Delta m^2_{ki}}{2E} \quad a \simeq 1$$

Let us introduce $$\frac{\Delta m^2_{ki}}{2E} = \frac{1}{L_{ki}}$$

FOR REACTOR AND ATMOSPHERIC NEUTRINOS $L_{12} \simeq 100 \text{ km} \; L_{23} \simeq 800 \text{ km}$

QM uncertainty of momentum $$(\Delta p)_{QM} \simeq \frac{1}{a}$$

We have $$L_{12}(L_{23}) \gg a \quad (p_i - p_k) \ll (\Delta p)_{QM}$$

PRODUCTION OF NEUTRINOS WITH DIFFERENT MASSES (MOMENTA) CAN NOT BE RESOVED

State of flavor neutrino

IN THE MATRIX ELEMENT OF THE DECAY DIFFERENCES IN NEUTRINO MOMENTA CAN BE NEGLECTED

$$\langle l^+ \nu_i \; b | \; S \; | a \rangle = \langle l^+ \nu_l \; b | \; S \; | a \rangle_{SM} \; U^*_{li},$$

$\langle l^+ \nu_l \; b | \; S \; | a \rangle_{SM}$ IS THE SM MATRIX ELEMENT OF THE PROCESS $a \to b + l^+ + \nu_l$

$$\langle l^+ \nu_i \; b | \; S \; | a \rangle = \langle l^+ \nu_l \; b | \; S \; | a \rangle_{SM} \; U^*_{li},$$

$$|f\rangle = |l^+\rangle |\nu_l\rangle |b\rangle \langle l^+ \nu_i \; b | \; S \; | a \rangle_{SM}$$

$$|\nu_l\rangle = \sum_i U^*_{li} |\nu_l\rangle$$

STATE OF FLAVOR NEUTRINO WITH MOMENTUM p

$$\langle \nu_{l'} | \nu_l \rangle = \delta_{l'l}$$

Evolution of flavor states

IF AT t-0 FLAVOR NEUTRINO ν_l WAS PRODUCED

$$|\nu_l\rangle_t = e^{-iH_0 t}\ |\nu_l\rangle = \sum_i e^{-iE_i t} U^*_{li}\ |\nu_i\rangle \quad E_i \simeq E + \frac{m_i^2}{2E}$$

FLAVOR NEUTRINOS ARE DETECTED IN PROCESSES LIKE $\nu_\mu + N \to \mu^- + X$

$$|\nu_l\rangle_t = \sum_{l'} |\nu_{l'}\rangle \sum_i U_{l'i}\ e^{-iE_i t}\ U^*_{li}$$

FOR DIFFERENT m_i^2 NEW STATES APPEAR IN THE SUM

PROBABILITY OF THE TRANSITION $\nu_l \to \nu_{l'}$

$$P(\nu_l \to \nu_{l'}) = |\sum_i U_{l'i}\ e^{-iE_i t}\ U^*_{li}|^2$$

Evolution of flavor states

CAN BE REWRITTEN

$$P(\nu_l \to \nu_{l'}) = |\delta_{l'l} + \sum_i U_{l'i}\,(e^{-i\frac{\Delta m^2_{pi} L}{2E}} - 1)\,U^*_{li}|^2, \quad E_i - E_p = \frac{\Delta m^2_{pi} L}{2E}$$

P IS ARBITRARY, FIXED $i \neq p$

WE USED $L \simeq t$ L IS SOURCE-DETECTOR DISTANCE

$$P(\nu_l \to \nu_{l'}) = \delta_{l'l} - 4 \sum_i |U_{li}|^2 (\delta_{l'l} - |U_{l'i}|^2) \sin^2 \Delta_{pi}$$

$$+ \quad 8\,\mathrm{Re} \sum_{i>k} e^{-i(\Delta_{pi} - \Delta_{pk}}\; U_{l'i} U^*_{li} U^*_{l'k} U_{lk}\; \sin\Delta_{pi} \sin\Delta_{pk}, \quad \Delta_{pi} = \frac{\Delta m^2_{pi} L}{4E}$$

Two-neutrino oscillations

$i, k = 1, 2$, choose $p = 1$, then $i = k = 2$.

$$P(\nu_l \to \nu_{l'}) = \delta_{l'l} - 4|U_{l2}|^2(\delta_{l'l} - |U_{l'2}|^2)\ \sin^2 \frac{\Delta m^2 L}{4E}.$$

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$P(\nu_l \to \nu_l) = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E} \qquad P(\nu_l \to \nu_{l'}) = \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}.\ \ l' \neq l$$

$$\frac{\Delta m^2 L}{4E} = 1.27 \frac{\Delta m^2(\mathrm{eV}^2) L(\mathrm{m})}{E(\mathrm{MeV})}$$

FIRST MINIMUM $\quad P(\nu_l \to \nu_l) \qquad L_{\mathrm{min}} = \frac{\pi E}{2.54 \Delta m^2}.$

ATMOSPHERIC NEUTRINOS 500 KM, REACTOR NEUTRINOS 60 KM

Three-neutrino oscillations-leading approximation

MASS LABELING $m_1 < m_2 < m_3$

SIX NEUTRINO OSCILLATION PARAMETERS $\theta_{12}, \theta_{23}, \theta_{13}$ and CP phase δ

Two mass-squared differences Δm^2_{12} and Δm^2_{23}

Two parameters are small: $\frac{\Delta m^2_{12}}{\Delta m^2_{123}} \simeq 3 \cdot 10^{-2}, \quad \sin^2\theta_{12} \simeq 2.5 \cdot 10^{-2}$

In the atmospheric range of $\frac{L}{E}$ $\left(\frac{\Delta m^2_{23}L}{4E} \geq 1\right)$ we can neglect $\frac{\Delta m^2_{12}L}{4E}$

Neglecting contribution of $]U_{e3}|^2 = \sin^2\theta_{13}$ we have

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta_{23} \sin^2 \frac{\Delta m^2_{23}L}{4E}$$

S-K, MINOS, K2K

Leading approximation

IN THE SOLAR RANGE $(\frac{\Delta m^2_{12} L}{4E} \geq 1)$ we can neglect $]U_{e3}|^2 = \sin^2\theta_{13}$

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m^2_{12} L}{4E}$$

KAMLAND

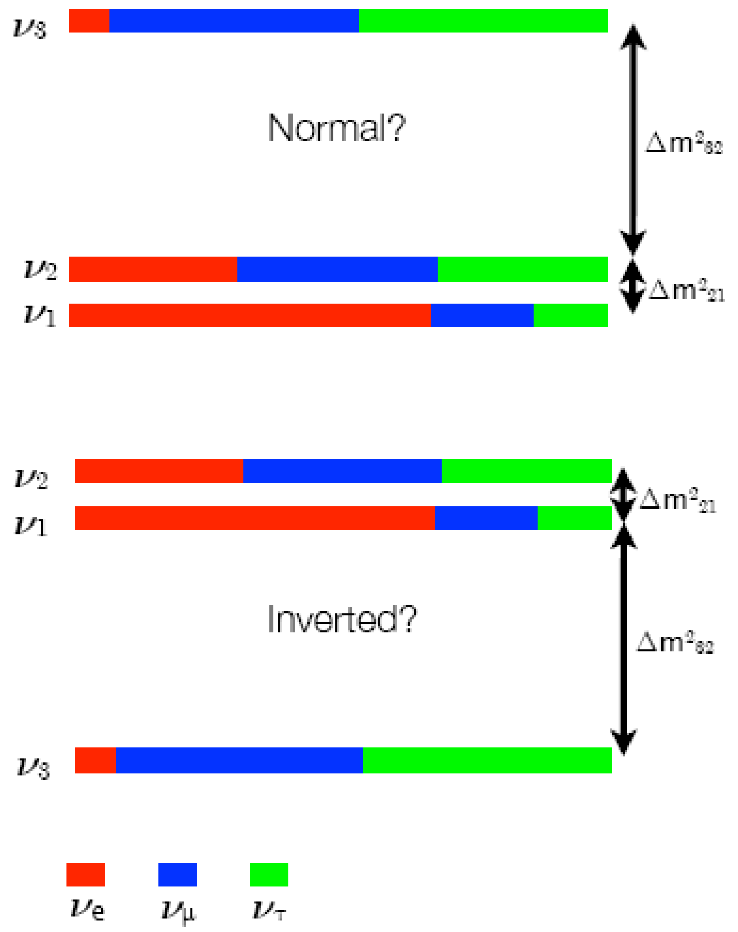
THE PROBABILITY OF THE REACTOR ANTINEUTRINO TO SURVIVE IN THE ATMOSPHERIC RANGE

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{23} L}{4E}$$

DAYA BAY, RENO, D-CHOOZ

IN THE CASE OF THREE NEUTRINOS TWO NEUTRINO MASS SPECTRA ARE POSSIBLE

Neutrino mass spectra



IN THE LEADING APPROXIMATION FOR NORMAL AND INVERTED MASS SPECTRA WE OBTAIN THE SAME EXPRESSIONS FOR NEUTRINO SURVIVAL PROBABILITIES

I\IN ORDER TO DISTINGUISH NORMAL AND INVERTED SPECTRA WE NEED TO BE SENSITIVE TO BEYOND THE LEADING APPROXIMATION TERMS

Let us introduce solar and atmospheric mass-squared differences

$$\Delta m_{12}^2 = \Delta m_S^2 \quad (N, I) \quad \Delta m_{23}^2 = \Delta m_A^2 \ (N) \quad |\Delta m_{13}^2| = \Delta m_A^2 \ (I)$$

NEXT PROBLEM OF NEUTRINO OSCILLATION PHYSICS TO DETERMINE THE CHARACTER

OF NEUTRINO MASS SPECTRUM

CAN BE SOLVED WITH ACCELERATOR, REACTOR, ATMOSPHERIC NEUTRINOS

Reactor antineutrinos in three-neutrino case

FROM GENERAL EXPRESSION

$$P^{\rm NS}(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \sin^2 \Delta_A$$
$$-(\cos^2 \theta_{13} \sin^2 2\theta_{12} + \sin^2 2\theta_{13} \cos^4 \theta_{12}) \ \sin^2 \Delta_S$$
$$-2 \sin^2 2\theta_{13} \cos^2 \theta_{12} \ \cos(\Delta_A + \Delta_S) \sin \Delta_A \sin \Delta_S.$$

$$P^{\rm IS}(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \sin^2 \Delta_A$$
$$-(\cos^2 \theta_{13} \sin^2 2\theta_{12} + \sin^2 2\theta_{13} \sin^4 \theta_{12}) \ \sin^2 \Delta_S$$
$$-2 \sin^2 2\theta_{13} \sin^2 \theta_{12} \ \cos(\Delta_A + \Delta_S) \sin \Delta_A \sin \Delta_S.$$

DIFFER BY THE CHANGE $\cos^2 \theta_{12} \leftrightarrows \sin^2 \theta_{12}$

Conclusion

NEUTRINO INTERACTION IS CC AND NC STANDARD MODEL INTERACTION

THREE MASS TERMS (D, M, D+M0 ARE POSSIBLE

NEUTRINOS WITH DEFINITE MASSES ARE D PARTICLES (D) OR M PARTICLES (M, D+M)

IN CC AND NC “MIXED” FLAVOR FIELDS ENTER

DUE TO HEISENBERG UNCERTAINTY RELATIN IN WEAK PROCESSES SMALL NEUTRINO MASS-SQUARED DIFFERENCES CAN NOT BE RESOLVED

MIXED FLAVOR ELECTRON, MUON AND TAU NEUTRINO STATES ARE PRODUCED AND DETECTED

SMALL NEUTRINO MASS-SQUARED DIFFERENCES CAN BE REVEALED IF NEUTRINO BEAMS PROPAGARE AT MACROSCOPICALLY LARGE DISTANCES

Conclusion

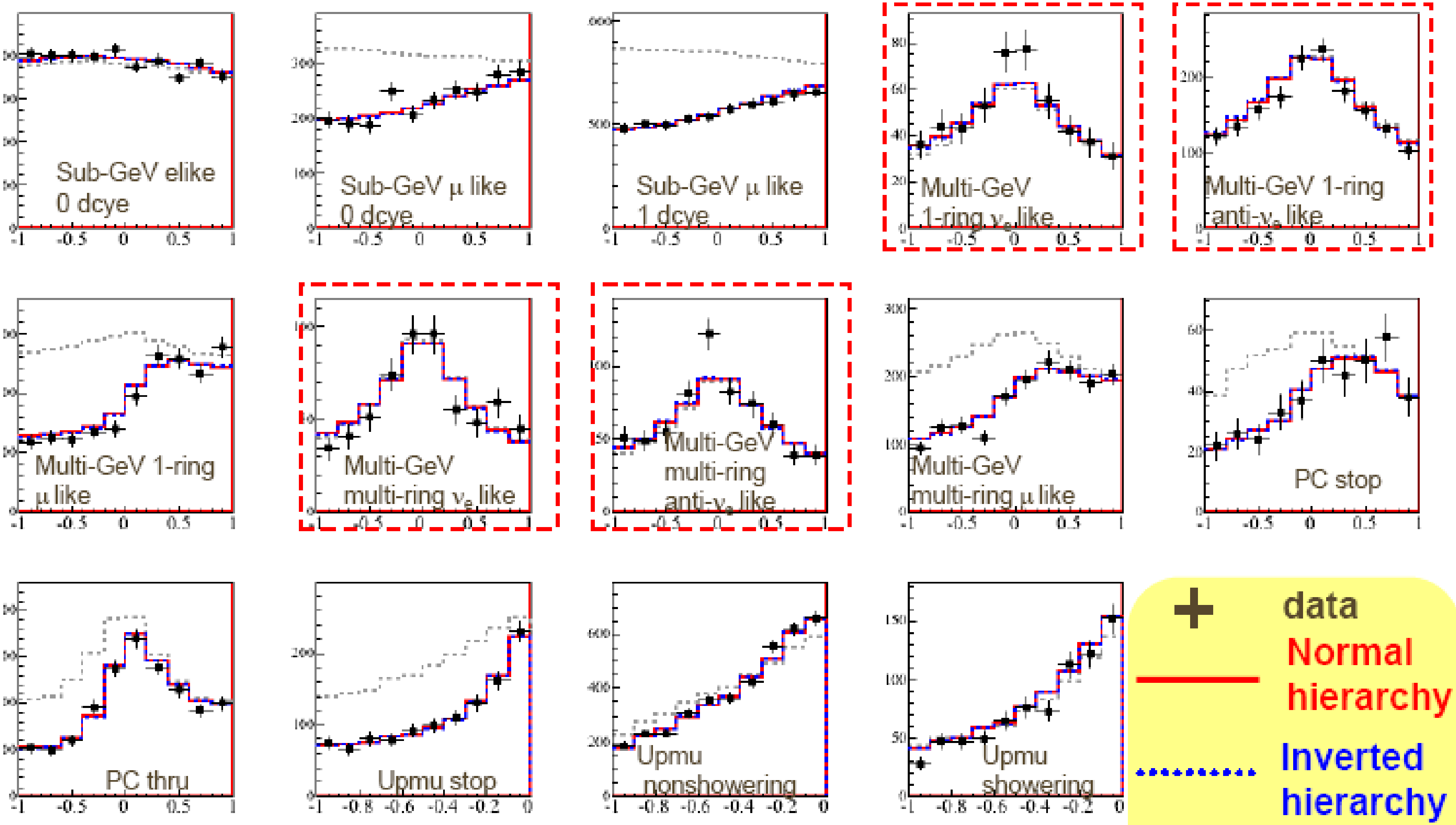
NEUTRINO OSCILLATIONS ARE DUE PHASES WHICH NEUTRINOS WITH DEFINITE MASSES GAIN

IN PROPAGATION

CONVENIENT PHENOMENOLOGICAL EXPRESSION FOR THE TRANSITION PROBABILITY IS DERIVED AND

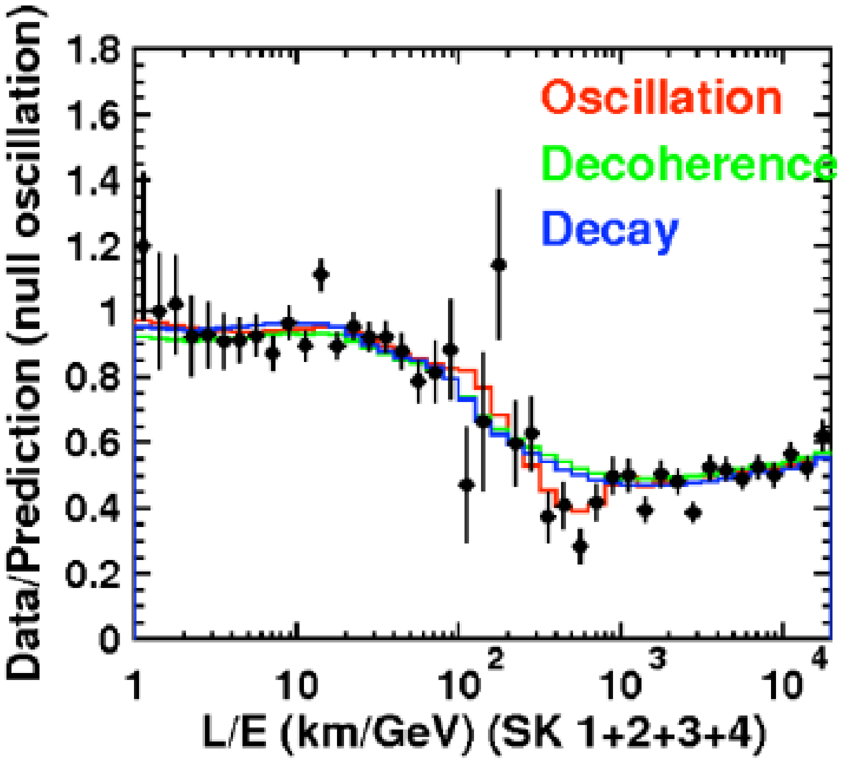
APPLIED TO DIFFERENT CASES

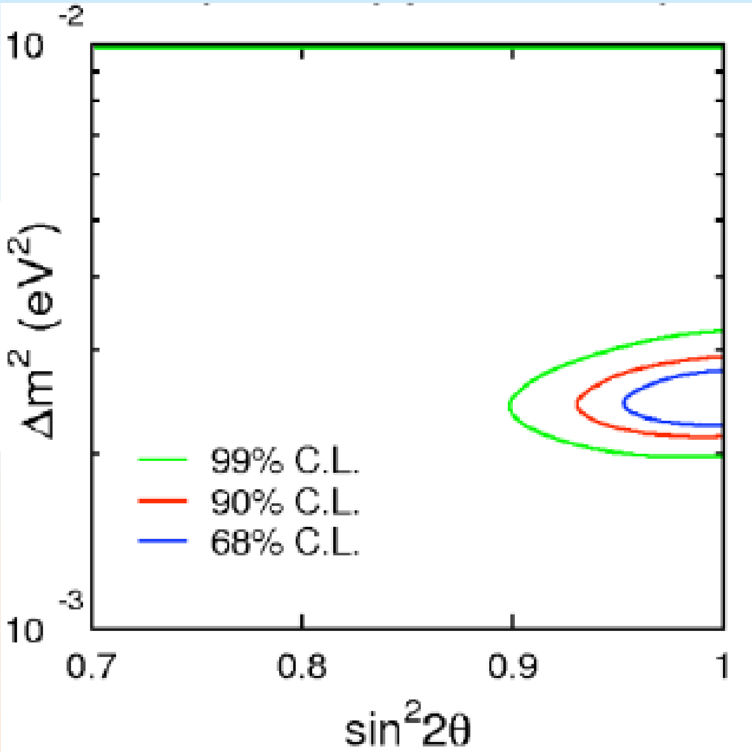
3 flavor analysis (SK1-4) with reactor constraint



- Fixed $\sin^2\theta_{13} = 0.025$

2 flavor L/E analysis update (SK1-4)





Neutrino decay

$$\chi^2_{\min} = 187.8/169(4.0\sigma)$$

Neutrino decoherence

$$\chi^2_{\min} = 194.8/169(4.8\sigma)$$

⟺

2 ν oscillation result

$$\sin^2 2\theta = 1.00(\geq 0.93(90\%CL))$$

$$\Delta m^2 = (2.5^{+0.27}_{-0.27}) \times 10^{-3} eV^2$$

$$\chi^2_{\min} = 171.7/169$$

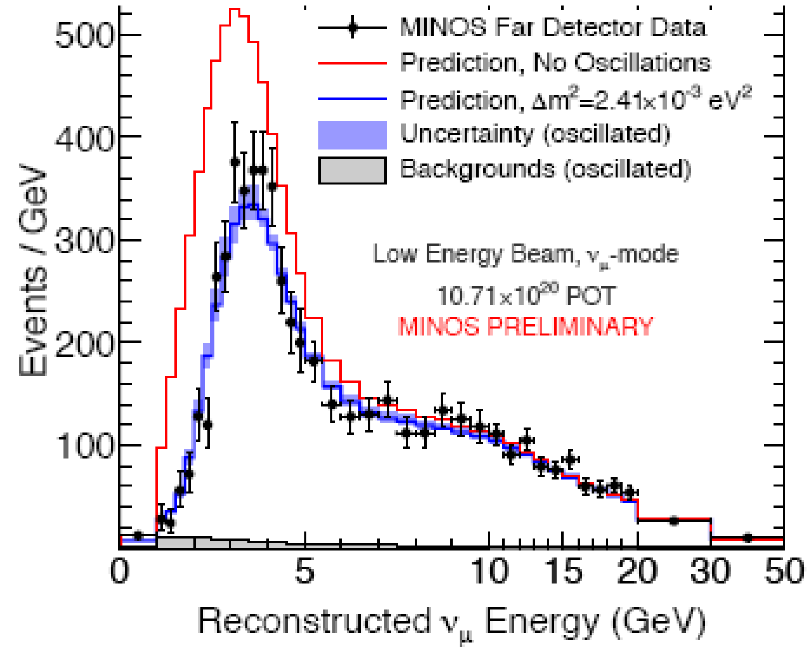
Muon Neutrinos at the Far Detector

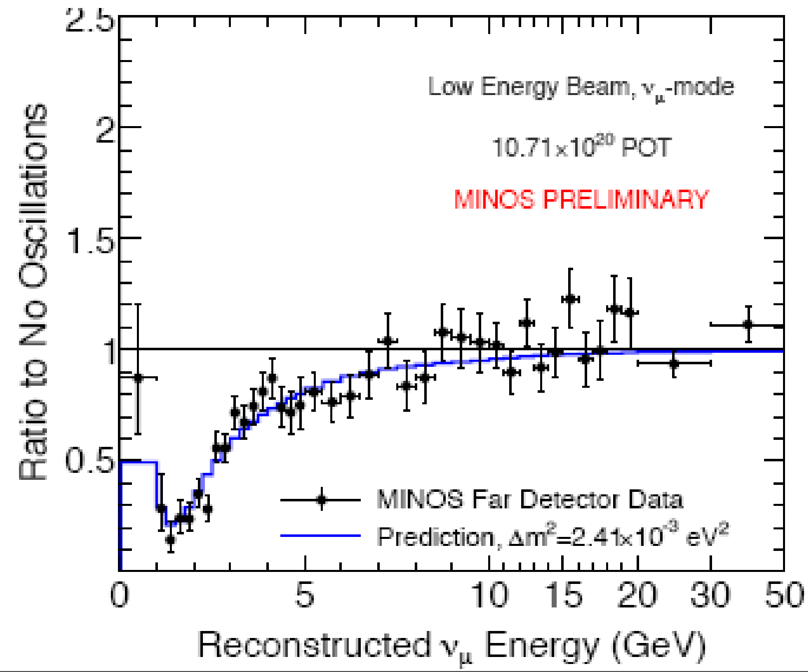
- Based on **ND** data we expect
    - 3564 **E**vents (no osc.)
- We observe
    - 2894 **E**vents
- The systematic uncertainties are still small compared to statistical uncertainty
- Best fit to neutrino beam

$$\left|\Delta m^2\right| = 2.41^{+0.11}_{-0.10} \times 10^{-3} eV^2$$

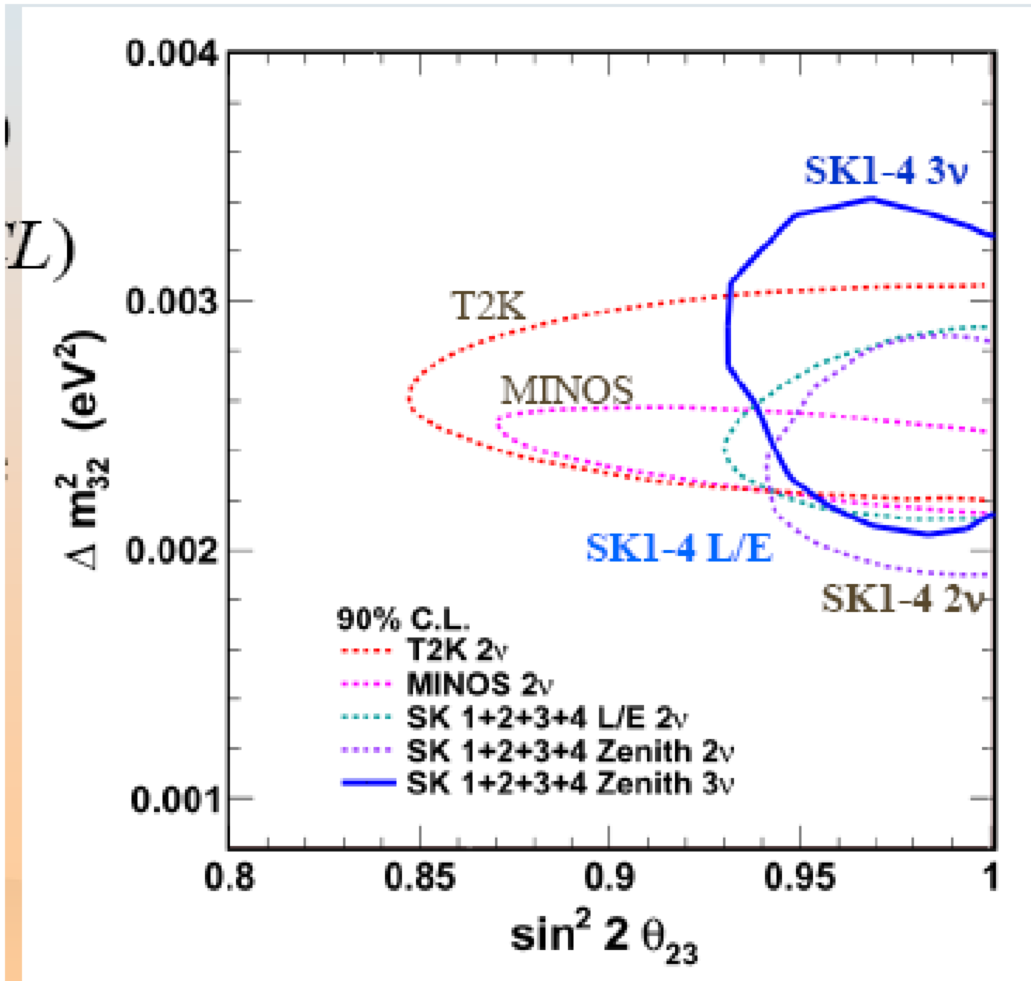
$$\sin^2(2\theta) = 0.94^{+0.04}_{-0.05}$$

See Poster

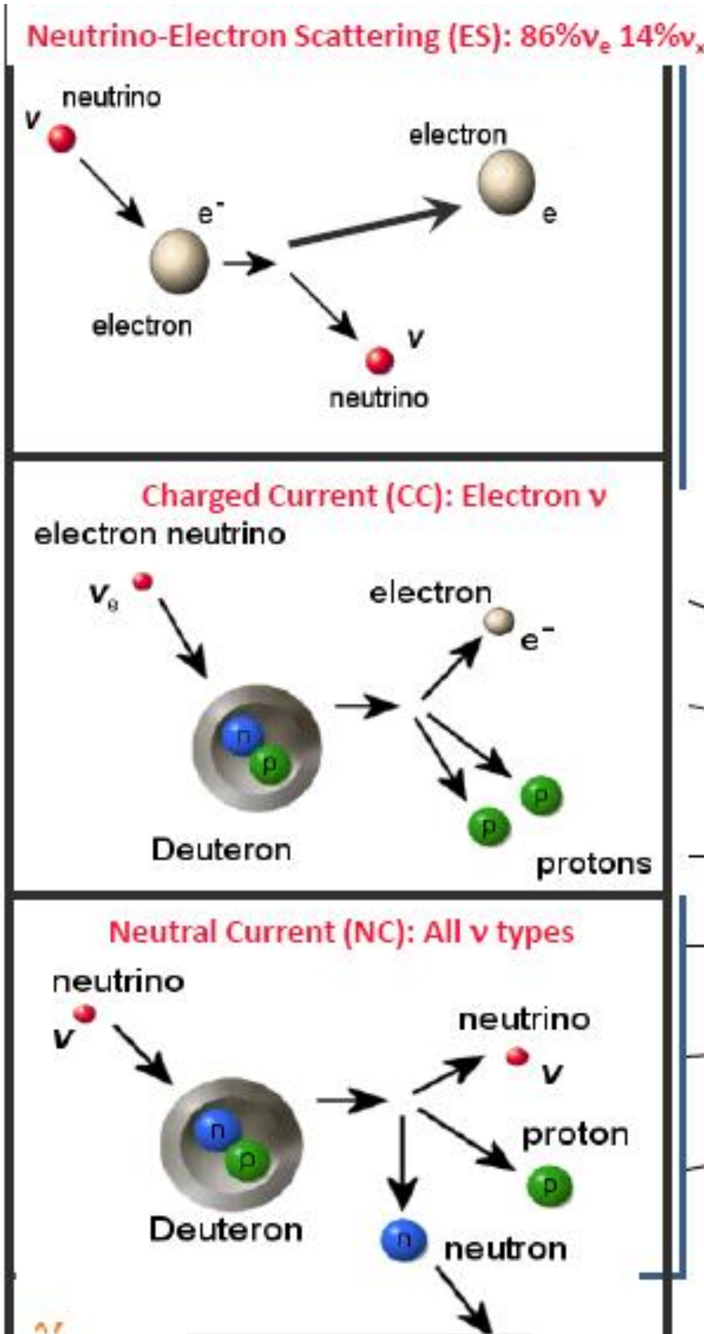


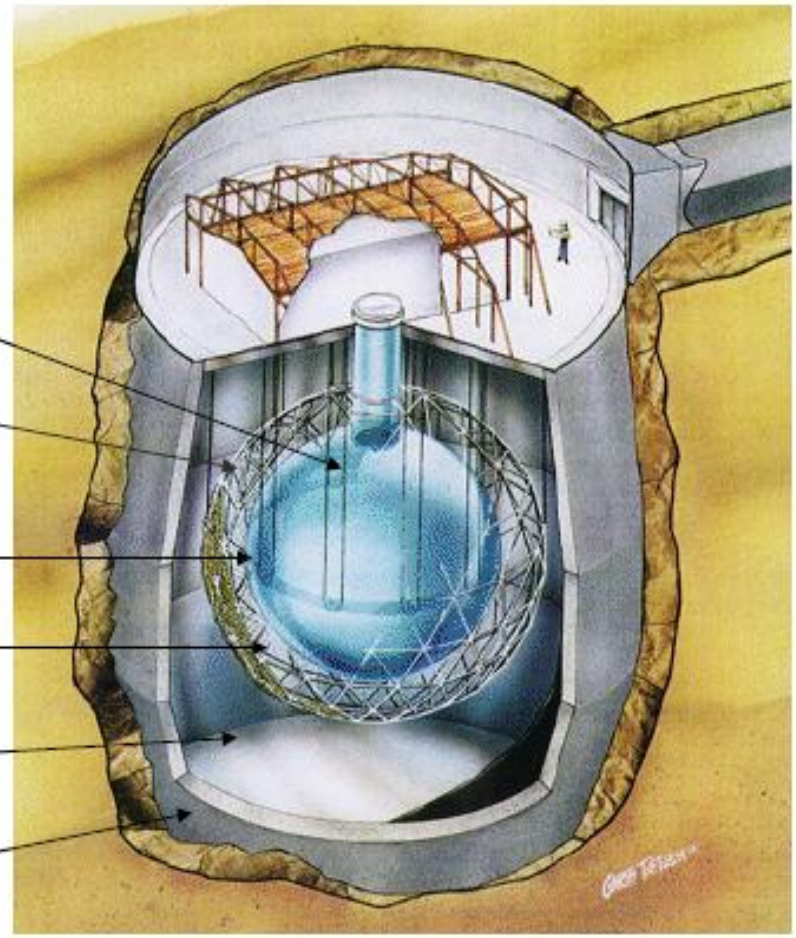


Summary: MINOS accelerator neutrinos

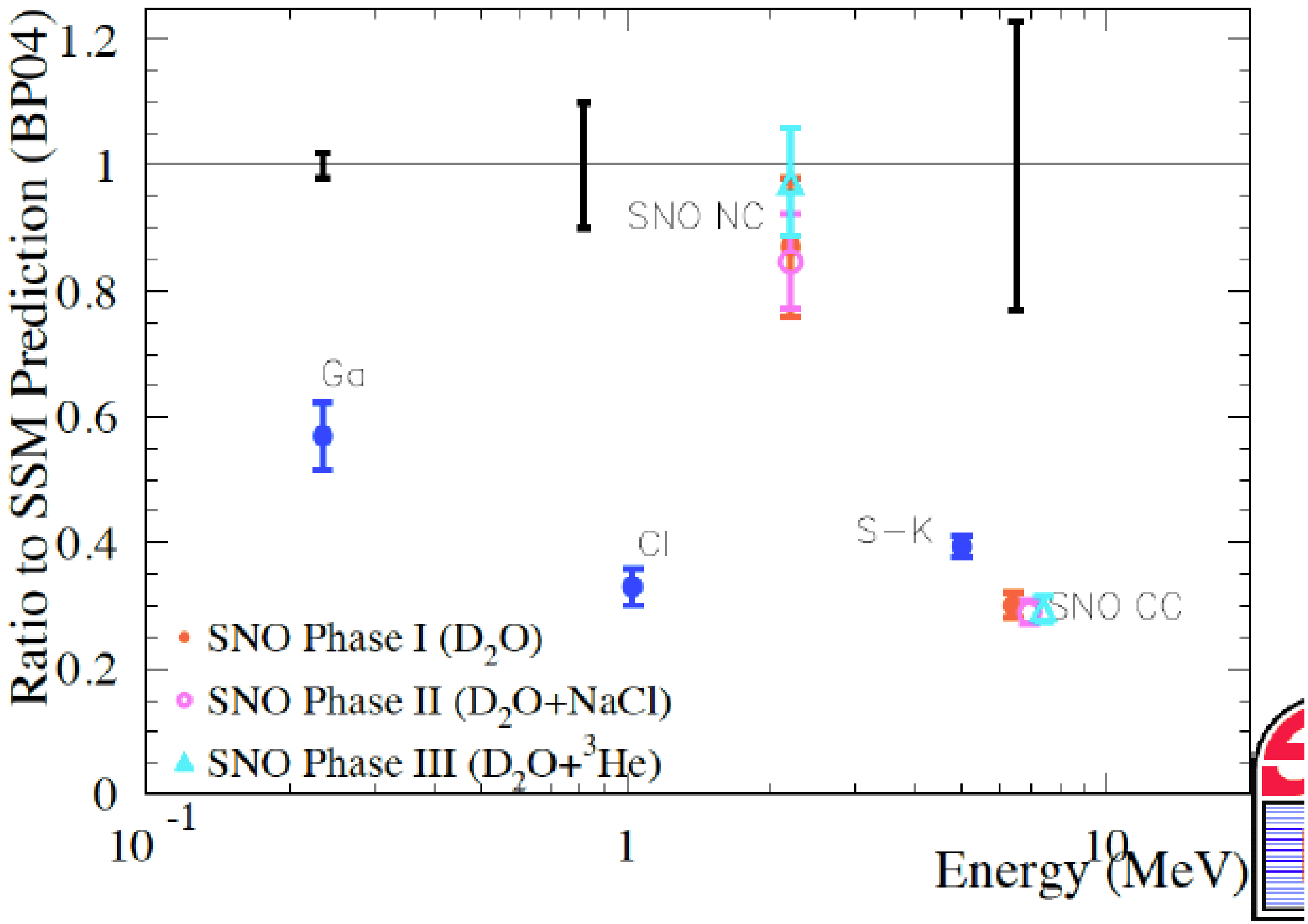


Sudbury Neutrino Observatory

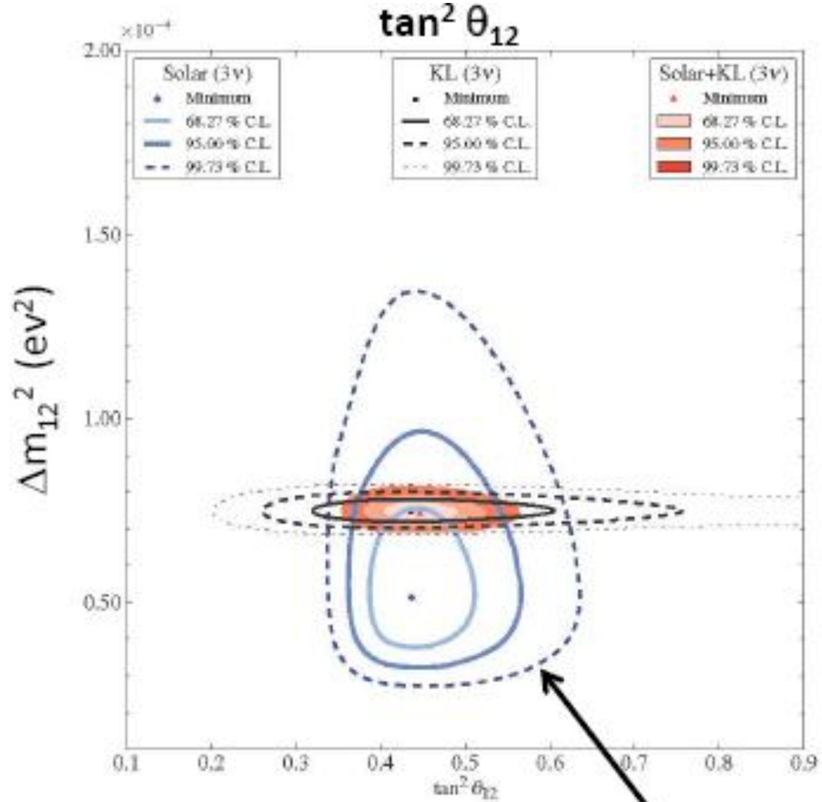




Solar Neutrino Problem Resolved

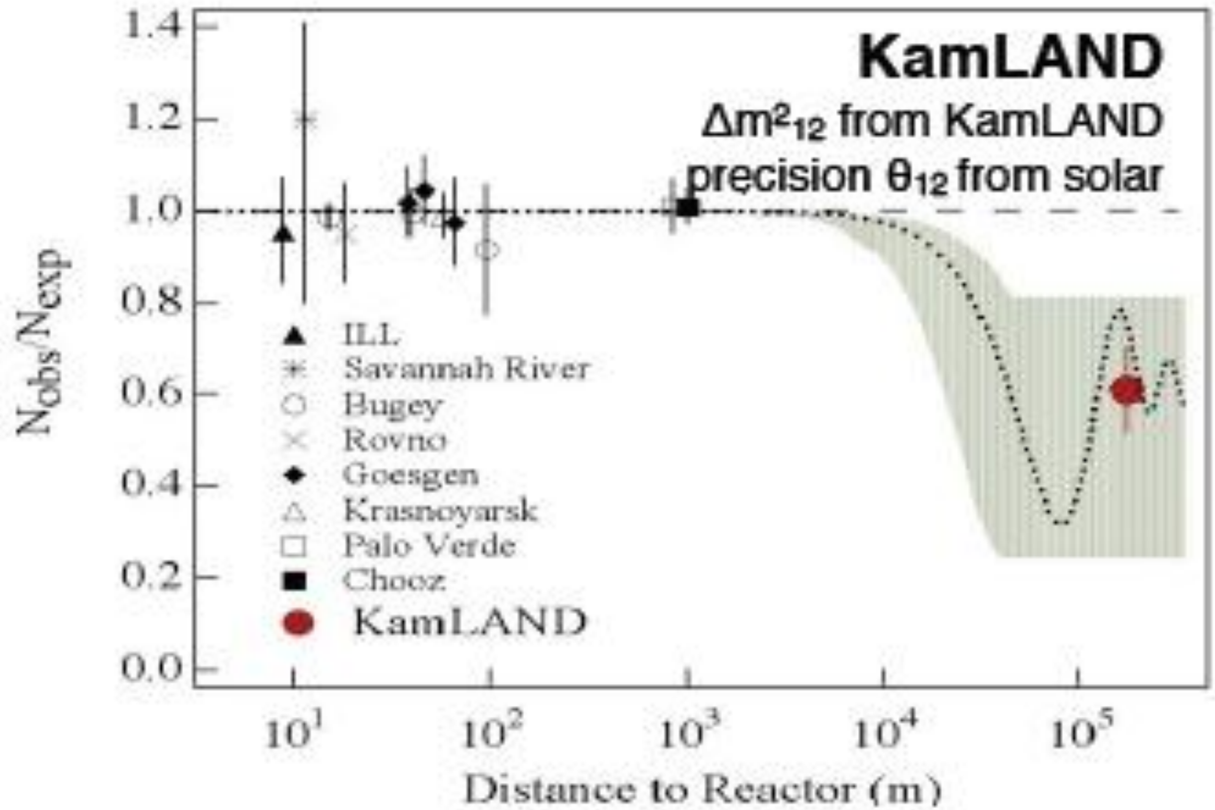


$\tan^2 \theta_{12}$



SNO defines θ_{12} to be non-maximal by more than 5 σ.

KamLAND reactor neutrinos



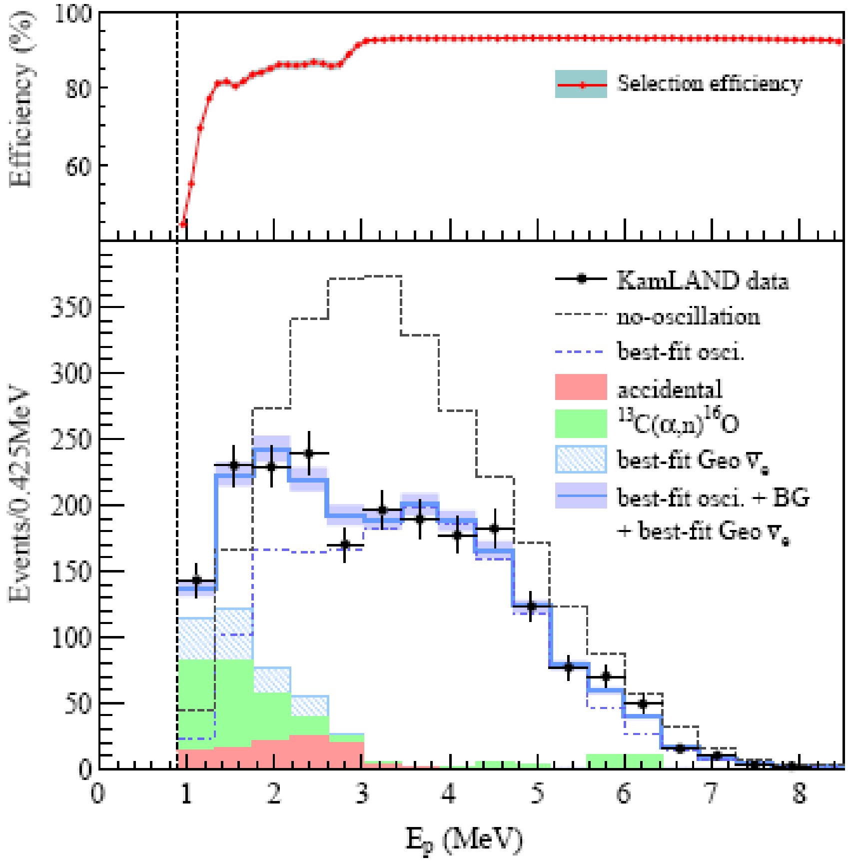
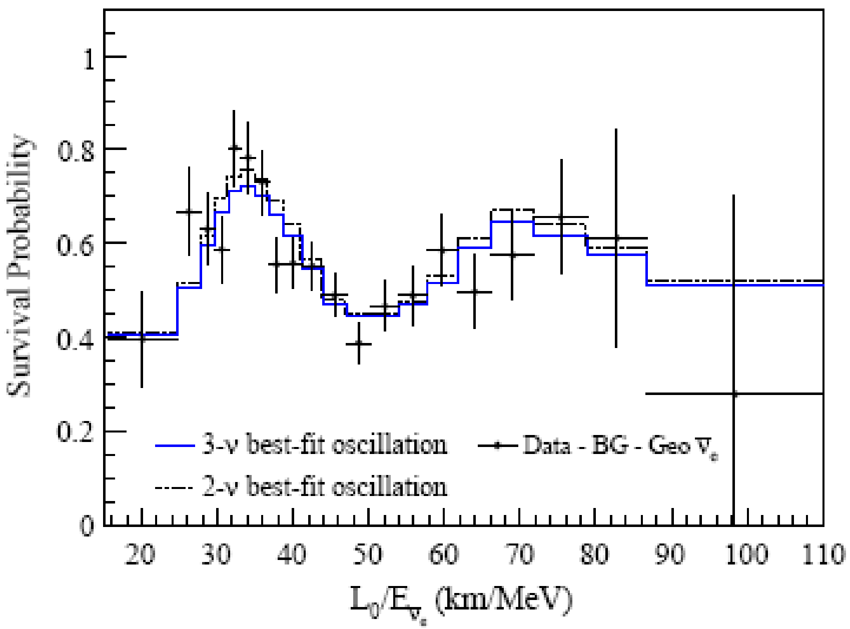


FIG. 1: Prompt energy spectrum of $\overline{\nu}_e$ candidate events above 0.9 MeV energy threshold (vertical dashed line). The data together with the background and reactor $\overline{\nu}_e$ contributions fitted from an unbinned maximum-likelihood three-flavor oscillation analysis are shown in the main panel. The number of geo-$\overline{\nu}_e$'s is unconstrained in the fit. The shaded background histograms are cumulative. The top panel shows the energy-dependent selection efficiency; each point is the weighted average over the five time periods described in the text.

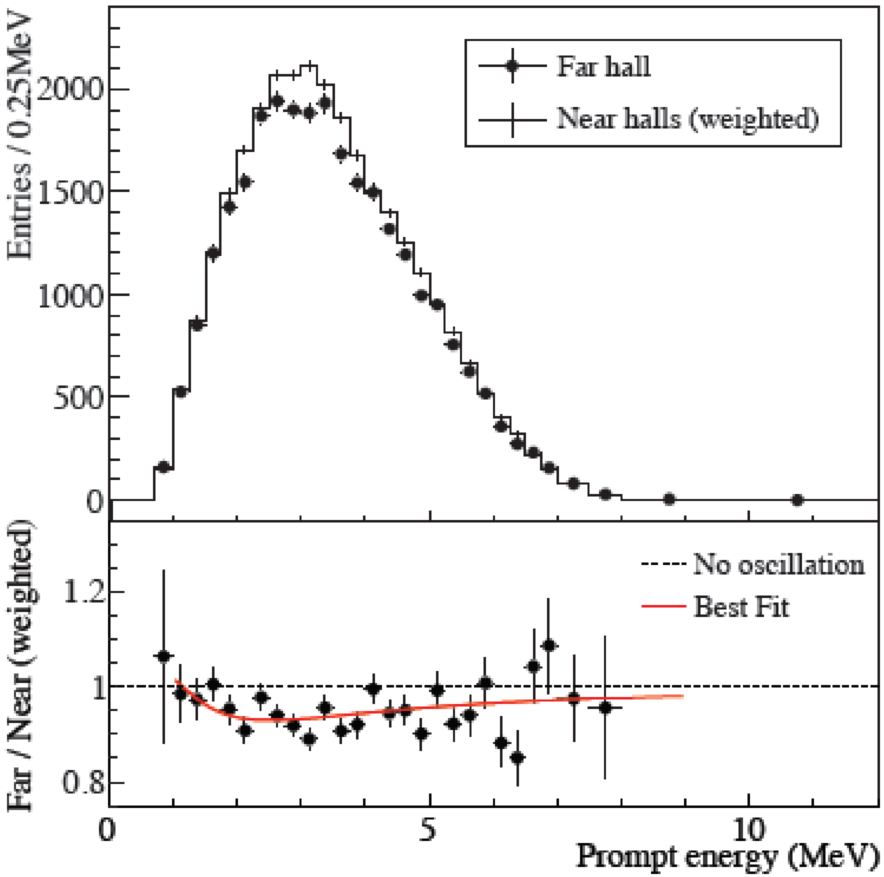
Kamland3



the best-fit oscillation parameter values are $\Delta m^2_{21} = 7.49^{+0.20}_{-0.20} \times 10^{-5}$ eV^2, $\tan^2\theta_{12} = 0.436^{+0.102}_{-0.081}$ and $\sin^2\theta_{13} = 0.032^{+0.037}_{-0.037}$ (< 0.094 at the 90% C.L.). The two-flavor oscillation treatment using Eq. (7), as presented pre-

Far vs. Near Comparison

Compare the far/near measured rates and spectra



$$R = \frac{Far_{measured}}{Far_{expected}} = \frac{M_4 + M_5 + M_6}{\sum_{i=4}^{6}(\alpha_i(M_1 + M_2) + \beta_i M_3)}$$

M_n are the measured rates in each detector. Weights α_i, β_i are determined from baselines and reactor fluxes.

R = 0.944 ± 0.007 (stat) ± 0.003 (syst)

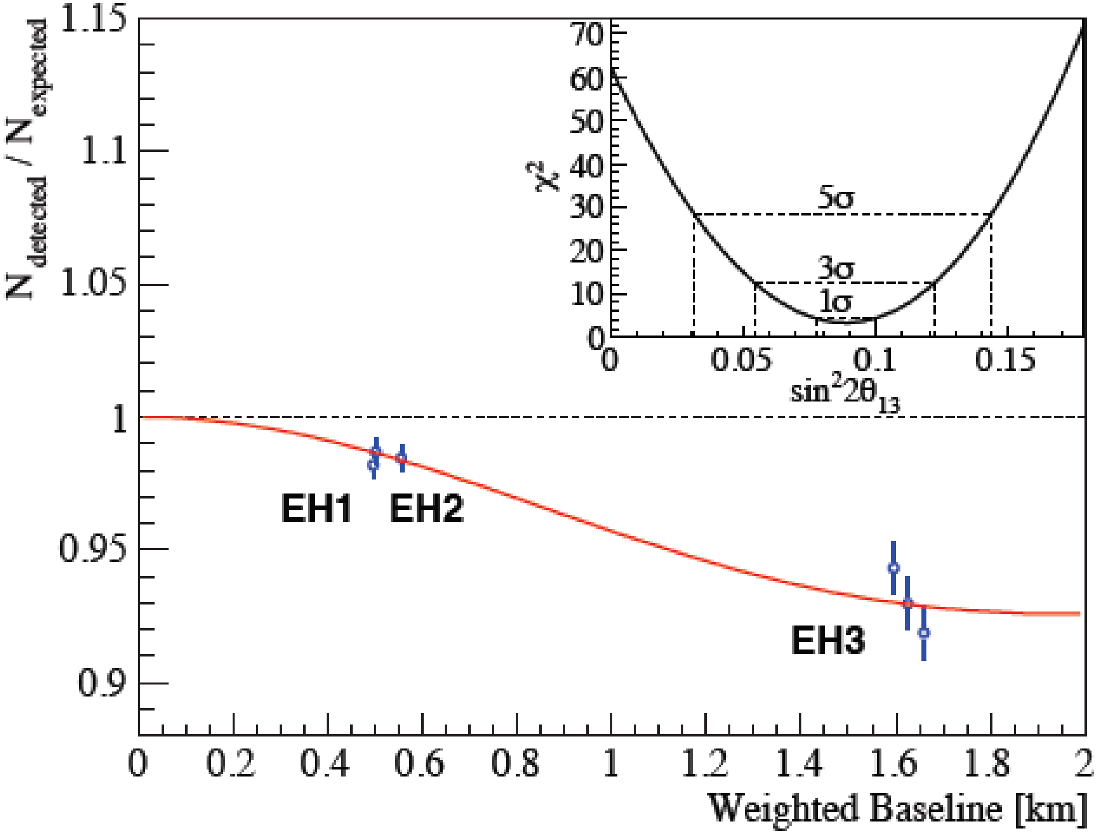
Clear observation of far site deficit.

Spectral distortion consistent with oscillation.*

* Caveat: Spectral systematics not fully studied; θ_{13} value from shape analysis is not recommended.

Rate Analysis

Estimate θ_{13} using measured rates in each detector.



Uses standard χ^2 approach.

Far vs. near relative measurement.
[Absolute rate is not constrained.]

Consistent results obtained by independent analyses, different reactor flux models.

Most precise measurement of $\sin^2 2\theta_{13}$ to date.

$\sin^2 2\theta_{13} = 0.089 \pm 0.010\ (\text{stat}) \pm 0.005\ (\text{syst})$

Future

Large value of θ_{13} opens the way to

I.Determination of the character of the neutrino mass spectrum (normal or inverted)

II. Measurement of the CP phase δ

Next fundamental problem:

Are neutrinos ν_i Dirac or Majorana particles ?

The answer can be abtained via investigation of **neutrinoless double β decay of even-even nuclei**

$$(A, Z) \to (A, Z+2) + e^- + e^=$$

Future

This is a process of the second order in G_F with virtual neutrino
The lepton part of the matrix element

$$G_F \frac{m_{\beta\beta}}{p^2}, \quad m_{\beta\beta} = \sum_i U_{ei}^2 m_i$$

From data of the Heidelberg- Moscow experiment

$$T_{\frac{1}{2}}(^{76}\text{Ge}) > 1.9 \cdot 10^{25}\text{years}$$

From these data $|m_{\beta\beta}| < (0.20 - 0.32)$ eV

Similar results were obtained in CUORICINO (^{130}Te), EXO (^{136}Xe) and other experiments

Sensitivity of future experiments

$$|m_{\beta\beta}| \simeq \text{a few } 10^{-2} \text{ eV}$$

Seesaw mechanism

Exist heavy Majorana leptons N_i, singlets of the $SU_L(2) \times U(1)$ group, which have the following lepton number violating Yukawa interactions with lepton and Higgs doublets

$$\mathcal{L} = -\sqrt{2} \sum_{i,l} Y_{li} \overline{L}_{lL} N_{iR} \tilde{H} + \text{h.c.}.$$

$$L_{lL} = \begin{pmatrix} \nu_{lL} \\ l_L \end{pmatrix}, \qquad H = \begin{pmatrix} H^{(+)} \\ H^{(0)} \end{pmatrix}$$

For the processes with virtual N_i generates the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = -\frac{1}{\Lambda} \sum_{l',l,i} \overline{L}_{l'L} \tilde{H} \sum_i (Y_{l'i} \frac{\Lambda}{M_i} Y_{li}) C \tilde{H}^T (\overline{L}_{lL})^T + \text{h.c.},$$

which does not conserve the total lepton number L

Λ has the dimension of mass and characterizes the scale of new physics

Seesaw mechanism

After spontaneous violation of the electroweak symmetry
the left-handed Majorana mass term

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2}\sum_{l',l} \overline{\nu}_{l'L}\ M^{L}_{l'l}\ (\nu_{lL})^{c} + \mathrm{h.c.} = -\frac{1}{2}\sum_{i} m_i \bar{\nu}_i \nu_i,$$

where

$$M^{L} = Y\frac{v^2}{M}Y^{T} = UmU^{T},$$

$\nu_i^c = \nu_i$ is the field of the Majorana neutrino with the mass m_i and

$$\nu_{lL} = \sum_i U_{li}\nu_{iL}.$$

The size of neutrino masses is determined by the seesaw factor $\frac{v^2}{M_i}$. From the existing data we can estimate that $M_i \simeq (10^{14} - 10^{15})$ GeV