THEORY OF NEUTRINO MASSES AND MIXING

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Topic of these Lectures

Models to explain Neutrino Masses and Mixing, considering different classes of symmetries and contexts...

...with a phenomenological perspective: no too many formal details...

...such that, at the end of the lectures, you will have good bases to understand papers and talks on this subject...

...and to get the tools to work on this topic, I will provide a series of exercises *(write at luca.merlo@ph.tum.de to get the solutions)*

Outline

- The Flavour Puzzle: quarks vs. leptons
- The Flavour Symmetries: which and where?
- Flavour Models at the GUT Scale
    - Continuous Symmetries: the Froggatt-Nielsen U(1)
    - Discrete Symmetries: the Altarelli-Feruglio A_4
- Minimal (Lepton) Flavour Violation
- Flavour Models at the Electroweak Scale
    - Multi-Higgs Models: The Ma-Rajasekaran A_4

The Flavour Puzzle: quarks vs. leptons

Notation

I will use Weil spinors instead of Dirac spinors:

$$\Psi = \Psi_L + \Psi_R \qquad \Psi_L \equiv \begin{pmatrix} 0 \\ \psi \end{pmatrix} \quad \Psi_R \equiv \begin{pmatrix} i\sigma_2 \psi^{c*} \\ 0 \end{pmatrix}$$

(mass term: $\overline{\Psi}_1 \Psi_2 = \psi_1^c \psi_2 + (\psi_1 \psi_2^c)^*$)

where the two spinors can transform independently under the Lorentz and SM gauge groups

→ Weil spinors are fundamental objects

The SM spectrum consists of three copies of the following fields:

		ℓ	e^c	q	u^c	d^c	H
$\mathcal{G}_{SM}$	$SU(3)_c$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\mathbf{1}$
	$SU(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$
	$U(1)_Y$	$-1/2$	$+1$	$+1/6$	$-2/3$	$+1/3$	$+1/2$

The SM Lagrangian

The most general renormalisable Lagrangian invariant under Lorentz and $\mathcal{G}_{SM}$:

$$\mathcal{L}_{\rm SM} = \mathcal{L}_K + \mathcal{L}_Y + V$$

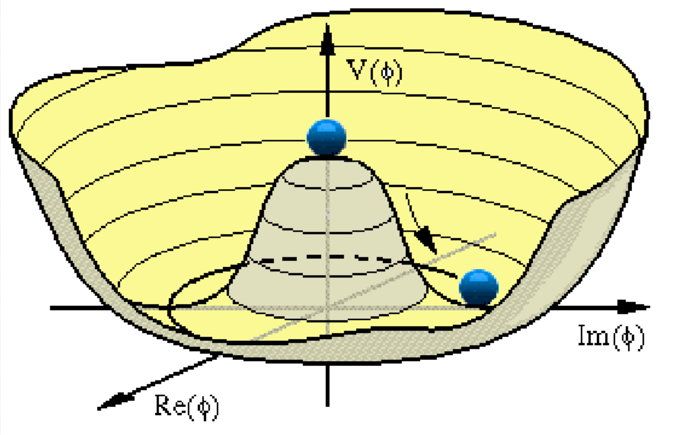
$$\mathcal{L}_K \supset i\ell^\dagger \sigma^\mu D_\mu \ell + i e^{c\dagger}\sigma^\mu D_\mu e^c + i q^\dagger \sigma^\mu D_\mu q + i u^{c\dagger}\sigma^\mu D_\mu u^c + i d^{c\dagger}\sigma^\mu D_\mu d^c$$

$$\longrightarrow \quad \mathcal{L}_{cc} = -\frac{g}{\sqrt{2}} W_\mu^- \left(e^\dagger \sigma^\mu \nu + d^\dagger \sigma^\mu u \right)$$

$$\mathcal{L}_Y = (Y_e)_{ij}\, e_i^c H^\dagger \ell_j + (Y_d)_{ij}\, d_i^c H^\dagger q_j + (Y_u)_{ij}\, u_i^c \widetilde{H}^\dagger q_j + \text{h.c.}$$

$$\widetilde{H} \equiv i\sigma^2 H^*$$

$V \longrightarrow$



$$\langle H \rangle \equiv \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$\longrightarrow \quad M_f = Y_f \frac{v}{\sqrt{2}} \qquad f = e, u, d$$

Quarks

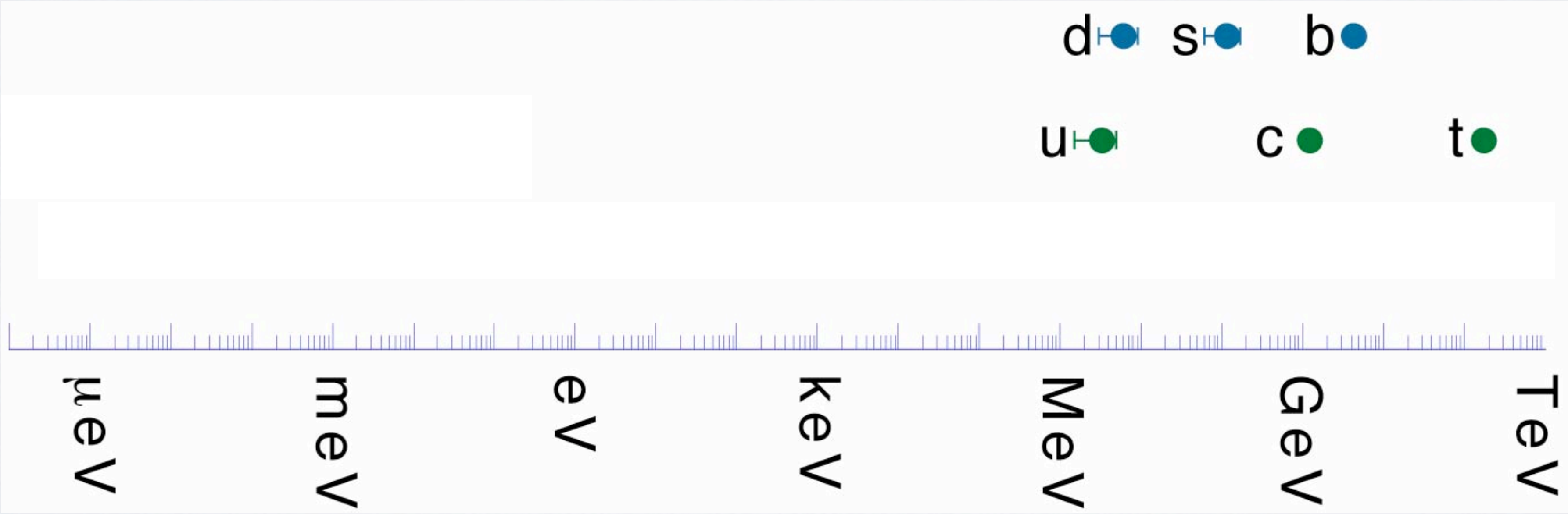
Going to the mass basis: bi-unitary transformations on the fields

$$V_{u^c}^\dagger \, M_u \, V_u = \mathrm{diag}(m_u,\, m_c,\, m_t)$$

$$V_{d^c}^\dagger \, M_d \, V_d = \mathrm{diag}(m_d,\, m_s,\, m_b)$$

$$\begin{cases} u' \equiv V_u^\dagger \, u \\ u^{c\prime} \equiv V_{u^c}^\dagger \, u^c \end{cases} \qquad \begin{cases} d' \equiv V_d^\dagger \, d \\ d^{c\prime} \equiv V_{d^c}^\dagger \, d^c \end{cases}$$

$$\longrightarrow \quad \mathcal{L}_Y \supset M_{d_i} \, d_i^{c\prime} \, d_i' + M_{u_i} \, u_i^{c\prime} \, u_i' + \mathrm{h.c.}$$



The CKM Matrix

These transformations affect the CC-Lagrangian:

$$\mathcal{L}_{cc} \supset -\frac{g}{\sqrt{2}} W_\mu^- \, d^\dagger \, \sigma^\mu \, u = -\frac{g}{\sqrt{2}} W_\mu^- \, d'^\dagger \, \sigma^\mu \, V_d^\dagger \, V_u \, u'$$

$$V \equiv V_u^\dagger \, V_d$$ **Cabibbo Kobayashi Maskawa**

Removing all the non-physical degrees of freedom:

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

Wolfenstein parametrisation

$$\lambda \approx 0.23 \quad A \approx 0.81 \quad \rho\left(1-\frac{\lambda^2}{2}\right) \approx 0.13 \quad \eta\left(1-\frac{\lambda^2}{2}\right) \approx 0.35$$

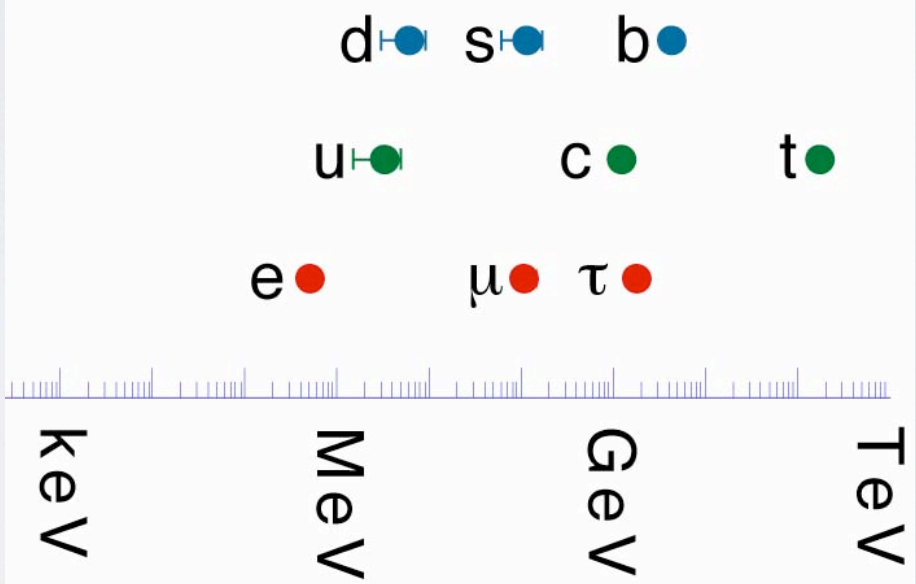
Leptons

For the charged leptons, the procedure is similar:

$$U_{e^c}^{\dagger}\, M_e\, U_e = \mathrm{diag}(m_e,\, m_\mu,\, m_\tau)$$

$$\begin{cases} e' \equiv U_e^{\dagger}\, e \\ e^{c\prime} \equiv U_{e^c}^{\dagger}\, e^c \end{cases}$$

$\longrightarrow$ $$\mathcal{L}_Y \supset M_{e_i}\, e_i^{c\prime}\, e_i'$$



But a mass term for neutrinos is forbidden in the SM:

$\longrightarrow$ **How to extend the SM to accommodate neutrino masses?**

There are two possibilities (without giving up gauge invariance):

1. Modify the particle content
2. Abandon the renormalisability and adopt an effective description

1. Modify the SM spectrum

Mirror the charged fermion sectors: introduce three copies of right-handed neutrinos, that are singlets under $\mathcal{G}_{SM}$ ⟶ $\nu^c \sim (1,1,0)$

Asking for L conservation, the Yukawa Lagrangian gets a new piece:

$$\mathcal{L}_Y = (Y_e)_{ij}\, e_i^c H^\dagger \ell_j + (Y_d)_{ij}\, d_i^c H^\dagger q_j + (Y_u)_{ij}\, u_i^c \widetilde{H}^\dagger q_j + (Y_\nu)_{ij}\, \nu_i^c \widetilde{H}^\dagger \ell_j + \text{h.c.}$$

$$\longrightarrow \quad M_\nu = Y_\nu \frac{v}{\sqrt{2}}$$

Going to the mass basis:

$$U_{\nu^c}^\dagger M_\nu U_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \qquad \begin{cases} \nu' \equiv U_\nu^\dagger \nu \\ \nu^{c\prime} \equiv U_{\nu^c}^\dagger \nu^c \end{cases}$$

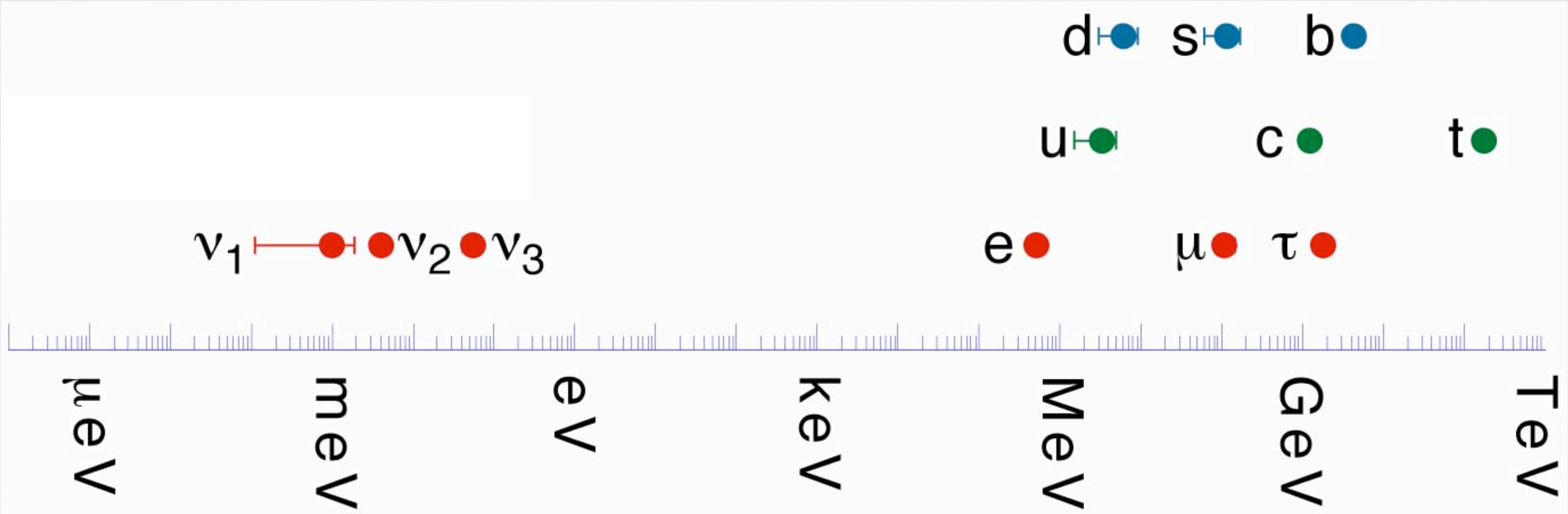
$$\longrightarrow \quad \mathcal{L}_Y \supset M_{\nu_i} \nu_i^{c\prime} \nu_i'$$

$$\longrightarrow \quad \mathcal{L}_{cc} \supset -\frac{g}{\sqrt{2}} W_\mu^- e^\dagger \sigma^\mu \nu = -\frac{g}{\sqrt{2}} W_\mu^- e'^\dagger \sigma^\mu U_e^\dagger U_\nu \nu'$$

$$U = U_e^\dagger U_\nu$$ **Pontecorvo Maki Nakagawa Sakata**

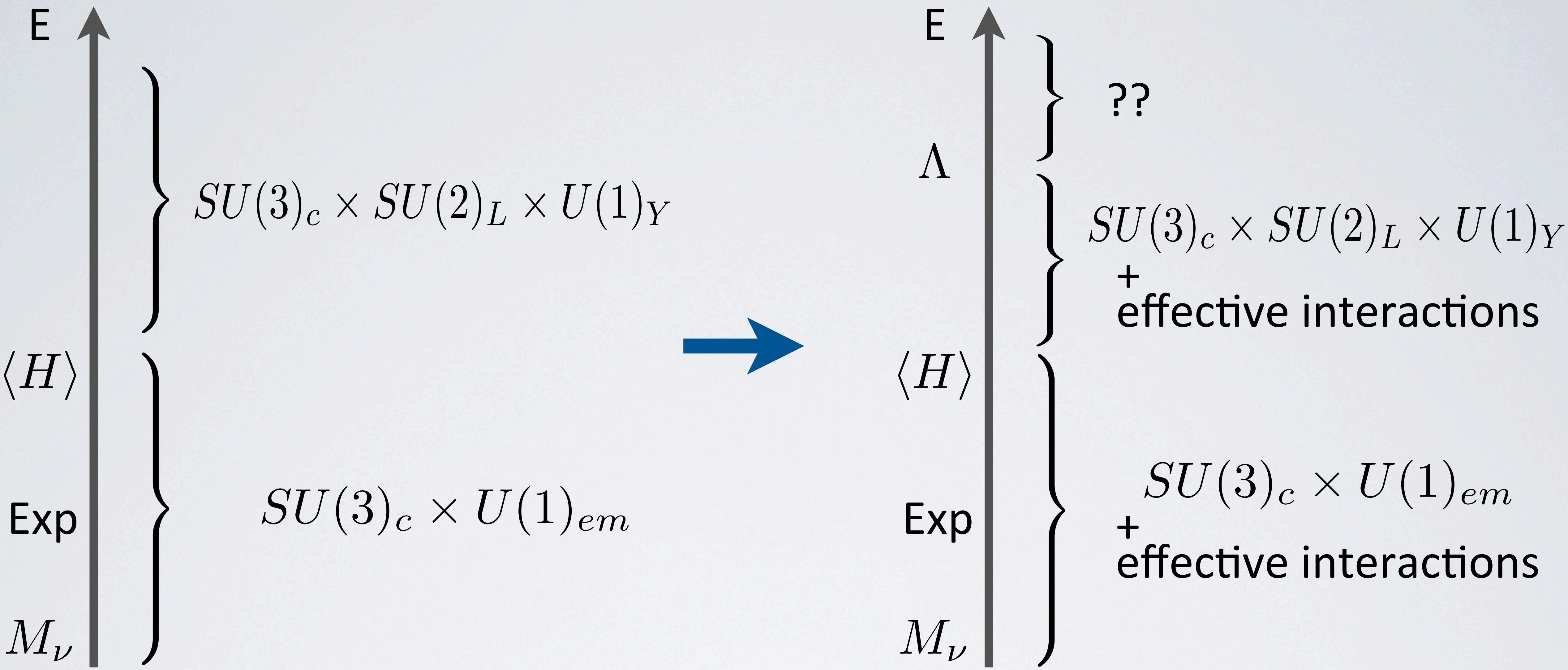
with 3 mixing angles and 1 phase, like the CKM matrix

If neutrinos are so similar to the other fermions, why are so light?



$$\frac{m_\nu}{m_t} \lesssim 10^{-12}$$

2. Effective Description



The theory is valid at all the energies. However, we expect a cut-off at the Planck scale to account for the gravitational interactions.

$$\mathcal{L}_{eff} = \mathcal{L}^{SM}_{d\leq 4} + \frac{\mathcal{L}_5}{\Lambda} + \frac{\mathcal{L}_6}{\Lambda^2} + \ldots$$

The new operators in the d>4 Lagrangian contribute to amplitudes for physical processes with terms of the type

$$\frac{\mathcal{L}_5}{\Lambda} \to \frac{E}{\Lambda} \qquad\qquad \frac{\mathcal{L}_6}{\Lambda} \to \frac{E^2}{\Lambda^2}$$

When $E \ll \Lambda$ the effects of the d>4 Lagrangian are tiny: indeed, the non-renormalisable effects are of the order

$$\left.\begin{aligned} E &\approx 10^2 \ \text{GeV} \\ \Lambda &\approx 10^{15} \ \text{GeV} \end{aligned}\right\} \qquad \frac{E}{\Lambda} \approx 10^{-13}$$

→

- The theory is not valid at all the energies, but has a cut-off
- New heavy physics could explain the lightness of neutrinos

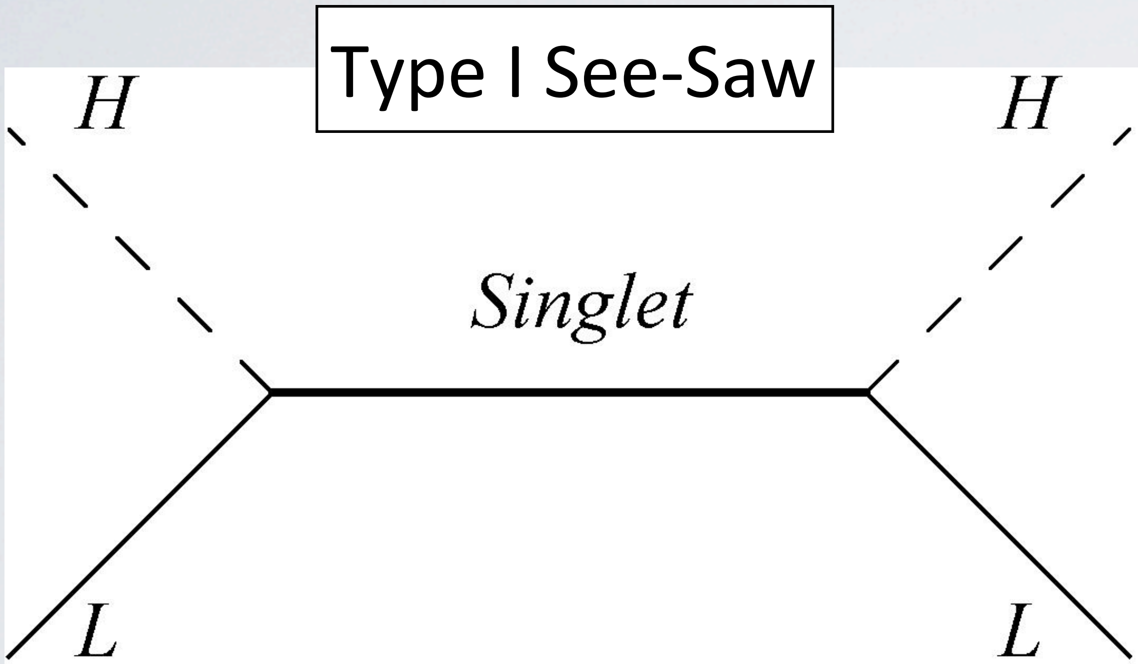
There is only one d=5 operator invariant under $\mathcal{G}_{SM}$: Weinberg Op.

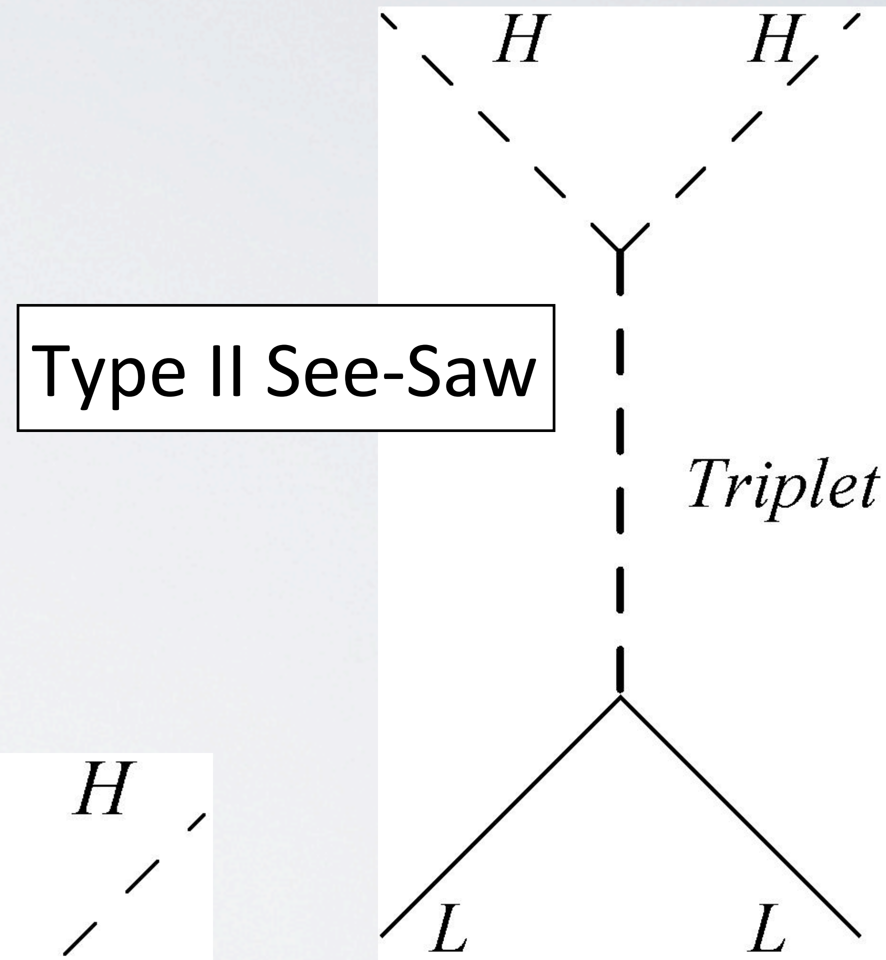
$$\frac{\mathcal{L}_5}{\Lambda} = (Y_\nu)_{ij} \frac{\left(\tilde{H}^\dagger \ell_i\right)\left(\tilde{H}^\dagger \ell_j\right)}{\Lambda} \to \frac{v}{2}\frac{v}{\Lambda}(Y_\nu)_{ij}\nu_i\nu_j$$

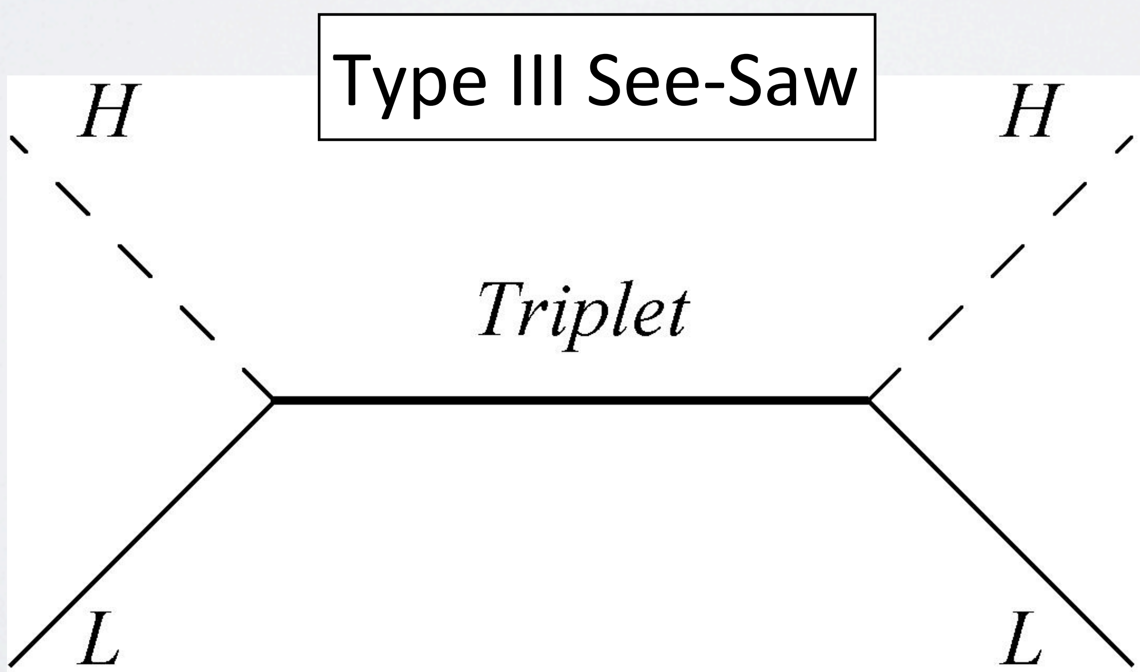
This operator violates L of two units!

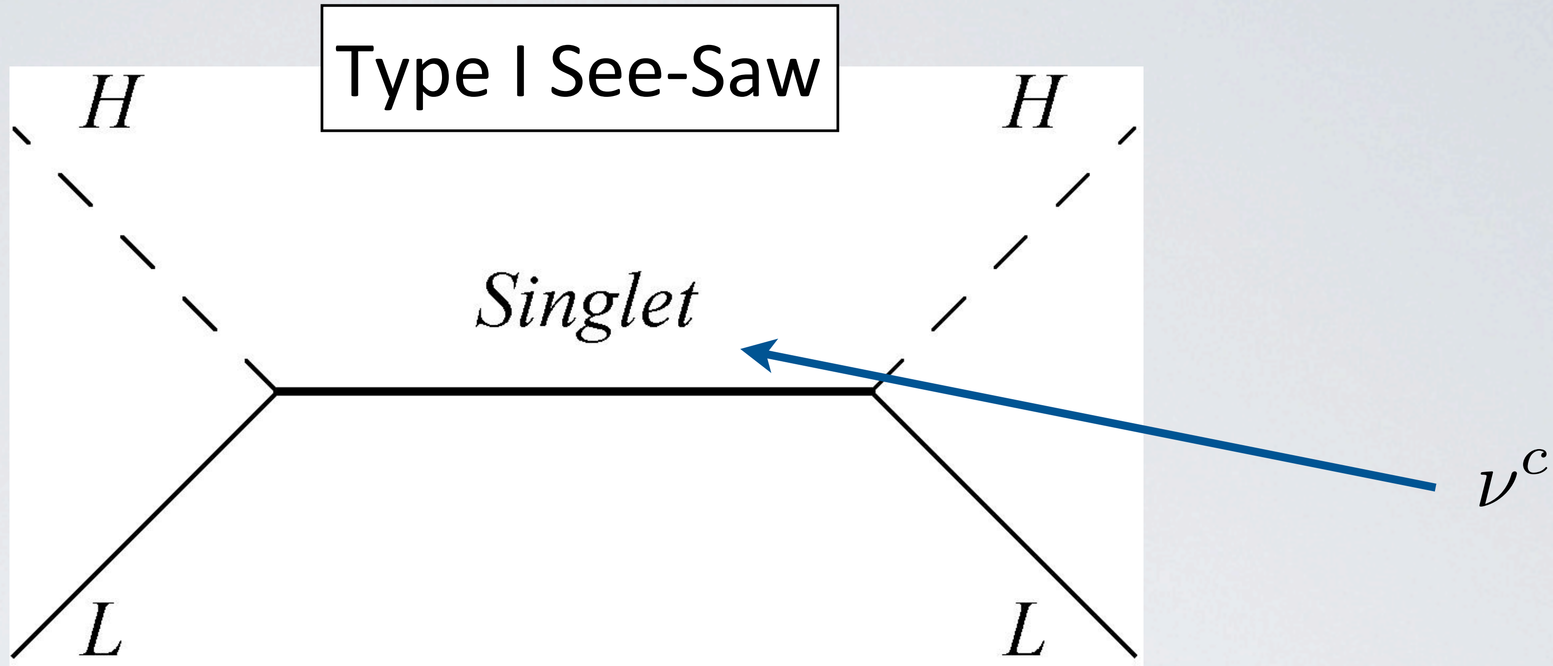
$$\frac{\mathcal{L}_5}{\Lambda} = (Y_\nu)_{ij} \frac{\left(\tilde{H}^\dagger \ell_i\right)\left(\tilde{H}^\dagger \ell_j\right)}{\Lambda} \rightarrow \frac{v}{2}\frac{v}{\Lambda}(Y_\nu)_{ij}\nu_i\nu_j$$

It can arise from several extensions of the SM (see lecture by S.Petcov)









$$\mathcal{L}_Y \supset \frac{1}{2}(M_{\nu^c})_{ij}\,\nu_i^c\nu_j^c + (Y_\nu)_{ij}\,\nu_i^c\widetilde{H}^\dagger\ell_j + \text{h.c.}$$

$$\longrightarrow \quad M_{(\nu,\nu^c)} = \begin{pmatrix} 0 & Y_\nu^T v/\sqrt{2} \\ Y_\nu v/\sqrt{2} & M_{\nu^c} \end{pmatrix}$$

Assuming $M_{\nu^c} \gg v$ and integrating out the fields ν^c:

$$\mathcal{L}_Y \supset -\frac{1}{2}\left(Y_\nu^T\, M_{\nu^c}^{-1}\, Y_\nu\right)_{ij}\left(\tilde{H}^\dagger\,\ell_i\right)\left(\tilde{H}^\dagger\,\ell_j\right) + \text{h.c.} + \ldots$$

This reproduces $\mathcal{L}_5$ with $\Lambda \leftrightarrow M_{\nu^c}$ and the light neutrino mass matrix is:

$$M_\nu = -Y_\nu^T\, M_{\nu^c}^{-1}\, Y_\nu \frac{v^2}{2}$$

$$\frac{\mathcal{L}_5}{\Lambda} = (Y_\nu)_{ij} \frac{\left(\tilde{H}^\dagger \ell_i\right)\left(\tilde{H}^\dagger \ell_j\right)}{\Lambda} \to \frac{v}{2}\frac{v}{\Lambda}(Y_\nu)_{ij}\nu_i\nu_j$$

It provides an explanation for the smallness of the neutrino masses, once the new physics enters at $\Lambda \approx 10^{15}\ \text{GeV}$

$$\longrightarrow \quad M_\nu = Y_\nu \frac{v}{\Lambda} v$$

Going to the mass basis:

$$U_\nu^T\, M_\nu\, U_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \qquad \nu' \equiv U_\nu^\dagger\, \nu$$

$$\longrightarrow \quad \mathcal{L}_Y \supset \frac{1}{2} M_{\nu_i}\, \nu_i'\, \nu_i'$$

$$\longrightarrow \quad \mathcal{L}_{cc} \supset -\frac{g}{\sqrt{2}} W_\mu^- e^\dagger \sigma^\mu \nu = -\frac{g}{\sqrt{2}} W_\mu^- e'^\dagger \sigma^\mu U_e^\dagger U_\nu \nu'$$

$$U = U_e^\dagger\, U_\nu$$ **Pontecorvo Maki Nakagawa Sakata**

with 3 mixing angles and **3 phases**, differently with respect to the CKM matrix: indeed the neutrino Majorana mass term prevents to absorb 2 phases

The PMNS Matrix

$$U = U_e^\dagger U_\nu$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \mathcal{P}$$

$$0 \leq \theta_{12}, \theta_{23}, \theta_{13} \leq \frac{\pi}{2} \qquad 0 \leq \delta < 2\pi$$

where the matrix $\mathcal{P}$ contains the physical Majorana phases:

$$\mathcal{P} = \begin{pmatrix} 1 & & \\ & e^{i\alpha} & \\ & & e^{i\beta} \end{pmatrix}$$

$$0 \leq \alpha, \beta \leq \pi$$

What we know about Oscillations

Fogli *et al.* 1205.5254

$$\Delta m^2_{\rm sol} \equiv m^2_{\nu_2} - m^2_{\nu_1} = (7.54^{+0.26}_{-0.22}) \times 10^{-5}\,{\rm eV}^2$$

$$\Delta m^2_{\rm atm} \equiv \left| m^2_{\nu_3} - m^2_{\nu_1} \right| = (2.43^{+0.06}_{-0.10})[2.42^{+0.07}_{-0.11}] \times 10^{-3}\,{\rm eV}^2$$

$$\sin^2\theta_{12} = 0.307^{+0.018}_{-0.016}$$

$$\sin^2\theta_{23} = 0.386^{+0.024}_{-0.021}[0.392^{+0.039}_{-0.020}]$$

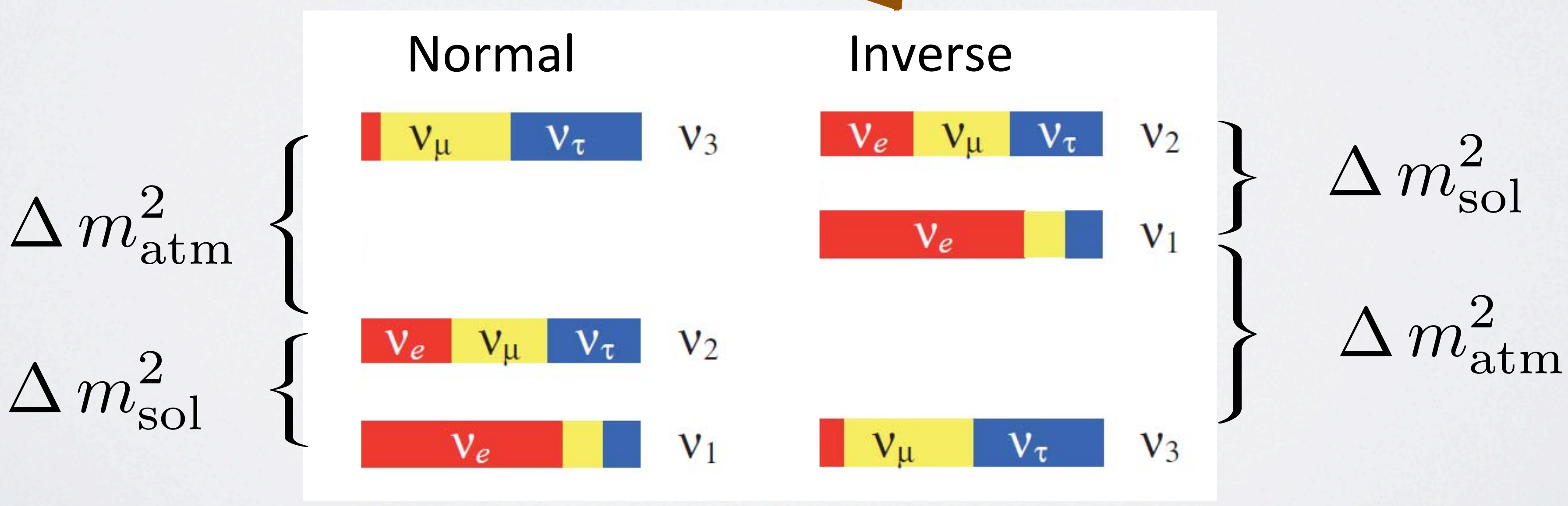
$$\sin^2\theta_{13} = 0.0241 \pm 0.0025[0.0244^{+0.0023}_{-0.0025}]$$

$$\delta = \pi(1.08^{+0.28}_{-0.31})[1.09^{+0.38}_{-0.26}]$$

solar

atmospheric

reactor



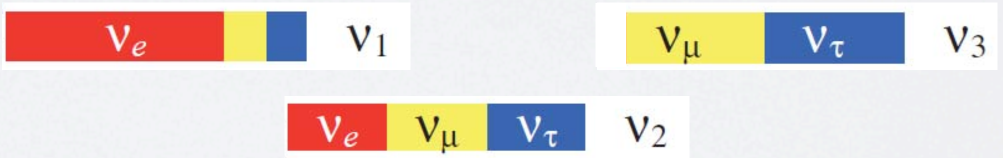
First Approximation Patterns

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \rightarrow V \approx \begin{pmatrix} 1 & \lambda & \lambda^{3\div 4} \\ \lambda & 1 & \lambda^2 \\ \lambda^{3\div 4} & \lambda^2 & 1 \end{pmatrix}$$

$$|U| \approx \begin{pmatrix} 0.82 & 0.55 & 0.16 \\ 0.36 & 0.70 & 0.61 \\ 0.44 & 0.45 & 0.77 \end{pmatrix} \rightarrow \; ??$$

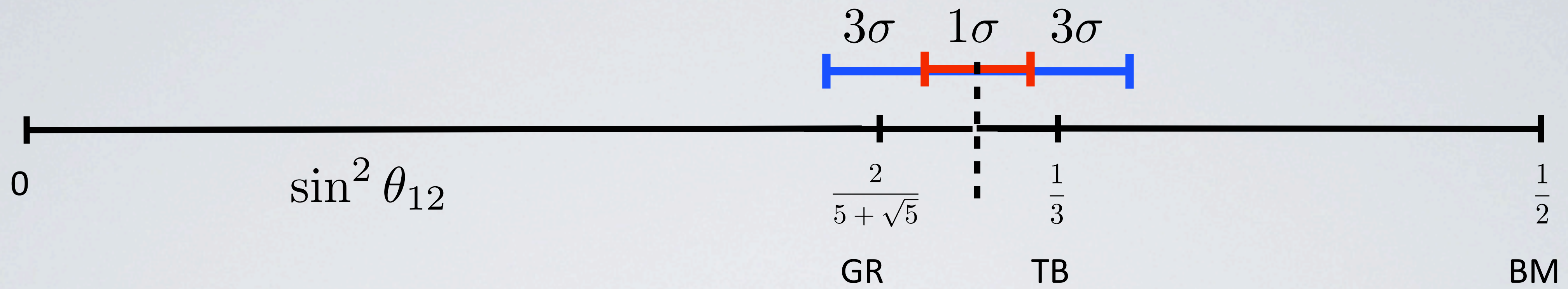
Trimaximal

Bimaximal



$$\rightarrow \quad U_{TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}$$

Tri-Bimaximal (TB) Pattern



TRI-BIMAXIMAL (TB) [Harrison, Perkins & Scott 2002; Xing 2002]

$$\sin^2\theta_{23} = \frac{1}{2} \quad \sin^2\theta_{13} = 0 \quad \sin^2\theta_{12} = \frac{1}{3} \quad \longrightarrow \quad \theta_{12} = 35.26^\circ$$

GOLDEN RATIO (GR) [Kajiyama, Raidal & Strumia 2007]

$$\sin^2\theta_{23} = \frac{1}{2} \quad \sin^2\theta_{13} = 0 \quad \tan\theta_{12} = \frac{1}{\phi} \quad \longrightarrow \quad \theta_{12} = 31.72^\circ$$

$$\phi \equiv \frac{1+\sqrt{5}}{2}$$

BIMAXIMAL (BM) [Vissani 1997; Barger *et al.* 1998]

$$\sin^2\theta_{23} = \frac{1}{2} \quad \sin^2\theta_{13} = 0 \quad \sin^2\theta_{12} = \frac{1}{2} \quad \longrightarrow \quad \theta_{12} = 45^\circ$$

Maybe related to the

Quark-Lepton Complementarity: $\pi/4 \approx \theta_{12} + \lambda$

[Smirnov; Raidal; Minakata & Smirnov 2004]

$$\longrightarrow \quad \theta_{12}^{Exp} \approx \theta_{12}^{BM} - \lambda$$

[Altarelli *et al.* 2009, Adelhart *et al.* 2010, Meloni 2011]

Symmetries of the TB Pattern

$$U_{TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}$$

The most general neutrino mass matrix that corresponds to the TB pattern is:

$$M_\nu^{TB} = U_{TB}\,\mathrm{diag}(m_{\nu_1},\, m_{\nu_2},\, m_{\nu_3})\, U_{TB}^T = \begin{pmatrix} x & y & y \\ y & z & x+y-z \\ y & x+y-z & z \end{pmatrix}$$

This mass matrix has two symmetries:

<u>magic symmetry</u>

$$S^T\, M_\nu^{TB}\, S = M_\nu^{TB}$$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$$(M_\nu^{TB})_{23} = (M_\nu^{TB})_{11} + (M_\nu^{TB})_{12} - (M_\nu^{TB})_{22}$$

<u>mu-tau symmetry</u>

$$A_{23}^T\, M_\nu^{TB}\, A_{23} = M_\nu^{TB}$$

$$A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$(M_\nu^{TB})_{12} = (M_\nu^{TB})_{13}$$
$$(M_\nu^{TB})_{22} = (M_\nu^{TB})_{33}$$

Exercises 1 & 2

- Construct the most general mass matrices corresponding to the GR and BM patterns.

- Identify the unitary matrices that leave such mass matrices invariant. Is there any common symmetry among the TB, GR and BM patterns?

The Flavour Symmetries: which and where?

Strategy

Charged Fermions	Neutrinos
large hierarchies	undetermined spectrum
relatively large masses	relatively small masses
small angles	(two) large angles

- Specific flavour structures (i.e. TB patter) are the result of the invariance under flavour symmetries. The strategy is to promote these symmetries of the mass matrices in symmetries of all the Lagrangian
- Flavour symmetries cannot be exact: the Yukawas do not show any symmetry at low-energy
- Starting from a Yukawa Lagrangian invariant under a Flavour Symmetry, masses and mixings arise only through a Symmetry Breaking Mechanism
- New Beyond SM physics that originates such mechanism is necessary
- One further degree of freedom:

type of flavour symmetry

Type of Flavour Symmetries

Local	Abelian	Continuous
$\Updownarrow$	$\Updownarrow$	$\Updownarrow$
Global	non-Abelian	Discrete

There are advantages and disadvantages for any possible choice!

In the following I will concentrate only on global flavour symmetries: on U(1) and SU(3) for the class of continuous symmetries and on A_4 for the class of discrete symmetries. These are the most common combinations studied in the literature.

(

Basics of Group Theory

Definition of a group G:

G is a set of elements with a multiplication rule

1. **Closure**:

   $$g_1 \in G,\ g_2 \in G \ \Rightarrow\ g_1\, g_2 \in G$$

2. **Associativity**:

   $$(g_1\, g_2)\, g_3 = g_1\, (g_2\, g_3)$$

3. **Unit element**:

   $$\forall g \in G,\ \ \exists\, I \in G \quad \text{such that} \quad I\, g = g$$

4. **Inverse element**:

   $$\forall g \in G,\ \ \exists\, g^{-1} \in G \quad \text{such that} \quad g^{-1} g = I$$

U(1)

U(1) is the group of the 1x1 unitary matrices. Only 1-dim irrep.
Geometrically it is the group of the rotations in the complex plane about the origin: $\theta \to e^{i\,\theta}$.

Given two fields, $g_1 \sim \theta_1$ and $g_2 \sim \theta_2$, the product of the fields transforms as $\theta_1 + \theta_2$:

⟶ and invariant under U(1) is such that it transforms as 0

$$g_1\, g_2\, \cdots\, g_n \quad \longrightarrow \quad \theta_1 + \theta_2 + \cdots + \theta_n = 0$$

SU(3)

SU(3) is the special unitary group of degree 3 and is the group of 3x3 unitary matrices with determinant 1.

Its generators are represented as traceless hermitian matrices:

$$\mathrm{tr}(\lambda_a) = 0 \qquad \lambda_a = \lambda_a^\dagger$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \qquad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

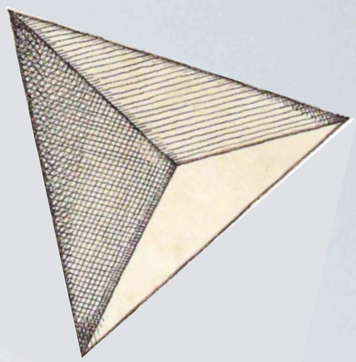
$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

If $g_1, g_2 \sim 3$ then $\bar{g}_1, \bar{g}_2 \sim \bar{3}$ and

$$3 \times 3 = \bar{3} + 6 \qquad 6 \times \bar{6} = 1 + 8 + \ldots$$

$$3 \times \bar{3} = 1 + 8 \qquad 8 \times 8 = 1 + 8 + \ldots$$

A_4



A_4 is the group of even permutations of 4 objects isomorphic to the group of the rotations which leave a regular tetrahedron invariant (Subgroup of SO(3)).

It has 12 elements and 4 representations: 3, 1, 1', 1''

It has two generators, S and T, that satisfy the relations:

$$S^2 = T^3 = (ST)^3 = 1$$

$$A_4 = \{1, S, T, ST, TS, T^2, ST^2, STS, TST, T^2S, TST^2, T^2ST\}$$

3d rep:

$$T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$1' \times 1' = 1''$$

$$1' \times 1'' = 1$$

$$1'' \times 1'' = 1'$$

$$3 \times 3 = 1 + 1' + 1'' + 3 + 3$$

$$\begin{cases} 1 = a_1 b_1 + a_2 b_2 + a_3 b_3 \\ 1' = a_1 b_1 + \omega^2 a_2 b_2 + \omega\, a_3 b_3 \\ 1'' = a_1 b_1 + \omega\, a_2 b_2 + \omega^2 a_3 b_3 \\ 3 \sim (a_2 b_3,\ a_3 b_1,\ a_1 b_2) \\ 3 \sim (a_3 b_2,\ a_1 b_3,\ a_2 b_1) \end{cases}$$

$$\omega \equiv e^{\frac{2\pi i}{3}}$$

Physics is independent on the chosen basis for the generators!

$$S^2 = T^3 = (ST)^3 = 1$$

3d rep:

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \qquad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$$1' \times 1' = 1''$$

$$1' \times 1'' = 1$$

$$1'' \times 1'' = 1'$$

$$3 \times 3 = 1 + 1' + 1'' + 3_S + 3_A$$

$$\left\{ \begin{aligned} 1 &= a_1 b_1 + a_2 b_3 + a_3 b_2 \\ 1' &= a_3 b_3 + a_1 b_2 + a_2 b_1 \\ 1'' &= a_2 b_2 + a_1 b_3 + a_3 b_1 \\ 3_S &= \frac{1}{3}\begin{pmatrix} 2a_1 b_1 - a_2 b_3 - a_3 b_2 \\ 2a_3 b_3 - a_1 b_2 - a_2 b_1 \\ 2a_2 b_2 - a_1 b_3 - a_3 b_1 \end{pmatrix} \\ 3_A &= \frac{1}{2}\begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_1 b_2 - a_2 b_1 \\ a_3 b_1 - a_1 b_3 \end{pmatrix} \end{aligned} \right.$$

)

Flavour Models at the GUT Scale:

The U(1) Flavour Symmetry

Recent reviews:
Altarelli *et al. 2012*
Buchmuller *et al.* 2011

Mainly based on
Froggatt & Nielsen 1979
Altarelli, Feruglio & Masina 2002

The Froggatt-Nielsen U(1)

The Froggatt-Nielsen U(1) model is a milestone in this context:

- formulated only in the quark sector (1979!!)
- the Flavour Symmetry is a global $U(1)_{FN}$
- new scalar field θ, called flavon, which develops a VEV

$$\langle\theta\rangle/\Lambda \approx \epsilon \ll 1$$

- the SM quarks are charged under $U(1)_{FN}$ as $FN(f) = n_f \geq 0$, while the flavon as a negative charge $FN(\theta) = -1$
- the corresponding non-renormalisable Lagrangian reads:

$$\mathcal{L}_Y = \left(\frac{\theta}{\Lambda}\right)^{n_{d_i^c}} \left(\frac{\theta}{\Lambda}\right)^{n_{q_j}} (Y_d)_{ij}\, d_i^c H^\dagger q_j + \\ + \left(\frac{\theta}{\Lambda}\right)^{n_{u_i^c}} \left(\frac{\theta}{\Lambda}\right)^{n_{q_j}} (Y_u)_{ij}\, u_i^c \widetilde{H}^\dagger q_j + \text{h.c.}$$

where $Y_{u,d} \approx \mathcal{O}(1)$ are free parameters

$$\mathcal{L}_Y = \left(\frac{\theta}{\Lambda}\right)^{n_{d_i^c}} \left(\frac{\theta}{\Lambda}\right)^{n_{q_j}} (Y_d)_{ij}\, d_i^c H^\dagger q_j + $$
$$+ \left(\frac{\theta}{\Lambda}\right)^{n_{u_i^c}} \left(\frac{\theta}{\Lambda}\right)^{n_{q_j}} (Y_u)_{ij}\, u_i^c \widetilde{H}^\dagger q_j + \text{h.c.}$$

when the symmetry is spontaneously broken by the VEV of θ, fermions receive different contributions in terms of ϵ. The Yukawa matrices are then given by:

$$y_u = F_{u^c} Y_u F_q \qquad\qquad y_d = F_{d^c} Y_d F_q$$

$$F_f = \begin{pmatrix} \epsilon^{n_{f1}} & 0 & 0 \\ 0 & \epsilon^{n_{f2}} & 0 \\ 0 & 0 & \epsilon^{n_{f3}} \end{pmatrix} \qquad (f = q, u^c, d^c)$$

Assuming $n_{f1} > n_{f2} > n_{f3} \geq 0$, we move to the physical basis:

$$(V_{u,d})_{ii} \approx 1 \qquad\qquad (V_{u,d})_{ij} \approx \frac{n_{q_i}}{n_{q_j}} < 1 \qquad (i < j)$$

→

$$V_{ud} \approx V_{cs} \approx V_{tb} \approx \mathcal{O}(1)$$
$$V_{ub} \approx V_{td} \approx V_{us} \times V_{cb}$$

} independently of the particular charge choice correct CKM with: $\begin{cases} n_q = (3,2,0) \\ \epsilon \approx 0.2 \end{cases}$ $\longrightarrow$ $V = \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$ (neglecting O(1) parameters)

$$\left[V \approx \begin{pmatrix} 1 & \lambda & \lambda^{3\div 4} \\ \lambda & 1 & \lambda^2 \\ \lambda^{3\div 4} & \lambda^2 & 1 \end{pmatrix} \right]$$

correct quark masses with: $\begin{cases} n_{u^c} = (4,1,0) \\ n_{d^c} = (1,0,0) \end{cases}$

$$\longrightarrow \quad M_u = \begin{pmatrix} \epsilon^7 & 0 & 0 \\ 0 & \epsilon^3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad M_d = \begin{pmatrix} \epsilon^4 & 0 & 0 \\ 0 & \epsilon^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(neglecting the mass and the O(1) parameters)

$$\left[M_u^{exp} = \begin{pmatrix} \epsilon^7 & 0 & 0 \\ 0 & \epsilon^3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad M_d^{exp} = \begin{pmatrix} \epsilon^4 & 0 & 0 \\ 0 & \epsilon^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

Lepton Sector

$$\mathcal{L}_Y = \left(\frac{\theta}{\Lambda}\right)^{n_{e_i^c}} \left(\frac{\theta}{\Lambda}\right)^{n_{\ell_j}} (Y_e)_{ij}\, d_i^c H^\dagger q_j +$$

$$+ \left(\frac{\theta}{\Lambda}\right)^{n_{\ell_i}} \left(\frac{\theta}{\Lambda}\right)^{n_{\ell_j}} (Y_\nu)_{ij} \frac{\left(\widetilde{H}^\dagger \ell_i\right)\left(\widetilde{H}^\dagger \ell_j\right)}{\Lambda_L} + \text{h.c.}$$

correct CKM with: $n_q = (3, 2, 0)$

correct quark masses with: $n_{u^c} = (4, 1, 0)$ $\quad n_{d^c} = (1, 0, 0)$

With a similar choice for the charged leptons, it is possible to explain the mass hierarchy:

$$\begin{cases} n_\ell = (2, 0, 0) \\ n_{e^c} = (3, 2, 0) \end{cases} \longrightarrow \quad M_e = \begin{pmatrix} \epsilon^5 & \epsilon^3 & \epsilon^3 \\ \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & 1 \end{pmatrix}$$

and in the physical basis:

$$U_e = \begin{pmatrix} 1 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & 1 \\ \epsilon^2 & 1 & 1 \end{pmatrix} \quad M_e = \begin{pmatrix} \epsilon^5 & 0 & 0 \\ 0 & \epsilon^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \left[M_e^{exp} = \begin{pmatrix} \epsilon^5 & 0 & 0 \\ 0 & \epsilon^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

This does not fit with neutrinos and the lepton mixings:

$$M_\nu = \begin{pmatrix} \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & 1 \\ \epsilon^2 & 1 & 1 \end{pmatrix} \longrightarrow U_\nu = \begin{pmatrix} 1 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & 1 \\ \epsilon^2 & 1 & 1 \end{pmatrix}$$

$$\longrightarrow U = U_e^\dagger U_\nu = \begin{pmatrix} 1 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & 1 \\ \epsilon^2 & 1 & 1 \end{pmatrix}$$

(neglecting O(1) parameters)

$$\theta_{23} \sim 45^\circ$$

$$\theta_{12} \sim \theta_{13} \sim 2^\circ$$

$$\left[\begin{array}{l} \theta_{23}^{exp} \sim (38.0 \pm 1.5)^\circ \\ \theta_{12}^{exp} \sim (34.0 \pm 1.0)^\circ \\ \theta_{13}^{exp} \sim (9.0 \pm 0.5)^\circ \end{array} \right]$$

$$\longrightarrow \begin{array}{l} m_1 \sim \epsilon^4 \\ m_2 \approx m_3 \sim 1 \end{array}$$

$$r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx 1 \qquad \left[r^{\text{exp}} \approx 0.03 \pm 0.003 \right]$$

$\longrightarrow$ Need to change the value of ϵ and/or the charges

$n_\ell = (2,1,0) \qquad n_{e^c} = (7,2,0) \qquad \epsilon \approx 0.4$

$$\longrightarrow \quad M_e = \begin{pmatrix} \epsilon^9 & \epsilon^8 & \epsilon^7 \\ \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \qquad M_\nu = \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}$$

and in the physical basis:

$$\longrightarrow \quad M_e = \begin{pmatrix} \epsilon^9 & 0 & 0 \\ 0 & \epsilon^3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \left[M_e^{exp} = \begin{pmatrix} \epsilon^9 & 0 & 0 \\ 0 & \epsilon^3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

$$\longrightarrow \quad m_1 \sim \epsilon^4\,, \qquad m_2 \sim \epsilon^2\,, \qquad m_3 \sim 1$$

$$r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx \epsilon^4 \sim 0.026 \qquad \left[r^{\rm exp} \approx 0.03 \pm 0.003 \right]$$

$$\longrightarrow \quad U_e \sim U_\nu = \begin{pmatrix} 1 & \epsilon & \epsilon^2 \\ \epsilon & 1 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \quad \longrightarrow \quad U = \begin{pmatrix} 1 & \epsilon & \epsilon^2 \\ \epsilon & 1 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}$$

$$\theta_{23} \sim \theta_{12} \sim 22^\circ$$

$$\theta_{13} \sim 9^\circ$$

$$\left[\begin{array}{l} \theta_{23}^{exp} \sim (38.0 \pm 1.5)^\circ \\ \theta_{12}^{exp} \sim (34.0 \pm 1.0)^\circ \\ \theta_{13}^{exp} \sim (9.0 \pm 0.5)^\circ \end{array} \right]$$

Summary

The use of the U(1) symmetry has several advantages:

- it is minimal: only 1 symmetry factor and only 1 new scalar field
- the U(1) is already present in the SM (hypercharge)
- the U(1) can be gauged to avoid the presence of the Goldstone boson due to the spontaneously symmetry breaking: in this case there will be the appearance of a new gauge boson, the Z', well studied in phenomenology
- the gauge U(1) is usually present in the low-energy theories that originate from GUT or Strings (constraints on the choice of charges)
- realistic models can be constructed respecting the symmetric principle and all the fermion mass hierarchies and mixings can be explained

On the other hand, the predictions are affected by O(1) coefficients and therefore the predictive power is rather weak.

Exercises 3 & 4

- Construct the mass matrices in the case of type I See-Saw if the charges of the fields are the following:

  a. $n_\ell = (1,0,0) \qquad n_{e^c} = (3,2,0) \qquad n_{\nu^c} = (2,1,0) \qquad \epsilon \approx 0.2$

  b. $n_\ell = (2,1,0) \qquad n_{e^c} = (5,3,0) \qquad n_{\nu^c} = (2,1,0) \qquad \epsilon \approx 0.4$

  c. $n_\ell = (2,0,0) \qquad n_{e^c} = (5,3,0) \qquad n_{\nu^c} = (1,-1,0) \qquad \epsilon \approx 0.4$

  and discuss the values of the charged lepton mass hierarchy, of the mixing angles and of r. Notice that this charges are compatible with SU(5) GUT unification.

- Identify the charges that give rise to the quark mass hierarchies and the CKM matrix taking as expanding parameter $\epsilon = 0.1$.

Flavour Models at the GUT Scale:

The A_4 Flavour Symmetry

Recent Reviews:
Altarelli *et al.* 2004
Mohapatra *et al.* 2006
Altarelli 2010
Altarelli *et al.* 2012

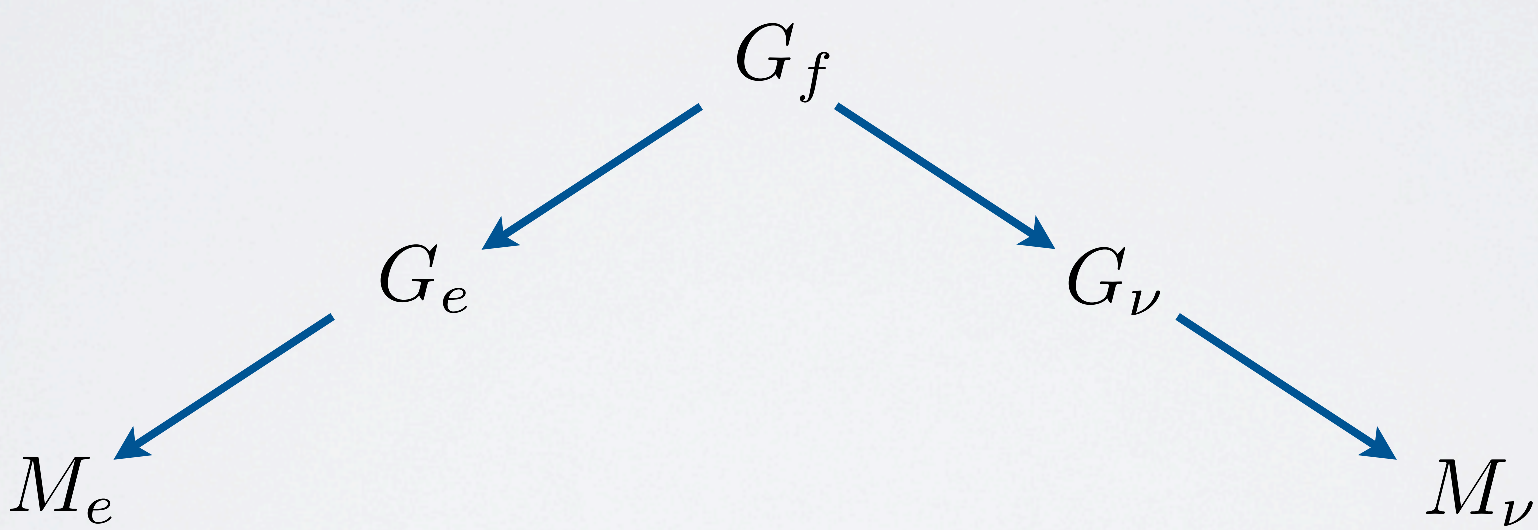
Mainly based on
Altarelli & Feruglio 2005

General Strategy

Wrt U(1), discrete symmetries have a more complicate structure: usually, non-trivial subgroups can be identified. It is possible to use this property to improve the predictive power and describe specific patterns (such as the TB).

Consider the flavour symmetry G_f, broken into two different subgroups:

- G_e in the charged lepton sector and G_ν in the neutrino sector
- M_e and M_ν are determined by G_e and G_ν, respectively

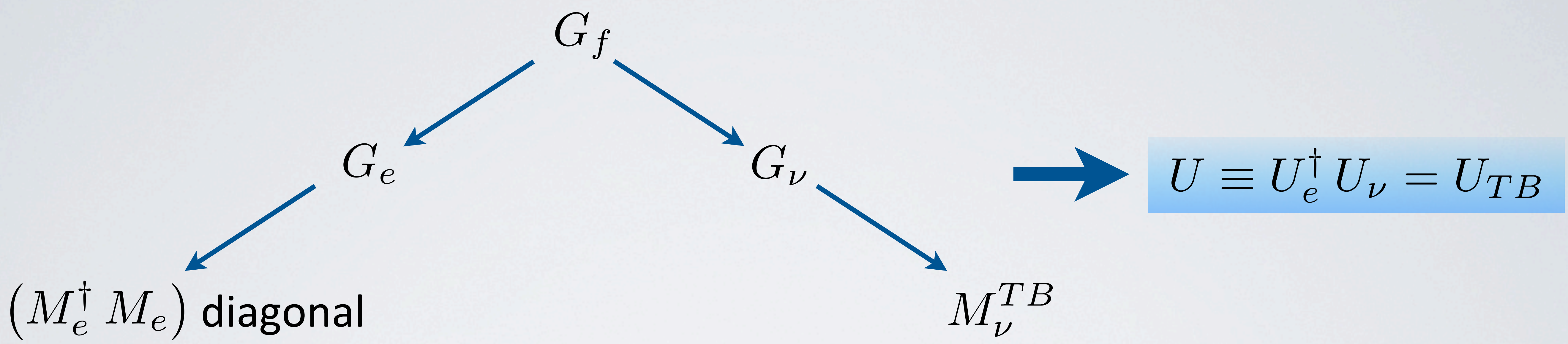


Imposing that $\left(M_e^\dagger M_e\right)$ is invariant under G_e and M_ν under G_ν, then U_e and U_ν, and therefore $U \equiv U_e^\dagger U_\nu$, are fully determined by G_e and G_ν.

$$U_e^\dagger \left(M_e^\dagger M_e\right) U_e = \left(M_e^\dagger M_e\right)^{diag}$$

TB from Symmetry Breaking

It is useful to work in the basis in which $(M_e^\dagger M_e)$ is diagonal. As a result, the PMNS matrix comes only from the neutrino sector:



A symmetry that forces $(M_e^\dagger M_e)$ to be diagonal can be (many possibilities):

$$G_e = \{1, T, T^2\} \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \qquad \omega \equiv e^{\frac{2\pi i}{3}}$$

that is called the Ciclic group Z₃, under which

$$\longrightarrow \quad T^\dagger (M_e^\dagger M_e) T = (M_e^\dagger M_e) = \mathrm{diag}(m_e^2, m_\mu^2, m_\tau^2)$$

(A bottom-up approach also works: given $(M_e^\dagger M_e)$ diagonal, which is a T that leaves $(M_e^\dagger M_e)$ invariant?)

For the neutrinos, we have already seen that the TB pattern

$$U_{TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}$$

arises from the following most general neutrino mass matrix:

$$M_\nu^{TB} = \begin{pmatrix} x & y & y \\ y & z & x+y-z \\ y & x+y-z & z \end{pmatrix}$$

that is invariant under transformations of the magic and mu-tau symmetries:

magic symmetry

$$S^T\, M_\nu^{TB}\, S = M_\nu^{TB}$$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

mu-tau symmetry

$$A_{23}^T\, M_\nu^{TB}\, A_{23} = M_\nu^{TB}$$

$$A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Notice that $S^2 = 1$ and $A_{23}^2 = 1$. This means that both $G_S = \{1, S\}$ and $G_{A_{23}} = \{1, A_{23}\}$ are parity groups Z_2.

$$G_f$$

$$G_e \equiv Z_3 \qquad\qquad G_\nu \equiv Z_2 \times Z_2$$

$\left(M_e^\dagger M_e\right)$ diagonal $\qquad\qquad M_\nu^{TB}$

and the generators of the groups are $T,\ S,\ A_{23}$.

The **minimal** choice for G_f such that the charged leptons are diagonal and the neutrinos are diagonalized by the TB pattern is A_4 as done by Altarelli and Feruglio (2005). A_4 has only two generators and 12 elements:

$$S^2 = T^3 = (ST)^3 = 1$$

and 4 irreducible representations: (2nd basis)

$\mathbf{1}$ $\quad S = 1 \quad T = 1$

$\mathbf{1'}$ $\quad S = 1 \quad T = \omega^2$

$\mathbf{1''}$ $\quad S = 1 \quad T = \omega$

$$\mathbf{3} \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \quad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

the same T and S that generate G_e and G_S

The Model

	Matter fields				Higgs	Flavons			
	ℓ	e^c	μ^c	τ^c	$h_{u,d}$	θ	φ_T	φ_S	ξ
A_4	3	1	$1''$	$1'$	1	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	1	ω	ω
$U(1)_{FN}$	0	2	1	0	0	-1	0	0	0

Change of notation for the Higgs!

Non-renormalisable Lagrangian invariant under SM and Flavour symmetries:

$$\mathcal{L}_e = y_e \frac{\theta^2}{\Lambda^2} e^c \left(\frac{\varphi_T}{\Lambda}\ell\right) h_d + y_\mu \frac{\theta}{\Lambda}\mu^c \left(\frac{\varphi_T}{\Lambda}\ell\right)' h_d + y_\tau \tau^c \left(\frac{\varphi_T}{\Lambda}\ell\right)'' h_d$$

$$\mathcal{L}_\nu = x_a \frac{\xi}{\Lambda} h_u\, h_u \frac{(\ell\,\ell)}{\Lambda_L} + x_b\, h_u\, h_u \left(\frac{\varphi_S}{\Lambda}\frac{\ell\ell}{\Lambda_L}\right)$$

$1' \times 1' = 1''$

$1' \times 1'' = 1$

$1'' \times 1'' = 1'$

$3 \times 3 = 1 + 1' + 1'' + 3 + 3$

Λ → Flavour cut-off

Λ_L → LN cut-off

$\frac{\theta^n}{\Lambda^n}$ → FN mechanism

$$\mathcal{L}_e = y_e \frac{\theta^2}{\Lambda^2} e^c \left(\frac{\varphi_T}{\Lambda}\ell\right) h_d + y_\mu \frac{\theta}{\Lambda}\mu^c \left(\frac{\varphi_T}{\Lambda}\ell\right)' h_d + y_\tau \tau^c \left(\frac{\varphi_T}{\Lambda}\ell\right)'' h_d$$

$$\mathcal{L}_\nu = x_a \frac{\xi}{\Lambda} h_u\, h_u \frac{(\ell\,\ell)}{\Lambda_L} + x_b\, h_u\, h_u \left(\frac{\varphi_S}{\Lambda}\frac{\ell\ell}{\Lambda_L}\right)$$

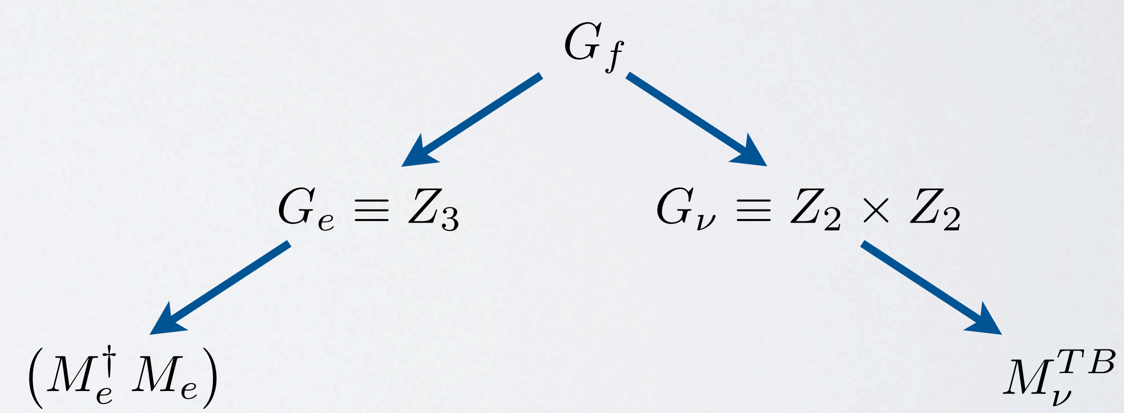
Under appropriate conditions (SUSY, Extra-D...) a natural minimum of the scalar potential is

$$\frac{\langle\varphi_T\rangle}{\Lambda} = (u,0,0)$$ ⟶ breaks A_4 down to $G_e \equiv Z_3$

$$\frac{\langle\varphi_S\rangle}{\Lambda} = c_b(u,u,u)$$ ⟶ breaks A_4 down to $G_S \equiv Z_2$

$$\frac{\langle\xi\rangle}{\Lambda} = c_a u$$

$$\frac{\langle\theta\rangle}{\Lambda} = \epsilon$$



We need to prevent that φ_S (φ_T) couples to the changed lepton (neutrino) sector: the additional Z_3 avoids the appearance of such terms.

After the EW and Flavour symmetry breakings:

$\epsilon \approx 0.5$ explains the mass hierarchy

$$\mathcal{L}_e \longrightarrow M_e = \begin{pmatrix} y_e\epsilon^2 & 0 & 0 \\ 0 & y_\mu\epsilon & 0 \\ 0 & 0 & y_\tau \end{pmatrix} u\, v_d$$

$$\mathcal{L}_\nu \longrightarrow M_\nu = \begin{pmatrix} a+\frac{2}{3}b & -\frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & \frac{2}{3}b & a-\frac{b}{3} \\ -\frac{b}{3} & a-\frac{b}{3} & \frac{2}{3}b \end{pmatrix} \frac{v_u^2}{\Lambda_L}$$

$$\left[M_\nu^{TB} = \begin{pmatrix} x & y & y \\ y & z & x+y-z \\ y & x+y-z & z \end{pmatrix} \right]$$

$$\begin{cases} a \equiv 2\, x_a\, c_a\, u \\ b \equiv 2\, x_b\, c_b\, u \end{cases}$$

2 complex parameters in the neutrino sector

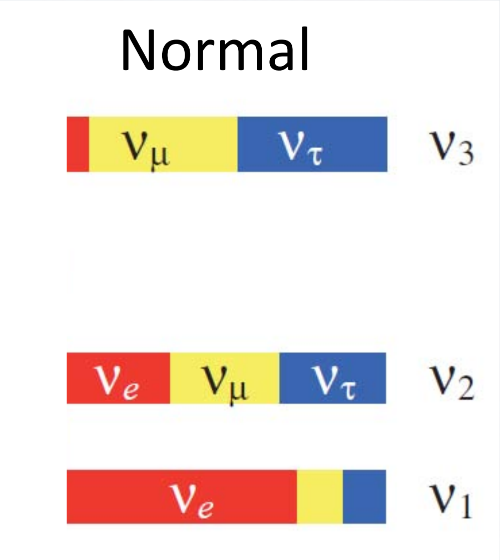
Notice that M_ν is also invariant under $G_{A_{23}}$: it does not come from the flavour symmetry, but it is an accidental symmetry!

- The mixing is the TB pattern:

$$U_{TB}^T\, M_\nu\, U_{TB} = \mathrm{diag}(a+b,\ a,\ -a+b)\,\frac{v_u^2}{\Lambda_L}$$

- Only normal hierarchical spectrum

$$r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx \frac{1}{35}$$



- Lower bounds on the lightest neutrino mass, sum of neutrino masses

$$m_1 \geq 0.014\,\mathrm{eV} \qquad \sum m_i \geq 0.09\,\mathrm{eV}$$

- Prediction for the 0ν2β-decay effective mass:

$$|m_{ee}|^2 = \left|a + \frac{2}{3}b\right|^2$$
$$= \frac{1}{9}\left(9m_1^2 + 5\Delta m^2_{sol} - \Delta m^2_{sol}\right)$$

Sub-Leading Corrections

Sub-leading corrections arise from higher-dimensional operators, suppressed by additional powers of the cut-off: they affect

$$\underbrace{M_e \qquad\qquad M_\nu \qquad\qquad \text{VEV alignment}}$$

due to the complete breaking of A_4:
φ_S (φ_T) couples to the charged lepton (neutrino) sector and no surviving subgroup is present at the NLO

- The mixing is perturbed:

$$\sin^2\theta_{12} \simeq \frac{1}{3} + \mathcal{O}(u) \qquad \sin^2\theta_{23} \simeq \frac{1}{2} + \mathcal{O}(u) \qquad \sin\theta_{13} \simeq \mathcal{O}(u)$$

- The predictions for $m_1, \sum m_i$ and $|m_{ee}|$ modified by $\mathcal{O}(u)$ terms

- u must be of the order of $u \sim 0.08$ to accommodate the value of the reactor angle

Summary

The use of the A_4 symmetry has several advantages:

- the predictive power is strong: all the three mixing angles
- with an additional U(1), it is possible to explain the charged lepton mass hierarchy (the U(1) can be substitute by other discrete syms)
- the spontaneous breaking of a discrete symmetry does not lead to the appearance of massless Goldstone bosons
- the discrete symmetries could be a leftover of Poincare symmetry in D>4 dimensions or could arise from string theories
- realistic models can be constructed respecting the symmetric principle
- extensions to quark exist and are working [Feruglio *et al.* 2007]

On the other hand:

- the spectrum is enriched of several scalar fields
- the cut-off is generically at the GUT scale: no directly testable
- misalignment problem: two triplets with two different VEVs. This is solved only in SUSY or Extra-D models

Exercises 5 & 6

- Write explicitly the NLO operators for the Altarelli-Feruglio model. Identify the corresponding corrections to the mass matrices and to the mixings (2nd basis with generator T diagonal).

- Construct the Altarelli-Feruglio model in the type I See-Saw context: wrt to the charges presented before,

	ν^c	φ_S	ξ
A_4	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$
Z_3	ω^2	ω^2	ω^2
$U(1)_{FN}$	0	0	0

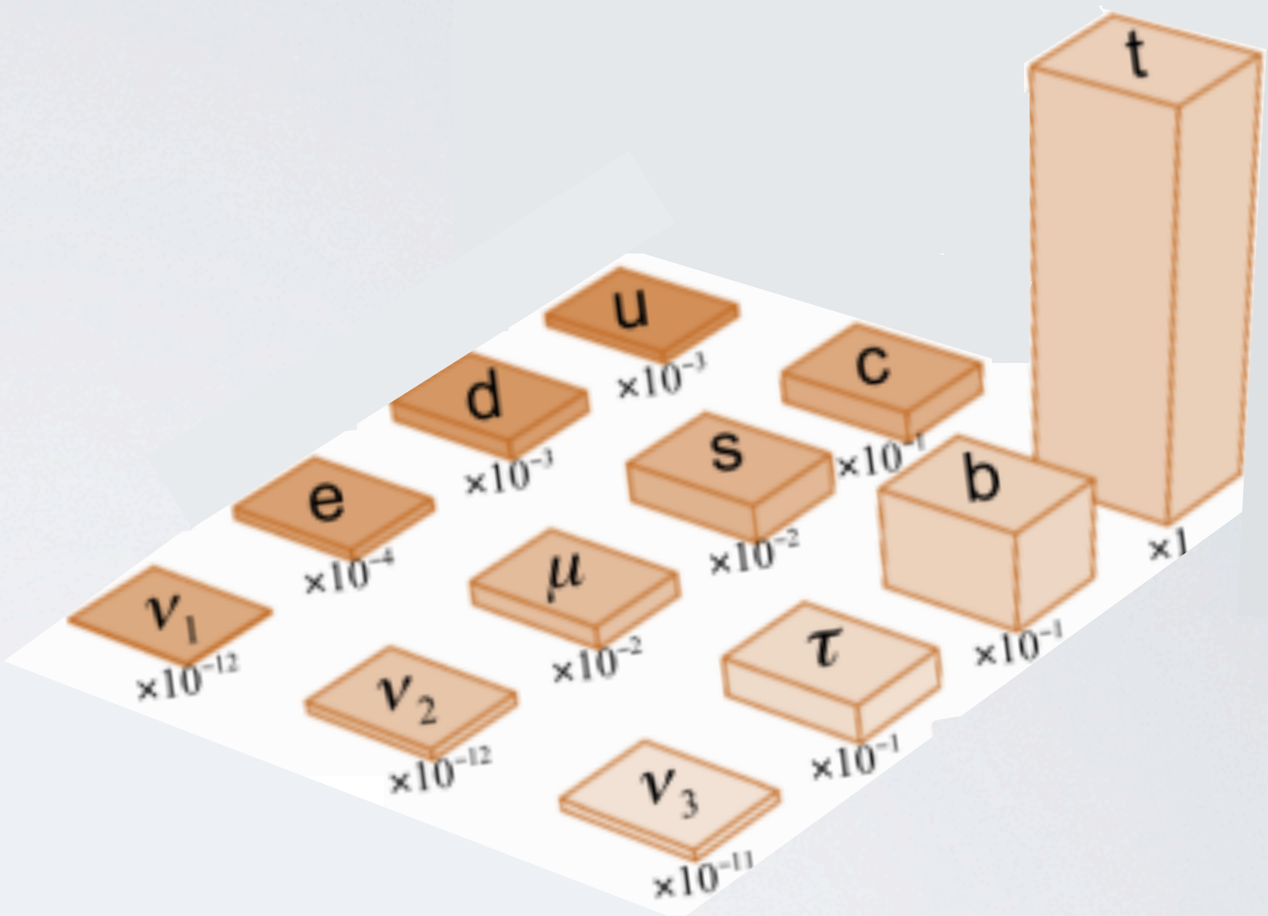
Compute the full Lagrangian, the lepton masses and the mixings.

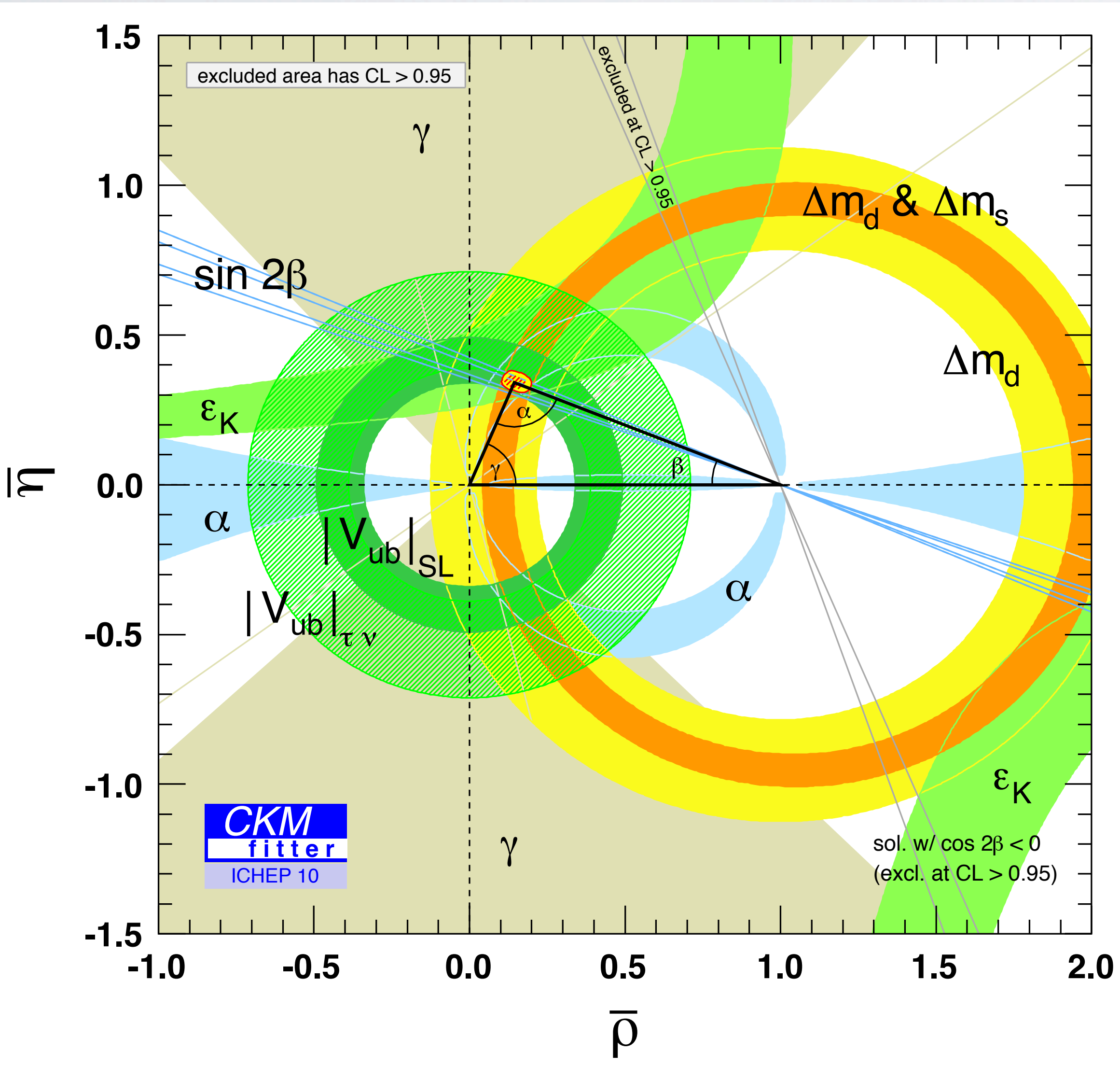
Minimal (Lepton) Flavour Violation: a bottom-up approach

Mainly based on
D'Ambrosio *et al.* 2002
Cirigliano *et al.* 2005

The Flavour Problem

The Flavour Puzzle:
can be solved with flavour symmetries.





FCNC:
automatically suppressed in the SM. But what with new physics?

New Physics

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum \frac{c_i^{(6)}}{\Lambda_{\rm NP}^2} O_i^{(6)}$$

[Isidori, Nir & Perez 2010]

Operator	Bounds on Λ in TeV ($c_{ij}=1$)		Bounds on c_{ij} ($\Lambda = 1$ TeV)		Observables
	Re	Im	Re	Im	
$(\bar{s}_L \gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	Δm_K; ϵ_K
$(\bar{s}_R\, d_L)(\bar{s}_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	Δm_K; ϵ_K
$(\bar{c}_L \gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	Δm_D; $\|q/p\|, \phi_D$
$(\bar{c}_R\, u_L)(\bar{c}_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	Δm_D; $\|q/p\|, \phi_D$
$(\bar{b}_L \gamma^\mu d_L)^2$	5.1×10^2	9.3×10^2	3.3×10^{-6}	1.0×10^{-6}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar{b}_R\, d_L)(\bar{b}_L d_R)$	1.9×10^3	3.6×10^3	5.6×10^{-7}	1.7×10^{-7}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar{b}_L \gamma^\mu s_L)^2$	1.1×10^2		7.6×10^{-5}		Δm_{B_s}
$(\bar{b}_R\, s_L)(\bar{b}_L s_R)$	3.7×10^2		1.3×10^{-5}		Δm_{B_s}

→ Generic Flavour Violation sources at the TeV scale are excluded

Minimal Flavour Violation

The SM flavour symmetry of the gauge interactions is

$$\mathcal{L}_K \supset i\ell^\dagger \sigma^\mu D_\mu \ell + i e^{c\dagger} \sigma^\mu D_\mu e^c + i q^\dagger \sigma^\mu D_\mu q + i u^{c\dagger} \sigma^\mu D_\mu u^c + i d^{c\dagger} \sigma^\mu D_\mu d^c$$

$$G_f = U(3)_q \times U(3)_{u^c} \times U(3)_{d^c} \times U(3)_\ell \times U(3)_{e^c}$$

$$\begin{cases} q \to U_q\, q & q \sim (3,\, 1,\, 1) \\ u^c \to u^c\, U^\dagger_{u^c} & u^c \sim (1,\, \bar{3},\, 1) \\ d^c \to d^c\, U^\dagger_{d^c} & d^c \sim (1,\, 1,\, \bar{3}) \end{cases} \qquad \begin{cases} \ell \to U_\ell\, \ell & \ell \sim (3,\, 1) \\ e^c \to e^c\, U^\dagger_{e^c} & e^c \sim (1,\, \bar{3}) \end{cases}$$

This flavour symmetry is not respected by the Yukawa interactions:

$$\begin{aligned} \mathcal{L}_Y &= e^c Y_e H^\dagger \ell + d^c Y_d H^\dagger q + u^c Y_u \widetilde{H}^\dagger q + \text{h.c.} \\ &\to e^c U^\dagger_{e^c} Y_e U_\ell H^\dagger \ell + d^c U^\dagger_{d^c} Y_d U_q H^\dagger q + u^c U^\dagger_{u^c} Y_u U_q \widetilde{H}^\dagger q + \text{h.c.} \end{aligned}$$

The formal invariance is recovered if the Yukawa matrices are promoted to auxiliary fields, called spurions, which transform as:

$$\begin{cases} Y_u \to U_{u^c}\, Y_u\, U_q^\dagger & Y_u \sim (\bar{3},\, 3,\, 1) \\ Y_d \to U_{d^c}\, Y_d\, U_q^\dagger & Y_d \sim (\bar{3},\, 1,\, 3) \end{cases} \qquad \begin{cases} Y_e \to U_{e^c}\, Y_e\, U_\ell^\dagger & Y_e \sim (\bar{3},\, 3) \end{cases}$$

Fermion masses and the CKM matrix are described by:

$$Y_u = \frac{\sqrt{2}}{v} \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} V$$

$$Y_d = \frac{\sqrt{2}}{v} \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

$$Y_e = \frac{\sqrt{2}}{v} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

where *V* stands for the CKM matrix.

Assuming that all the dim-6 operators are constructed by SM fermions and the spurions, any FCNC process is kept under control, with a NP at a scale of few TeV.

The fermions masses and the CKM are ONLY DESCRIBED and NOT EXPLAINED, because the background values of the spurions are simply ASSUMED. It needs an higher energy theory to explain it. [Alonso *et al.* 2011]

Neutrinos

Assuming that neutrino masses arise from the Weinberg Operator:

- the flavour symmetry is again G_f (no new fields)
- G_f is not respected by the Weinberg Operator

$$\frac{\mathcal{L}_5}{\Lambda_L} = \frac{1}{\Lambda_L} \left(\tilde{H}^\dagger \ell\right)^T Y_\nu \left(\tilde{H}^\dagger \ell\right) \to \frac{1}{\Lambda_L} \left(\tilde{H}^\dagger \ell\right)^T U_\ell^T Y_\nu U_\ell \left(\tilde{H}^\dagger \ell\right)$$

- G_f is formally restored if

$$Y_\nu \sim (8,\, 1) \qquad\qquad Y_\nu \to U_\ell^* \, Y_\nu \, U_\ell^\dagger$$

- neutrino masses and the PMNS arise if

$$Y_\nu = \frac{\Lambda_L}{v^2} U^* \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^\dagger$$

where *U* is the PMNS matrix

Again, the neutrino masses and the PMNS are ONLY DESCRIBED but NOT EXPLAINED.

Exercise 7

- Introducing 3 RH neutrinos the flavour symmetry is enlarged:
  - determine the new G_f arising from the kinetic terms
  - determine the quantities that do not preserve the symmetry
  - identify which quantities must be promoted to spurions to formally restore G_f and determine their transformation properties
  - is there a unique possibility to describe the neutrino masses and the PMNS matrix?

Flavour Models at the Electroweak Scale

Mainly based on
Ma & Rajasekaran 2001

Scale of Symmetry Breaking

<u>Large Scale Symmetry Breaking</u>

- The FS is broken at a very large scale $E \gg v$ by scalar fields that are singlets under the SM gauge group but transform non trivially under the FS:

$$\underbrace{\varphi \sim 3,\ \chi \sim 2,\ \xi \sim 1}_{\text{flavons}}$$

- The Lagrangian of the model is usually non-renormalisable: the LO terms describe the most relevant contributions to masses and mixings, while the NLO account for corrections

<u>EW Scale Symmetry Breaking</u>

- The gauge and the flavour syms are broken together and by the same objects: Multi-Higgs scenario

$$\underbrace{H_1,\ H_2,\ H_3,\ \ldots}$$

doublets of $SU(2)_L$
non-trivial rep. of the FS

- The Lagrangian of the model is usually renormalisable for the charged fermions and up to dim=5 for the neutrinos (Weinberg Op.)

The model

The full symmetry of the model is $A_4 \times L$:

Fermions:

$\ell \sim (3,1)$

$\nu^c \sim (3,0)$

$e^c \sim (1,-1)$

$\mu^c \sim (1'',-1)$

$\tau^c \sim (1',-1)$

Scalars:

$H \sim (3,0)$

$\eta \sim (1,1)$

doublets under $SU(2)_L$

Lagrangian:

$$\mathcal{L} \supset \frac{1}{2} M \, \nu^{cT} \nu^c + y_\nu \, \nu^{cT} \left(\tilde{\eta}^\dagger \, \ell\right)_3 +$$

$$+ y_e \, e^c \left(H^\dagger \ell\right) + y_\mu \, \mu^c \left(H^\dagger \ell\right)' + y_\tau \, \tau^c \left(H^\dagger \ell\right)''$$

$$1' \times 1' = 1''$$

$$1' \times 1'' = 1$$

$$1'' \times 1'' = 1'$$

$$3 \times 3 = 1 + 1' + 1'' + 3 + 3$$

$$\mathcal{L} \supset \frac{1}{2} M \nu^{cT} \nu^c + y_\nu \nu^{cT} (\tilde{\eta}^\dagger \ell)_3 + \\ + y_e e^c (H^\dagger \ell) + y_\mu \mu^c (H^\dagger \ell)' + y_\tau \tau^c (H^\dagger \ell)''$$

Once the neutral components of the scalar fields get vevs:

$$\langle H^0 \rangle = (v, v, v)/\sqrt{2} \qquad \langle \eta^0 \rangle = u/\sqrt{2}$$

Charged Leptons: (1nd basis with generator S diagonal)

$$M_e = \begin{pmatrix} y_e & y_e & y_e \\ y_\mu & y_\mu \omega & y_\mu \omega^2 \\ y_\tau & y_\tau \omega^2 & y_\tau \omega \end{pmatrix} \frac{v}{\sqrt{2}}$$

that is diagonalized by a LH transformation:

$$M_e U_e = M_e^{diag} = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \frac{v}{\sqrt{2}} \qquad U_e = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}$$

Charged lepton mass hierarchy is NOT explained!

$$\mathcal{L} \supset \frac{1}{2} M \, \nu^{cT} \nu^c + y_\nu \, \nu^{cT} \left(\tilde{\eta}^\dagger \, \ell\right)_3 +$$

$$+ \, y_e \, e^c \, \left(H^\dagger \ell\right) + y_\mu \, \mu^c \, \left(H^\dagger \ell\right)' + y_\tau \, \tau^c \, \left(H^\dagger \ell\right)''$$

Once the <u>neutral</u> components of the scalar fields get vevs:

$$\langle H^0 \rangle = (v, v, v)/\sqrt{2} \qquad\qquad \langle \eta^0 \rangle = u/\sqrt{2}$$

<u>Neutrinos:</u>

$$M_{(\nu,\nu^c)} = \begin{pmatrix} 0 & Y_\nu^T u/\sqrt{2} \\ Y_\nu u/\sqrt{2} & M_{\nu^c} \end{pmatrix}$$

$$M_{\nu^c} = \begin{pmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{pmatrix} \qquad Y_\nu = \begin{pmatrix} y_\nu & 0 & 0 \\ 0 & y_\nu & 0 \\ 0 & 0 & y_\nu \end{pmatrix}$$

$$\longrightarrow \quad M_\nu = \frac{y_\nu^2 \, u^2}{2M} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

completely degenerate neutrino masses

UPMNS:

$$U_e = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \qquad U_\nu \quad \text{generic}$$

$$\longrightarrow \quad U \equiv U_e^\dagger U_\nu = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} U_\nu$$

Since all the neutrino masses are degenerate, then **the neutrino mixings are unspecified**!

Proposed Solution: add arbitrary soft terms (symmetry breaking terms) that break the degeneracy and fix the mixings. These terms are of the type:

$$\left(M_{\nu^c}^{soft}\right)_{ij} \nu_i^c \nu_j^c$$

In this way it is possible fit the data.

It does NOT follow the symmetry principle!

Where do these terms come from?
Why exactly these terms and not others?
...

The Scalar Potential

The key ingredient of the model is the vacuum alignment:

$$\langle H^0 \rangle = (v, v, v)/\sqrt{2} \qquad\qquad \langle \eta^0 \rangle = u/\sqrt{2}$$

Focussing on H:

$$\begin{aligned} V[\Phi_a] =& \mu^2(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2 + \Phi_3^\dagger\Phi_3) + \lambda_1(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2 + \Phi_3^\dagger\Phi_3)^2 + \\ &+ \lambda_3(\Phi_1^\dagger\Phi_1\Phi_2^\dagger\Phi_2 + \Phi_1^\dagger\Phi_1\Phi_3^\dagger\Phi_3 + \Phi_2^\dagger\Phi_2\Phi_3^\dagger\Phi_3) + \\ &+ \lambda_4(\Phi_1^\dagger\Phi_2\Phi_2^\dagger\Phi_1 + \Phi_1^\dagger\Phi_3\Phi_3^\dagger\Phi_1 + \Phi_2^\dagger\Phi_3\Phi_3^\dagger\Phi_2) + \\ &+ \frac{\lambda_5}{2}\left[e^{i\epsilon}\left[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_3)^2 + (\Phi_3^\dagger\Phi_1)^2\right] + \text{h.c.}\right] \end{aligned}$$

different contractions of 4 triplets

Condition for a stable minimum away from the origin:

$$\mu^2 < 0 \qquad\qquad \lambda_i \in \text{Real}$$

Condition for a potential bounded from below:

$$\lambda_1 > 0 \qquad\qquad \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 \cos\epsilon > 0$$

Condition for a stationary point (minimum or maximum):

$$\frac{\partial V[\Phi]}{\partial \Phi_i} = 0$$

Condition for a physical minimum (all the physical masses are non-negative):

$$\text{Eigenvalues}\left\{\frac{\partial^2 V[\Phi]}{\partial \Phi_i \partial \Phi_j}\right\} \geq 0$$

→ one of the solutions is indeed:

$$\langle H^0 \rangle = (v, v, v)/\sqrt{2} \qquad v \equiv v_{EW}/\sqrt{3}$$

A true physical minimum must pass also the following requirements:

- Perturbative Unitarity: additional scalars contribute to the gauge boson scattering (that is unitarised by the Higgs particle in the SM)
- Z and $W^\pm$ decays into scalar fields: these decays are well measured
- Consistency with the electroweak precision tests: TSU parameters
- Study the presence of tree-level flavour changing neutral currents

Strong constraints on the parameter space!

Summary

The model is the simplest of this kind in terms of fields content and of symmetries, but...

- no explanation of charged lepton mass hierarchies
- no symmetric explanation of neutrino masses and mixing

On the other hand the scalar content is well defined: the VEV is indeed a physical minimum of the scalar potential and all the phenomenological bound are respected

(Similar results in the quark sector)

→ The possibility of explaining masses and mixing using a multi-Higgs approach is still viable but new strategies should be followed.

Exercises 8 & 9

- Determine all the possible minima of the scalar potential:

$$
\begin{aligned}
V[\Phi_a] =&\mu^2(\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2+\Phi_3^\dagger\Phi_3)+\lambda_1(\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2+\Phi_3^\dagger\Phi_3)^2+\\
&+\lambda_3(\Phi_1^\dagger\Phi_1\Phi_2^\dagger\Phi_2+\Phi_1^\dagger\Phi_1\Phi_3^\dagger\Phi_3+\Phi_2^\dagger\Phi_2\Phi_3^\dagger\Phi_3)+\\
&+\lambda_4(\Phi_1^\dagger\Phi_2\Phi_2^\dagger\Phi_1+\Phi_1^\dagger\Phi_3\Phi_3^\dagger\Phi_1+\Phi_2^\dagger\Phi_3\Phi_3^\dagger\Phi_2)+\\
&+\frac{\lambda_5}{2}\left[e^{i\epsilon}\left[(\Phi_1^\dagger\Phi_2)^2+(\Phi_2^\dagger\Phi_3)^2+(\Phi_3^\dagger\Phi_1)^2\right]+\text{h.c.}\right]
\end{aligned}
$$

  distinguishing among real and complex VEVs.

- Modified the previous scalar potential involving also the field $\eta\sim(1,1)$ and determine the conditions for it to get a VEV.

Conclusions

- There is no THEORY of NEUTRINO MASSES and MIXINGS, only ideas!
- The Froggatt-Nielsen U(1) provides a valid strategy, but the predictive power is rather weak.
- Discrete Flavour Models at the GUT scale (i.e. AF model) have a strong predictive power and are able to describe lepton mass hierarchies and mixings. However, they require a much richer heavy spectrum and a more complicated symmetry content (auxiliary symmetries as Z_n).
- Minimal (Lepton) Flavour Violation is only a working context, where FCNCs are under control, but it is not a model of flavour: it needs an higher energy theory to explain it.
- Discrete Flavour Models at the Electroweak scale (i.e. MR model) have rather simple spectrum and symmetry content, but give up in describing charged lepton masses, need *ad hoc* terms for the neutrinos and are strongly constrained due to too large FCNCs.

Thank you

Backup

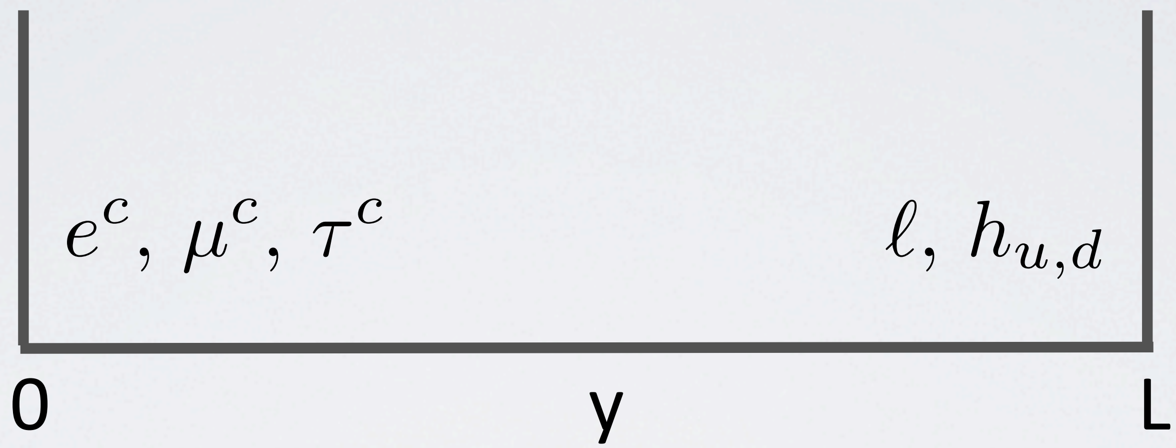
AF: Vacuum Alignment - Extra D

$$\frac{\langle\varphi_T\rangle}{\Lambda} = (u,0,0) \qquad \frac{\langle\varphi_S\rangle}{\Lambda} = c_b(u,u,u) \qquad \frac{\langle\xi\rangle}{\Lambda} = c_a u$$

is NOT a minimum of the most general renormalisable scalar potential: mainly due to the mixed quartic term among φ_T and φ_S

$\langle\varphi_T\rangle = (v_T, 0, 0)$

local minimum of V_0



$\langle\varphi_S\rangle = (v_S, v_S, v_S)$

$\langle\xi\rangle = v_\xi$

local minimum of V_L

The extra-D keeps separated φ_T and φ_S, solving the problem of the mixed quartic terms

ν masses arise from local operators at y=L: $h_u h_u \left(\frac{\varphi_S}{\Lambda}\frac{\ell\ell}{\Lambda_L}\right) \qquad h_u h_u \frac{\xi}{\Lambda}\frac{(\ell\ell)}{\Lambda_L}$

charged lepton masses arise from non-local operators:

$$\frac{(f^c\varphi_T F)\,\delta(y)}{\sqrt{\Lambda}} \qquad -MF^cF \qquad \frac{(F^c\ell)\,h_d\,\delta(y-L)}{\sqrt{\Lambda}}$$

integrating out the balk fermions F $\longrightarrow$ $\frac{(f^c\varphi_T\ell)\,h_d}{\Lambda}e^{-ML}$

AF: Vacuum Alignment - SUSY

The proposed vacuum alignment is a natural minimum of the scalar potential in the SUSY context:

	θ	φ_T	φ_S	ξ	$\tilde{\xi}$	φ_0^T	φ_0^S	ξ_0
A_4	1	3	3	1	1	3	3	1
Z_3	1	1	ω	ω	ω	1	ω	ω
$U(1)_R$	0	0	0	0	0	2	2	2

The complete superpotential is: $w = w_\ell + w_\nu + w_d$

$\mathcal{L}_{e,\nu} \longrightarrow w_{e,\nu}$

$$w_d = M(\varphi_0^T \varphi_T) + g(\varphi_0^T \varphi_T \varphi_T) + g_1(\varphi_0^S \varphi_S \varphi_S) + g_2 \xi (\varphi_0^S \varphi_S) + $$
$$+ g_3 \xi_0 (\varphi_S \varphi_S) + g_4 \xi_0 \xi^2 + g_5 \xi_0 \xi \tilde{\xi} + g_6 \xi_0 \tilde{\xi}^2$$

The minimum of the scalar potential is at:

$$\frac{\langle \varphi_T \rangle}{\Lambda} = (u,0,0) \qquad \frac{\langle \varphi_S \rangle}{\Lambda} = c_b(u,u,u) \qquad \frac{\langle \xi \rangle}{\Lambda} = c_a u \qquad \frac{\langle \tilde{\xi} \rangle}{\Lambda} = 0$$

$$u = -\frac{3}{2}\frac{M}{g} \qquad c_b^2 = -\frac{g_4}{3 g_3} c_a^2$$

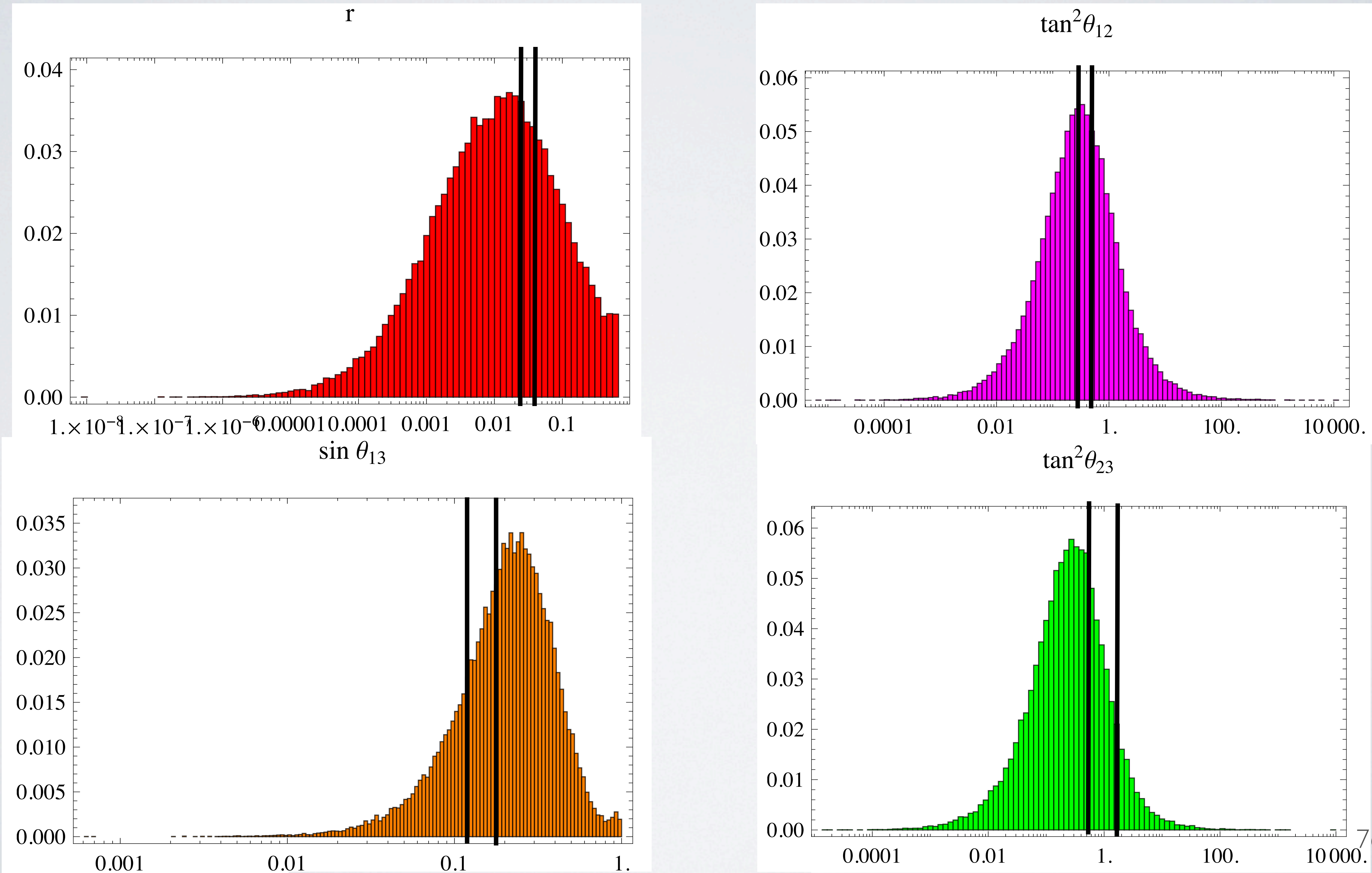
c_a undetermined

U(1)

[Altarelli, Feruglio, Masina & LM 2012]

Consider a simple U(1) as flavour symmetry, in a SU(5) inspired context:

$$\Psi_{10} = (5,3,0) \qquad \Psi_{\overline{5}} = (2,1,0)$$



Anarchy vs. Hierarchy?

[Altarelli, Feruglio, Masina & LM 2012]