Pontecorvo summer school
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Neutrinoless double-beta decay and double-electron capture

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OUTLINE

- *Introduction*
- *Effective Majorana neutrino mass, CP phases*
  (ν–mixing without and with sterile neutrinos)
- *$0\nu\beta\beta$-decay NMEs*
- *Competing $0\nu\beta\beta$-decay mechanisms*
- *0νECEC-decay*
- *Other LNV processes*
- *Bosonic neutrinos and $2\nu\beta\beta$-decay*
- *Transitional magnetic moment of Majorana neutrinos*
- *Conclusion*

Theory of neutrinoless double beta decay,
J.D. Vergados, H. Ejiri and F. Šimkovic, Rep. Progr. Phys.75 (2012) 106301.

Desperate idea of Pauli (80 years ago)

A letter to Tuebingen

“Liebe Radioaktive Damen and Herren!” (L. Meitner, H. Geiger)

4th December 1930

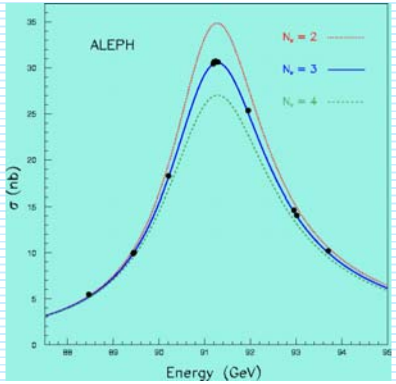
Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and Li^6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...

Fundamental properties of neutrinos

After 55 years we know

- **3 families of light (V-A) neutrinos: ν_e, ν_μ, ν_τ**
- **ν are massive: we know mass squared differences**
- **relation between flavor states and mass states (neutrino mixing) only partially known**



Claim for evidence of the $0\nu\beta\beta$-decay

H.V. Klapdor-Kleingrothaus et al.,NIM A 522, 371 (2004); PLB 586, 198 (2004)

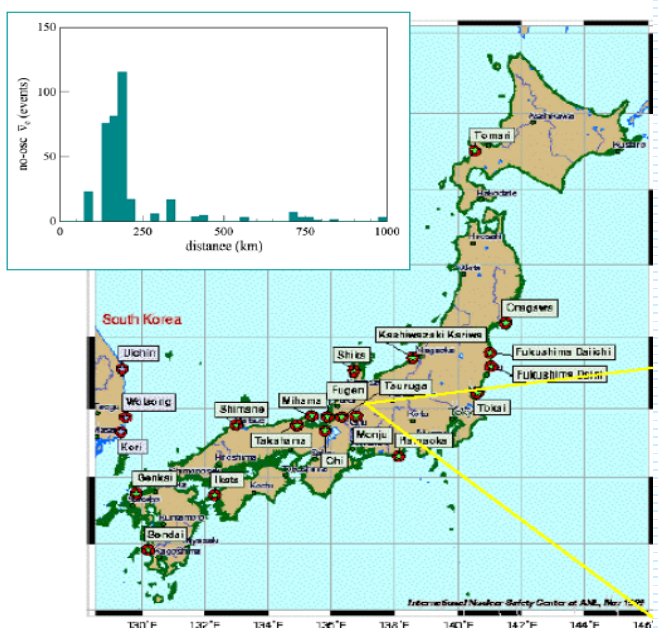
- **Absolute ν mass scale from the $0\nu\beta\beta$-decay. (cosmology, ^{3}H, ^{187}Rh ?)**
- **ν's are their own antiparticles – Majorana.**

No answer yet

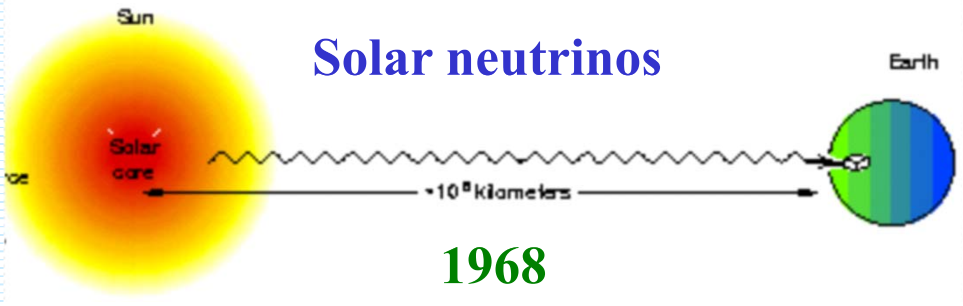
- **Is there a CP violation in ν sector? (leptogenesis)**
- **Are neutrinos stable?**
- **What is the magnetic moment of ν?**
- **Sterile neutrinos?**
- **Statistical properties of ν? Fermionic or partly bosonic?**

Compelling evidence about new physics beyond the SM.

Reactor neutrinos

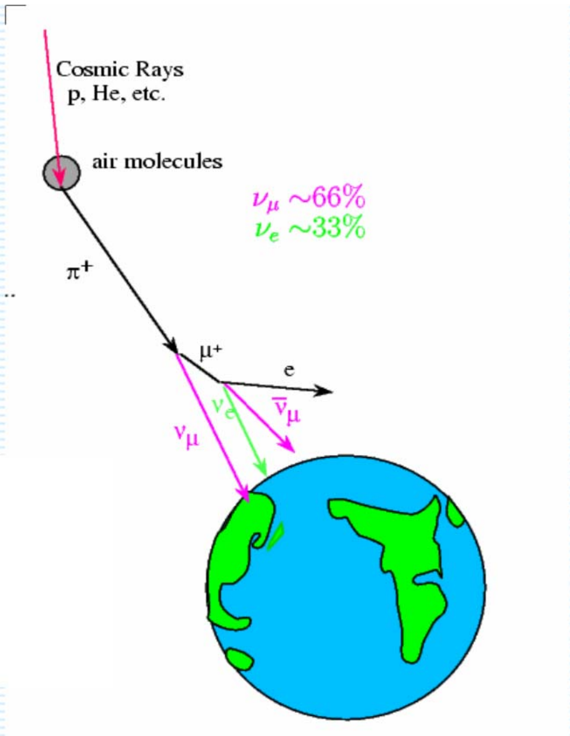


Solar neutrinos



1968

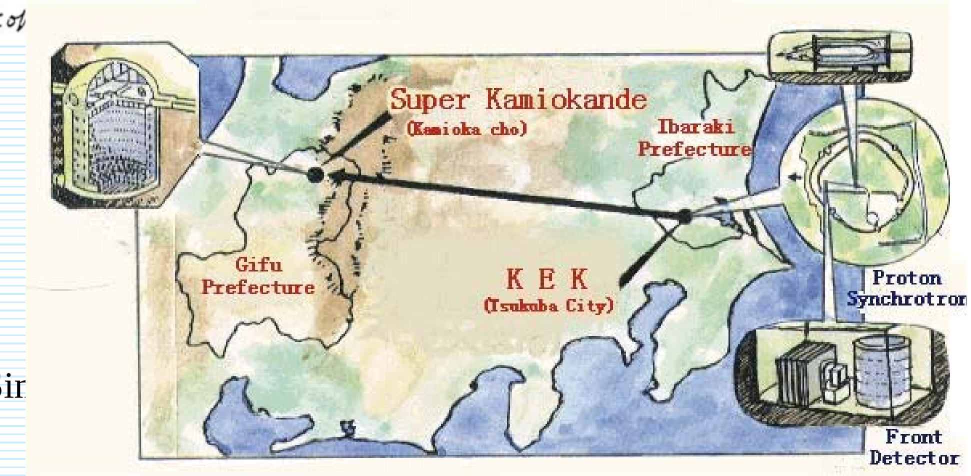
Atmospheric neutrinos



1957

JOINT INSTITUTE FOR NUCLEAR RESEARCH JINR

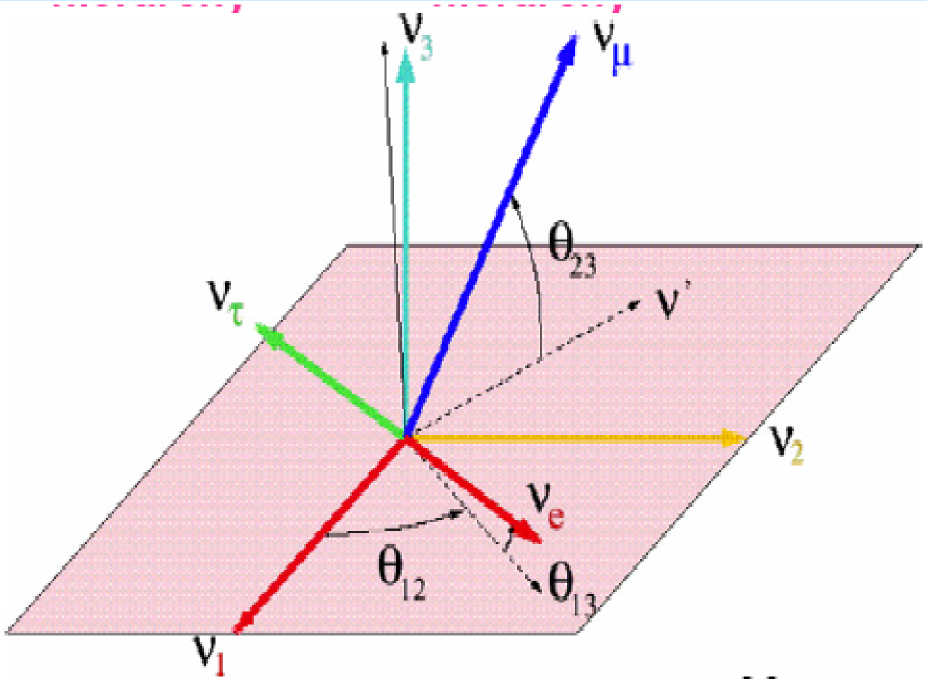
Accelerator neutrinos



Mixing of 3 light Neutrinos

Pontecorvo -Maki-Nakagawa-Sakata matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



Flavor eigenstates

Mass eigenstates

Mass matrix

$$\begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{\mu e} & M_{\mu\mu} & M_{\mu\tau} \\ M_{\tau e} & M_{\tau\mu} & M_{\tau\tau} \end{pmatrix}$$

$$m_{\beta\beta} = \Sigma\, U_{ei} U_{ei}\, m_i$$

Is there analogy between lepton mixing matrix and quark mixing?

PMNS Lepton Mixing Matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

CKM Quark Mixing Matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Large off diagonal elements

$$\begin{pmatrix} 0.7 & 0.7 & < 0.2\ e^{i\delta_{13}} \\ -0.5 & 0.5 & 0.7 \\ 0.5 & -0.5 & 0.7 \end{pmatrix}$$

CP violating Phases:
δ_{13}, δ_{CKM}

$$\begin{pmatrix} 0.97 & 0.22 & 0.003\ e^{i\delta_{CKM}} \\ -0.22 & 0.97 & 0.04 \\ 0.01 & -0.04 & 0.999 \end{pmatrix}$$

Disperity and challange for quark-lepton unified theories

PMNS for Majorana neutrinos

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta 13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

Quark-Lepton Complementarity

QLC- relations:

H. Minakata, A.S.
Phys. Rev. D70: 073009 (2004)
[hep-ph/0405088]

$\theta^{l}_{12} + \theta^{q}_{12} \sim \pi/4$

$\theta_{12} + \theta_{C}$ **= 46.5° +/- 1.3°**

$\theta^{l}_{23} + \theta^{q}_{23} \sim \pi/4$

$\theta_{23} + \theta_{23}$ **= 43.9° + 5.1/-3.6°**

Qualitatively correlation:

- **2-3 leptonic mixing is close to maximal because 2-3 quark mixing is small**
- **1-2 leptonic mixing deviates from maximal substantially because 1-2 quark mixing is relatively large**

$\theta_{13} \neq 0$: How Big or How Small?

Convincing flavor theory has been lacking—it is at present impossible to predict fermion masses, flavor mixing angles and CP phases fundamental level ⇒ the flavor problem

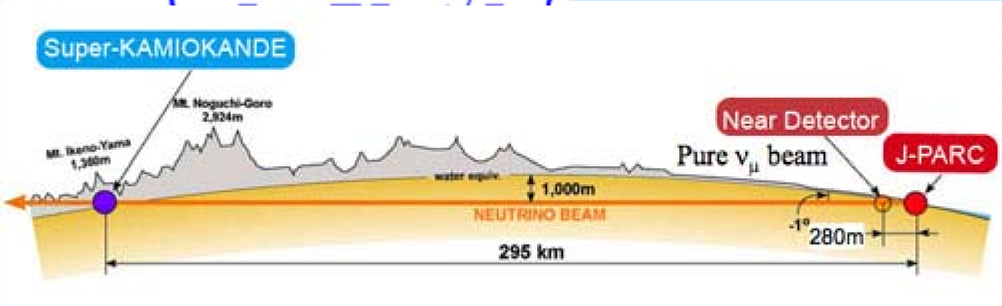
Bi-maximal mixing

$$U_{bm} = \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \sqrt{\frac{1}{2}} \\ \dots & \dots & \dots \end{pmatrix}$$

As dominant structure? Zero order?

$U_{bm} = U_{23}^m U_{12}^m$

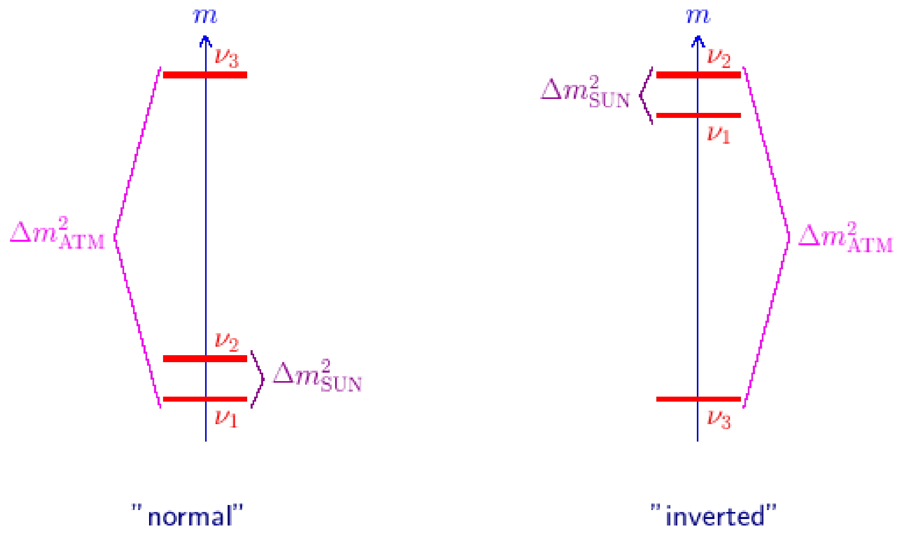
θ_{13} has a role!



T2K: $0.03 < \sin^2 2\theta_{13} < 0.34$ (June 2011)
DOOBLE CHOOZ: $\sin^2 2\theta_{13} = 0.085 \pm 0.051$ (Nov. 2011)
Daya Bay: $\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst})$ (Febr. 2011)
RENO : $\sin^2 2\theta_{13} = 0.103 \pm 0.013(\text{stat}) \pm 0.011(\text{syst})$ (March 2012)

Neutrinos mass spectrum

mass differences:
$|\Delta m^2_{sol}| = 7.65\ 10^{-5}\ \mathrm{eV}^2$
$\Delta m^2_{atm} = 2.43\ 10^{-3}\ \mathrm{eV}^2$

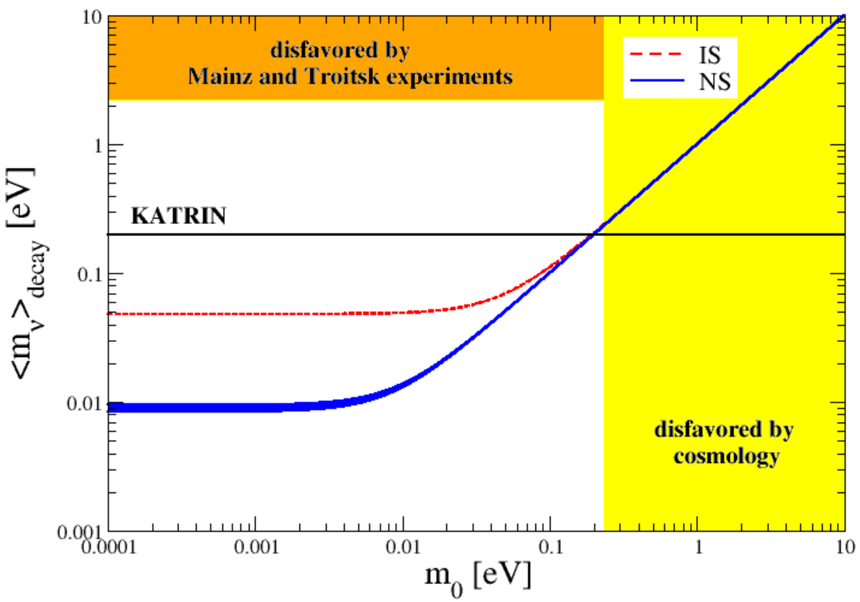


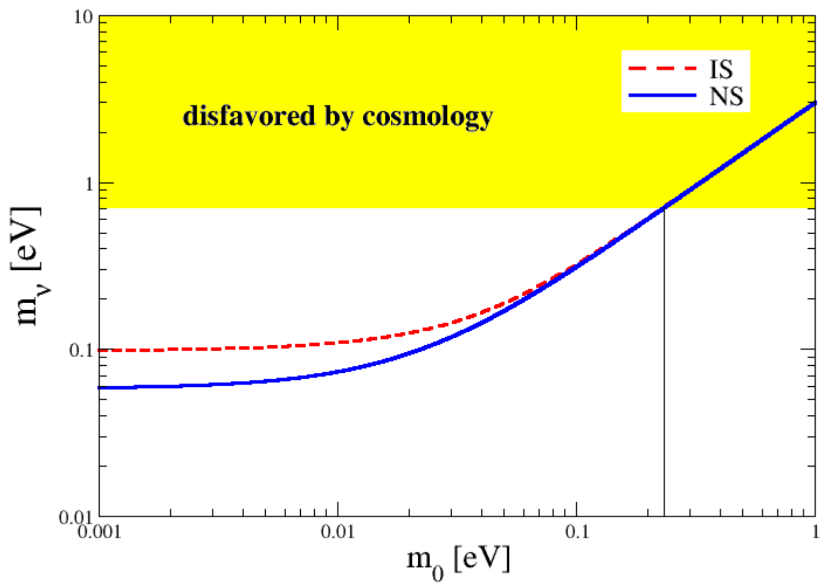
Tritium decay

$$m_\beta = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2\ m_i^2}$$

Cosmology

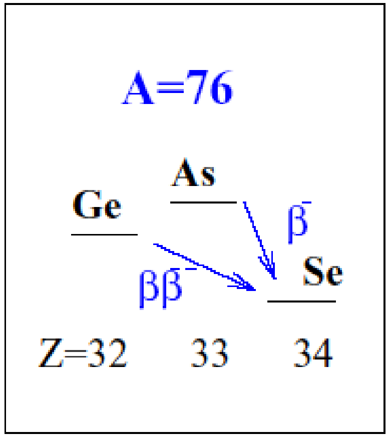
$$\sum_{i=1}^{3} m_i$$

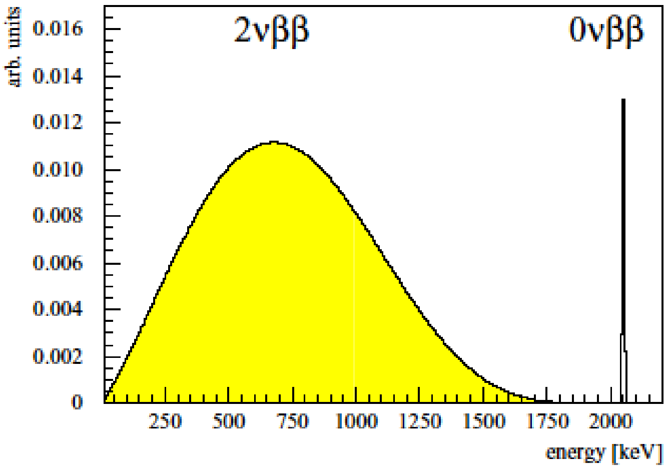


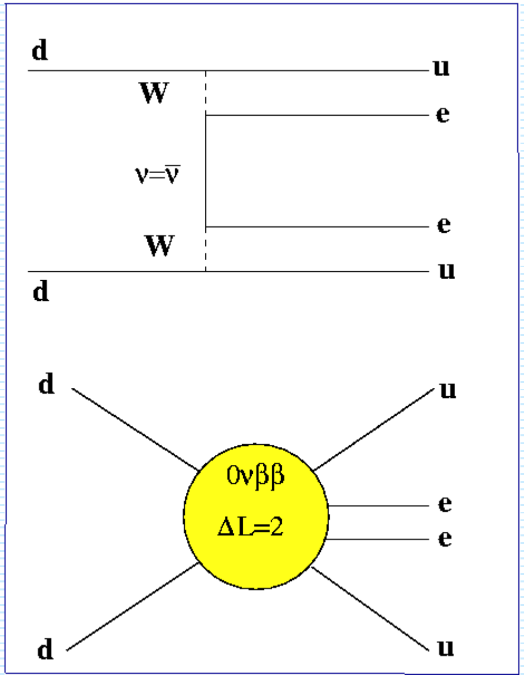


Neutrinoless Double-Beta Decay

$$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$$

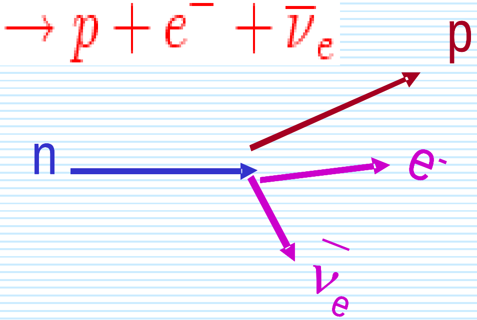






1934 First double beta decay calculation

$n \rightarrow p + e^- + \overline{\nu}_e$



Fermi, Z. Physik 88 (1934) 161

Fermi 4-fermion contact interaction, Lagrangian of interaction (in analogy with electrodynamics):

$$\mathcal{L}(x) = -\frac{G_F}{\sqrt{2}}\left[\overline{\phi}_p(x)\gamma^\mu\phi_n(x)\right]\left[\overline{\phi}_e(x)\gamma^\mu\phi_\nu(x)\right]$$

G_F = Fermi coupling constant = (1.16637±0.000001) 10^{-5} GeV^{-2}

$$(A, Z) \rightarrow (A, Z+2) + 2e^- + 2\overline{\nu}_e$$

1935
Q-value about 10 MeV
T1/2 ≈ 10^{17} years

Eugene Wigner

Maria-Goeppert Mayer

What is the nature of neutrinos?

The answer to the question whether neutrinos are their own antiparticles is of central importance, not only to our understanding of neutrinos, but also to our understanding of the origin of mass.

$\nu \Rightarrow$

GUT's

Only the $0\nu\beta\beta$-decay can answer this fundamental question

Analogy with kaons: K_0 and $\overline{K}_0$

Analogy with π_0

1937 Beginning of Majorana neutrino physics

Ettore Majorana discoveres the possiility of existence of truly neutral fermions

Charged fermion (electron) + electromagnetic field

$$(i\gamma^\mu\partial_\mu - e\gamma^\mu A_\mu - m)\Psi = 0$$

$$(i\gamma^\mu\partial_\mu + e\gamma^\mu A_\mu - m)\Psi^c = 0$$

$\Psi^c = \Psi$ **forbidden**

Neutral fermion (neutrino) + electromagnetic field

$$(i\gamma^\mu\partial_\mu - m)\nu = 0$$

$$(i\gamma^\mu\partial_\mu - m)\nu^c = 0$$

$\nu^c = \nu$ **allowed**

Majorana condition

Symmetric Theory of Electron and Positron
Nuovo Cim. 14 (1937) 171

Here is the beginning of Nonstandard Neutrino Properties

Majorana Neutrino $\longleftrightarrow$ **No Conserved Lepton Number**

$$L_e, \quad L_\mu, \quad L_\tau, \quad L = L_e + L_\mu + L_\tau$$

L= -1 $\longleftarrow$ $\nu^c = \nu$ $\longrightarrow$ **L= +1**

Conserved Lepton Number $\underset{\text{Theorem}}{\overset{\text{Noether}}{\rightleftarrows}}$ **Global Gauge Invariance**

Dirac mass term

$$\mathcal{L}^D = -m_D\left(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L\right)$$

invariant under

$\nu_L \rightarrow e^{i\Lambda}\nu_L$ $\quad$ $\nu_R \rightarrow e^{i\Lambda}\nu_R$

$\overline{\nu_R} \rightarrow e^{-i\Lambda}\overline{\nu_R}$ $\quad$ $\overline{\nu_R} \rightarrow e^{-i\Lambda}\overline{\nu_R}$

Majorana mass term

$$\mathcal{L}^M = -m_M\left(\overline{\nu_L}\nu_L^c + \overline{\nu_L^c}\nu_L\right)$$

not invariant under

$\nu_L \rightarrow e^{i\Lambda}\nu_L$ $\quad$ $\nu_L^c \rightarrow e^{-i\Lambda}\nu_L^c$

$\overline{\nu_L} \rightarrow e^{-i\Lambda}\overline{\nu_L}$ $\quad$ $\overline{\nu_L^c} \rightarrow e^{i\Lambda}\overline{\nu_L^c}$

Majorana Neutrino = Truly Neutral Fermion

ν_L AND ν_R $\longrightarrow$ <u>Dirac-Majorana Mass Term</u>

$$\begin{aligned}\mathcal{L}^{D+M} &= \mathcal{L}_L^M + \mathcal{L}_R^M + \mathcal{L}^D \\ &= -\frac{1}{2}\left(\overline{\nu_L^c} \quad \overline{\nu_R}\right)\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + H.c. \\ &= \frac{1}{2} N_L^T C^\dagger M N_L + H.c.\end{aligned}$$

$$M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$$

$$N_L = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$$

diagonalization

$\downarrow$

fields with definite mass

$$N_L = U n_L, \quad n_L = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix} \quad \Longrightarrow \quad U^T M U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

$$\mathcal{L}^{D+M} = \frac{1}{2}\sum_{k=1,2} m_k \nu_{kL}^T C^\dagger \nu_{kL} + h.c. = -\frac{1}{2}\sum_{k=1,2} m_k \overline{\nu_k}\nu_k$$

$$\nu_k = \nu_{kL} + \nu_{kL}^c$$

Massive neutrinos are Majorana!

Light ν-exchange 0νββ–decay mechanism

S.M. Bilenky, S. Petcov, Rev. Mod. Phys. 59, 671 (1987)

Majorana condition

$$C\, \overline{\chi_k}^T(x) \;=\; \xi_k\; \chi_k(x)$$

Majorana particle propagator

$$\begin{aligned} < \chi_\alpha(x_1)\overline{\chi}_\beta(x_2) > &= \frac{-1}{(2\pi)^4}\int \left(\frac{1}{\gamma p - im}\right)_{\alpha\beta} e^{ip(x_1-x_2)}dp \\ &= S_{\alpha\beta}(x_1 - x_2) \end{aligned}$$

$$\begin{aligned} < \chi(x_1)\chi^T(x_2) > &= -\xi S(x_1 - x_2)C \\ < \overline{\chi}^T(x_1)\overline{\chi}(x_2) > &= \xi C^{-1} S(x_1 - x_2) \end{aligned}$$

Weak β-decay Hamiltonian

$$\mathcal{H}^\beta_W = \frac{G_F}{\sqrt{2}}\, \bar{e}\gamma_\alpha(1+\gamma_5)\nu_e\, j_\alpha + h.c.$$

Neutrino mixing

$$\nu_{eL} \;=\; \sum_k U^L_{lk}\;\chi_{kL}$$

S-matrix term

$$S^{(2)} = -\frac{(-i)^2}{2}4\left(\frac{G_F}{\sqrt{2}}\right)^2 \int N\left[\overline{e_L}(x_1)\gamma_\alpha < \nu_{eL}(x_1)\nu^T_{eL}(x_2) > \gamma^T_\beta \overline{e_L}^T(x_2)\right] \times$$
$$T\left(j_\alpha(x_1)j_\beta(x_2)e^{-i\int \mathcal{H}_{str}(x)dx}\right) dx_1 dx_2$$

Contraction of ν-fields

$$< \nu_{eL}(x_1)\nu_{eL}{}^T(x_2) > = -\sum_k \left(U^L_{ek}\right)^2 \xi_k \frac{1+\gamma_5}{2} S_k(x_1 - x_2)\frac{1+\gamma_5}{2}C$$
$$= \frac{i}{(2\pi)^4}\sum_k \left(U^L_{ek}\right)^2 \xi_k m_k \int \frac{e^{iq(x_1-x_2)}dq}{q^2+m_k^2}\frac{1+\gamma_5}{2}C$$

Effective mass of Majorana neutrinos

$$m_{\beta\beta} = \sum_k \left(U^L_{ek}\right)^2 \xi_k m_k$$

0νββ-decay matrix element

$$< f|S^{(2)}|i > \; = \; m_{\beta\beta}\left(\frac{G_F}{\sqrt{2}}\right)^2 N_{p_1}N_{p_2}\overline{u}(p_1)\gamma_\alpha(1+\gamma_5)\gamma_\beta C\overline{u}^T(p_2) \times$$
$$\int e^{-ip_1x_1}e^{-ip_2x_2}\frac{-i}{(2\pi)^4}\int\frac{e^{iq(x_1-x_2)}dq}{q^2} \times$$
$$< A'|T\left[J_\alpha(x_1)J_\beta(x_2)\right]|A > dx_1dx_2 - (p_1 \leftrightarrow p_2)$$

Use of completness **1=Σ$_n$|n><n|**

$$< A'|J_\alpha(x_1)J_\beta(x_2)|A > \; = \; \sum_n < A'|J_\alpha(0,\vec{x}_1)|n >< n|J_\beta(0,\vec{x}_2)|A > \times$$
$$e^{-i(E'-E_n)x_{10}}e^{-i(E_n-E)x_{20}}$$

After integration over time variables

$$< f|S^{(2)}|i > \; = \; im_{\beta\beta}\left(\frac{G_F}{\sqrt{2}}\right)^2 N_{p_1}N_{p_2}\overline{u}(p_1)\gamma_\alpha(1+\gamma_5)\gamma_\beta C\overline{u}^T(p_2)$$
$$\times\int d\vec{x}_1 d\vec{x}_2 e^{-i\vec{p}_1\cdot\vec{x}_1}e^{-i\vec{p}_2\cdot\vec{x}_2}\frac{1}{(2\pi)^3}\int\frac{e^{i\vec{q}\cdot(\vec{x}_1-\vec{x}_2)}d\vec{q}}{\vec{q}^2} \times$$
$$\sum_n\left(\frac{< A'|J_\alpha(0,\vec{x}_1)|n >< n|J_\beta(0,\vec{x}_2)|A >}{E_n+q_0+p_{20}-E}+\right.$$
$$\left.\frac{< A'|J_\beta(0,\vec{x}_1)|n >< n|J_\alpha(0,\vec{x}_2)|A >}{E_n+q_0+p_{10}-E}\right)$$
$$\times \; 2\pi\delta(E'+p_{10}+p_{20}-E)$$

Approximations and simplifications

1) Non-relativistic impulse approx. for nuclear current

$$J_\alpha(0,\vec{x}) = \sum_n \tau_n^+ (\delta_{\alpha 4} + i g_A (\vec{\sigma})_k \delta_{\alpha k}) \delta(\vec{x} - \vec{x}_n)$$

2) Long-wave approximation for lepton wave functions

$$e^{-i\vec{p}_1\cdot\vec{x}_1 - i\vec{p}_2\cdot\vec{x}_2} \longrightarrow 1$$

3) Closure approximation

$$E_n \longrightarrow \; < E_n >$$

$$< f|S^{(2)}|i > = \overline{u}(p_1)\gamma_\alpha(1+\gamma_5)\gamma_\beta C \overline{u}^T(p_2) A_{\alpha\beta}, \quad A_{\alpha\beta} = A_{\beta\alpha}$$

contribute

Hadron part is symmetric

$$J_\alpha(0,\vec{x}_1) J_\beta(0,\vec{x}_2) = J_\beta(0,\vec{x}_2) J_\alpha(0,\vec{x}_1)$$

$$\gamma_\alpha\gamma_\beta = \delta_{\alpha\beta} + \frac{1}{2}(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha)$$

0νββ-decay matrix element

$$\begin{aligned} < f|S^{(2)}|i > \quad &= \quad i\, m_{\beta\beta} \left(\frac{G_F}{\sqrt{2}}\right)^2 N_{p_1} N_{p_2} \overline{u}(p_1)(1-\gamma_5) C \overline{u}^T(p_2) \frac{1}{R} \\ &\times \left(M_F - g_A^2 M_{GT}\right) \delta(p_{10} + p_{20} + M' - M) \end{aligned}$$

Nuclear matrix elements

$$M_F = < A'|\sum_{n,m}\tau_n^+\tau_m^+ h(|\vec{x}_n - \vec{x}_m|)|A >$$

$$M_F = < A'|\sum_{n,m}\tau_n^+\tau_m^+ h(|\vec{x}_n - \vec{x}_m|)\vec{\sigma}_n\cdot\vec{\sigma}_m|A >$$

Neutrino exchange potential

$$h(|\vec{x}_n - \vec{x}_m|) = \frac{1}{2\pi^2}\int\frac{e^{i\vec{q}\cdot\vec{x}}d\vec{q}}{q_0(q_0+ < E_n > -(E+E')/2)} \approx \frac{1}{|\vec{x}|}$$

Differential 0νββ-decay rate

$$d\Gamma_{0\nu} = \frac{1}{2}\frac{G_F^4 m_e^5}{(2\pi)^5}|m_{\beta\beta}|^2\frac{1}{R^2}|M_F - g_A^2 M_{GT}|^2(1-\cos\theta) F^2(Z)(\varepsilon_0 - \varepsilon + 1)^2(\varepsilon+1)d\varepsilon\sin\theta d\theta$$

$$F(Z) = \frac{2\pi\alpha(Z+2)}{1 - exp[-2\pi\alpha(Z+2)]}$$

$$\varepsilon_0 = \frac{1}{m_e}(M - M' - 2m_e)$$

Full 0νββ-decay rate

$$\Gamma_{0\nu} = \frac{1}{2}\frac{G_F^4 m_e^5}{(2\pi)^5}|m_{\beta\beta}|^2\frac{1}{R^2}|M_F - g_A^2 M_{GT}|^2 F^2(Z) \times\frac{1}{15}\left(\varepsilon_0^5 + 10\varepsilon_0^4 + 40\varepsilon_0^3 + 60\varepsilon_0^2 + 30\varepsilon_0\right)$$

The answer to the question whether neutrinos are their own antiparticles is of central importance, not only to our understanding of neutrinos, but also to our understanding of the origin of mass.

$$\frac{1}{T_{1/2}} = G^{0\nu}(E_0, Z)|M'^{0\nu}|^2|\langle m_{\beta\beta}\rangle|^2 \;, \qquad m_{\beta\beta} = \sum_{i=1}^{3} U_{ei}^2 \; m_i$$

Absolute ν mass scale

Normal or inverted Hierarchy of ν masses

CP-violating phases

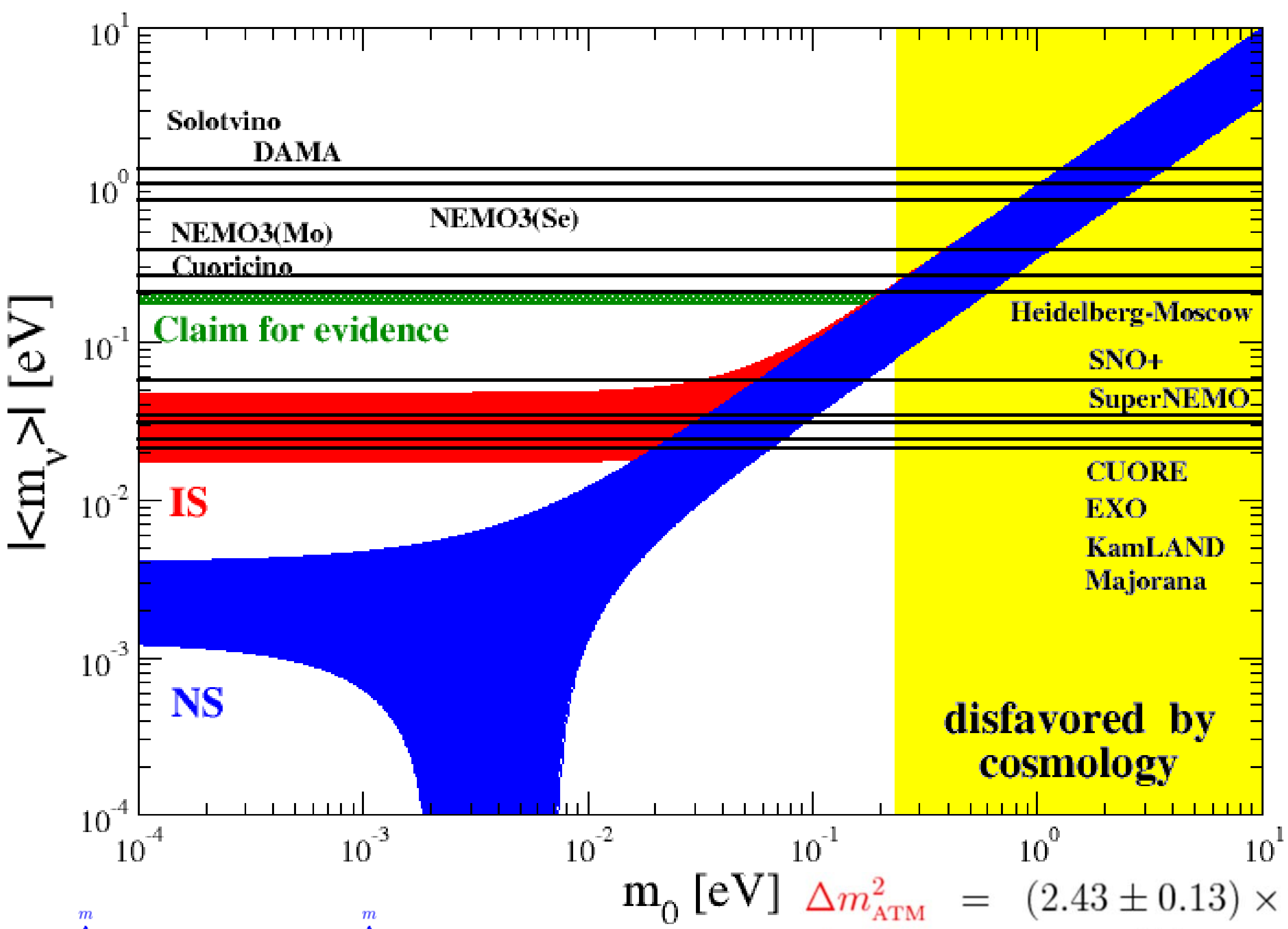
$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

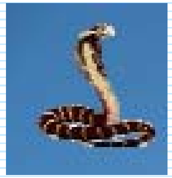
$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

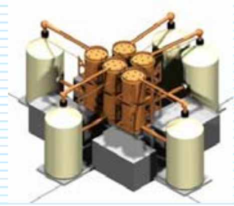
An accurate knowledge of the nuclear matrix elements, which is not available at present, is however a pre-requisite for exploring neutrino properties.

Daya Bay: $\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$ (March 2012)

$$|m_{\beta\beta}{}^{(3\ \nu)}| \;=\; |c_{12}^2 c_{13}^2 e^{2i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{2i\alpha_2} m_2 + s_{13}^2 m_3|$$







QRPA NMEs CB-Bonn g_A=1.25

$$\Delta m^2_{\mathrm{ATM}} = (2.43 \pm 0.13) \times 10^{-3}\ \mathrm{eV}^2\ (MINOS)$$

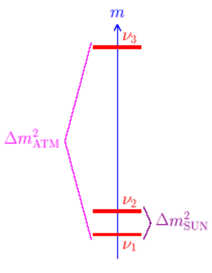
$$\Delta m^2_{\mathrm{SUN}} = (7.65^{+0.13}_{-0.20}) \times 10^{-5}\ \mathrm{eV}^2\ (global\ fit)$$

$$\sin^2\theta_{23} = 0.50^{+0.07}_{-0.06}\ (global\ fit)$$

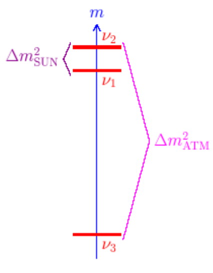
$$\tan^2\theta_{12} = 0.452^{+0.035}_{-0.033}\ (KAMLAND)$$

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016\ (stat)\ \pm 0.005\ (syst)\ (Daya\ Bay)$$

$$\sin^2 2\theta_{13} = 0.103 \pm 0.013\ (stat)\ \pm 0.011\ (syst)\ (Reno)$$



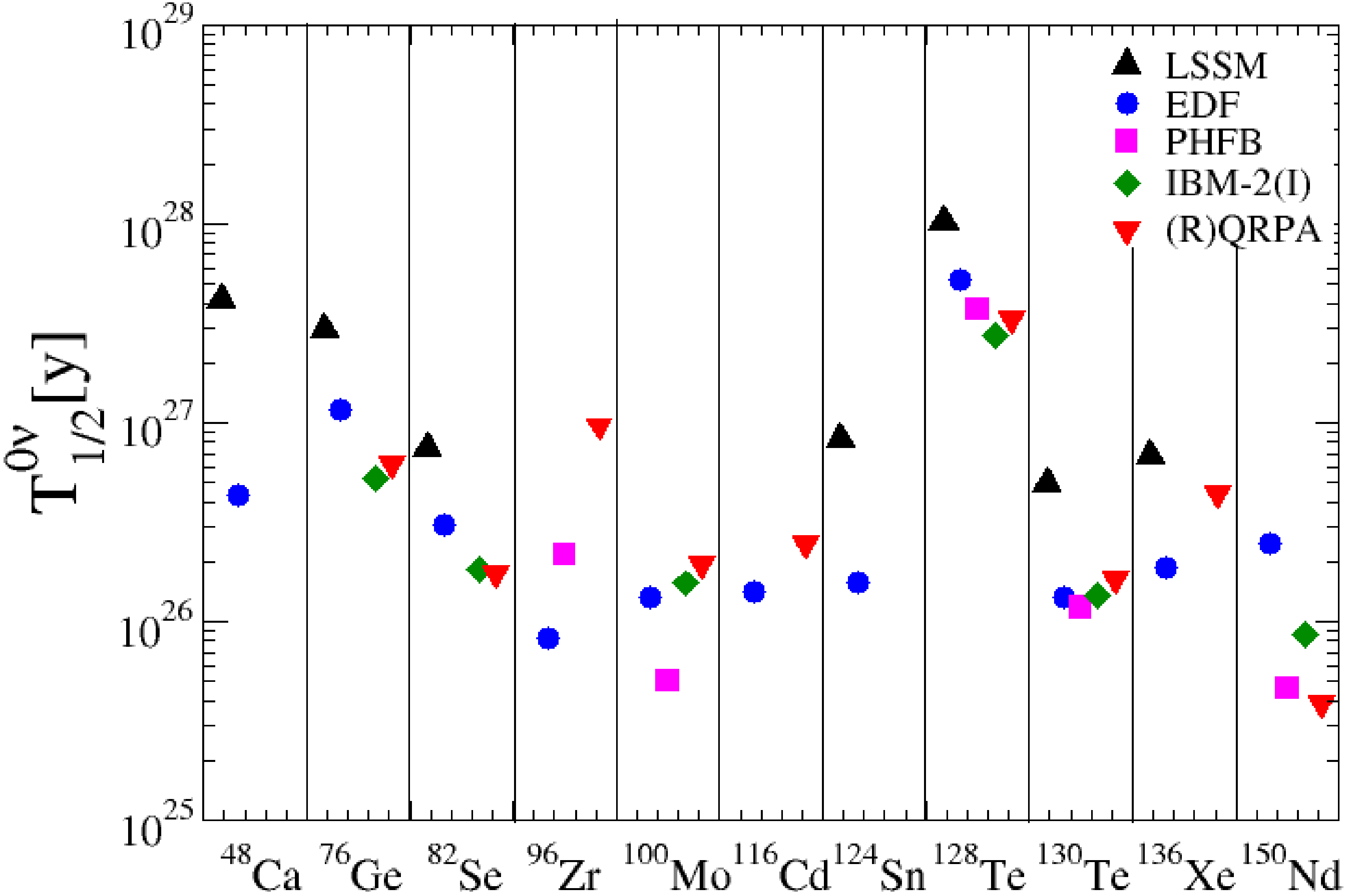
"normal"



"inverted"

0νββ-decay half-life

$m_{\beta\beta}$=50 meV



Can we measure CP phases in the $0\nu\beta\beta$-decay?

Majorana phases

$$P = diag(e^{-i\alpha_1/2}, e^{-i\alpha_2/2}, e^{-i\alpha_3/2})$$
$$\alpha_3/2 = \delta$$

$$|m_{\beta\beta}| = |c_{12}^2 c_{13}^2 e^{i\alpha_1} m_1 + s_{12}^2 c_{13}^2 e^{i\alpha_2} m_2 + s_{13}^2 m_3|$$

Measured quantity

$$\begin{aligned} |m_{\beta\beta}|^2 &= c_{12}^4 c_{13}^4 m_1^2 + s_{12}^4 c_{13}^4 m_2^2 + s_{13}^4 m_3^2 \\ &+ 2c_{12}^2 s_{12}^2 c_{13}^4 m_1 m_2 \cos(\alpha_1 - \alpha_2) \\ &+ 2c_{12}^2 c_{13}^2 s_{13}^2 m_1 m_3 \cos\alpha_1 + 2s_{12}^2 c_{13}^2 s_{13}^2 m_2 m_3 \cos\alpha_2 . \end{aligned}$$

Normal hierarchy

$$\begin{aligned} m_1 &\ll \sqrt{\Delta m^2_{\mathrm{SUN}}} \\ m_2 &\simeq \sqrt{\Delta m^2_{\mathrm{SUN}}} \\ m_3 &\simeq \sqrt{\Delta m^2_{\mathrm{ATM}}} \end{aligned}$$

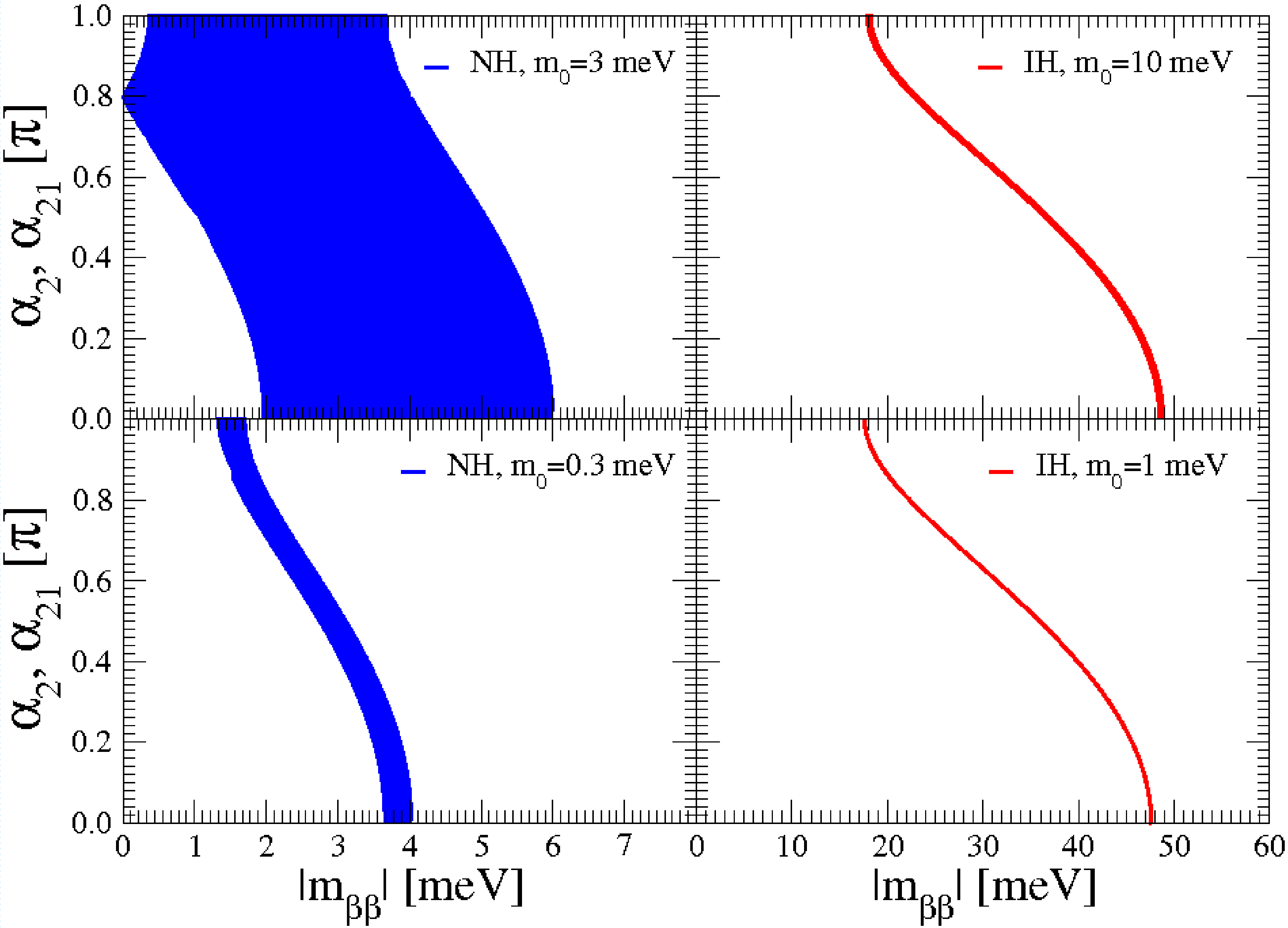
Inverted hierarchy

$$\begin{aligned} m_3 &\ll \sqrt{\Delta m^2_{\mathrm{ATM}}} \\ m_1 &\simeq m_2 \simeq \sqrt{\Delta m^2_{\mathrm{ATM}}} \end{aligned}$$

Assuming lightest neutrino mass to be zero

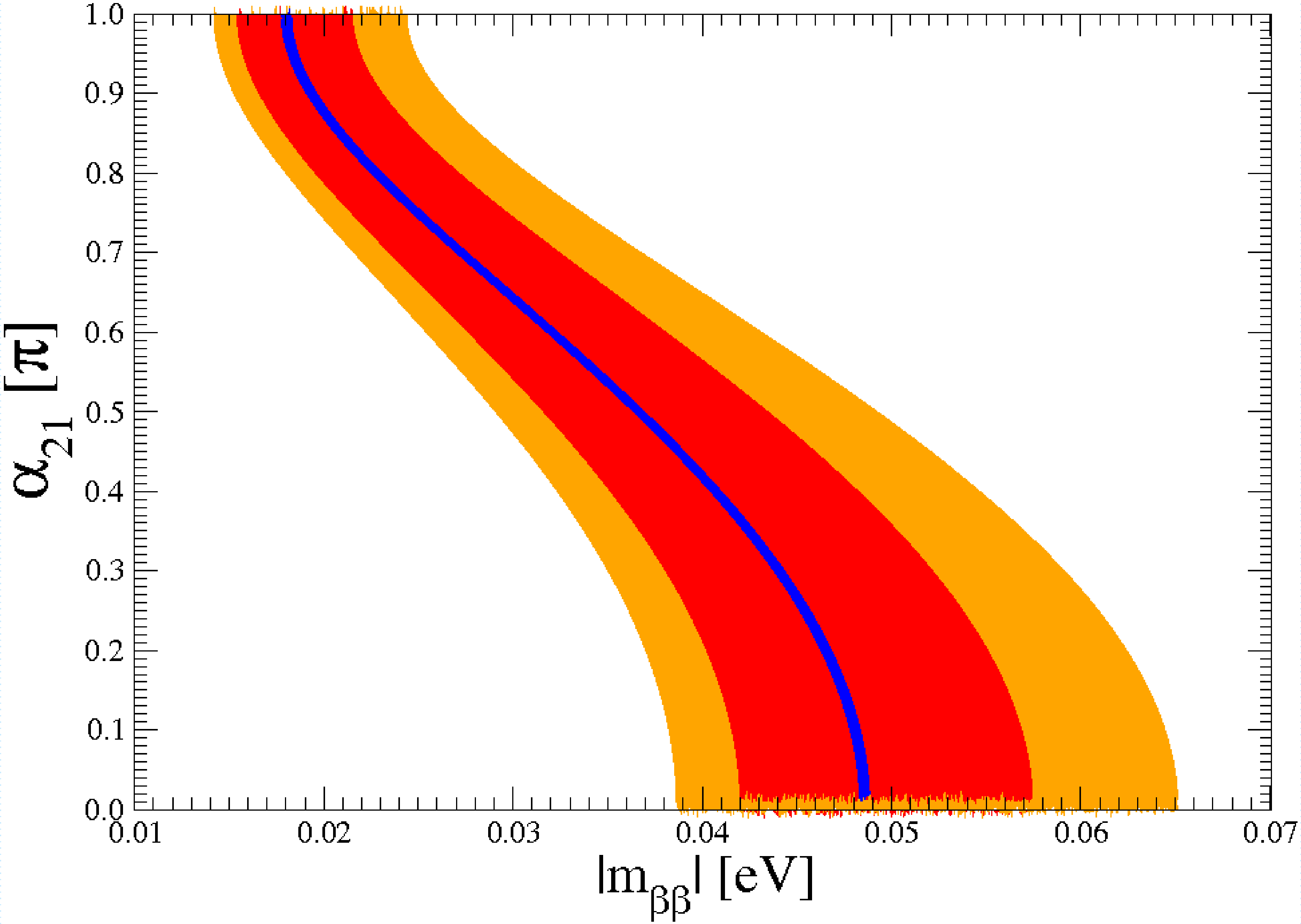
$$\cos\alpha_2 \simeq \frac{|m_{\beta\beta}|^2 - s_{12}^4 c_{13}^4 \Delta m^2_{\mathrm{SUN}} - s_{13}^4 \Delta m^2_{\mathrm{ATM}}}{2 s_{12}^2 c_{13}^2 s_{13}^2 \sqrt{\Delta m^2_{\mathrm{SUN}} \Delta m^2_{\mathrm{ATM}}}}$$

$$\cos\alpha_{12} = \frac{|m_{\beta\beta}|^2 - c_{13}^4 (1 - 2 s_{12}^2 c_{12}^2) \Delta m^2_{\mathrm{ATM}}}{2 c_{12}^2 s_{12}^2 c_{13}^4 \Delta m^2_{\mathrm{ATM}}}$$


NH, m_0=3 meV
IH, m_0=10 meV
NH, m_0=0.3 meV
IH, m_0=1 meV
α_2, α_21 [π]
α_2, α_21 [π]
|m_ββ| [meV]
|m_ββ| [meV]
0.0
0.2
0.4
0.6
0.8
1.0
0 1 2 3 4 5 6 7
0 10 20 30 40 50 60

$$|m_{\beta\beta}| = \frac{1}{\sqrt{T^{0\nu}_{1/2} G^{0\nu}(Q_{\beta\beta}, Z)} |M'^{0\nu}|}$$

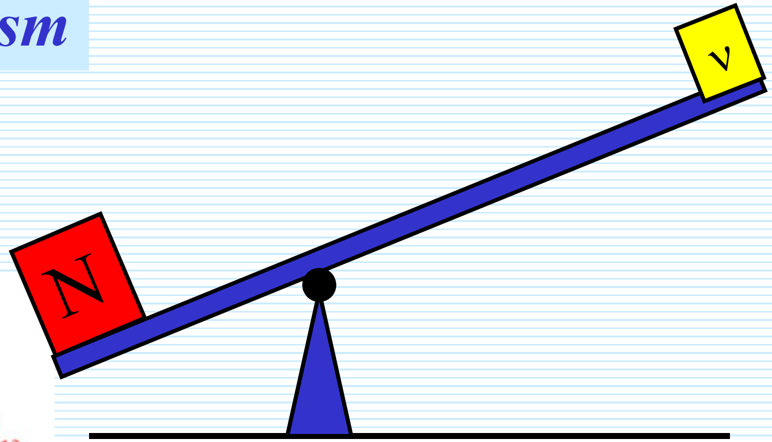
$$\frac{\sigma_{\beta\beta}}{|m_{\beta\beta}|^{obs}} = \sqrt{\frac{1}{4}\left(\frac{\sigma_{exp}}{T^{0\nu-obs}_{1/2}}\right)^2 + \left(\frac{\sigma_{th}}{|M'^{0\nu}|}\right)^2}$$



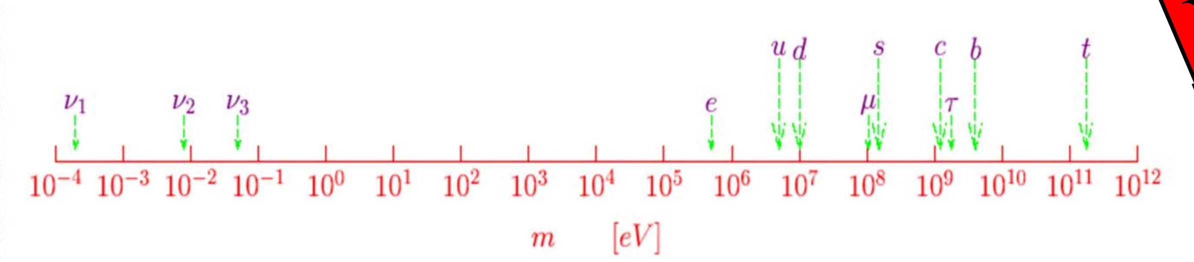
Sterile neutrinos

Assumption $M_R \gg m_D$

See-Saw mechanism



$$\begin{pmatrix} \overline{\nu}_L & \overline{(\nu_R)^c} \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix}$$



Left-right symmetric models SO(10)

$$\nu_{eL} = \sum_{i=1}^{light} \underbrace{U_{ei}}_{\text{large}} \chi_{iL} + \sum_{i=1}^{heavy} \underbrace{U_{ei}}_{\text{small}} N_{iL}$$

$$(\nu_{eR})^c = \sum_{i=1}^{light} \underbrace{V_{ei}}_{\text{small}} \chi_{iL} + \sum_{i=1}^{heavy} \underbrace{V_{ei}}_{\text{large}} N_{iL}$$

Probability of Neutrino Oscillations

As N increases, the formalism gets rapidly more complicated!

N	Δm_{ij}^2	θ_{ij}	CP
2	1	1	0
3	2	3	1
6	5	15	10

Physical Parameters in N x N Mixing Matrix

2 (N (N-1)/2) +N = N²

N x N Unitary Mixing Matrix => N^2 parameters

$$\begin{cases} \frac{N(N-1)}{2} & \text{Mixing Angles} \\ \\ \frac{N(N+1)}{2} & \text{Phases} \end{cases}$$

Weak Charged Current: $j_\rho^{CC\dagger} = 2\sum_\alpha \overline{l_{\alpha L}}\gamma_\rho \nu_{\alpha L} = 2\sum_{\alpha,k} \overline{l_{\alpha L}}\gamma_\rho U_{\alpha k}\nu_{kL}$

Lagrangian is invariant under global phase transformations of Dirac fields

$$\left.\begin{array}{l} l_\alpha \to e^{i\theta_\alpha} l_\alpha \\ \\ \nu_k \to e^{i\Phi_k}\nu_k \end{array}\right\} \Longrightarrow \begin{cases} j_\rho^{CC\dagger} \to 2\sum_{\alpha,k} \overline{l_{\alpha L}} e^{-i\theta_\alpha}\gamma_\rho U_{\alpha k} e^{i\Phi_k}\nu_{kL} \\ \qquad = 2\sum_{\alpha,k} \overline{l_{\alpha L}} \underbrace{e^{-i(\theta_e-\Phi_1)}}_{1} \underbrace{e^{-i(\theta_\alpha-\theta_e)}}_{N-1} \gamma_\rho U_{\alpha k} \underbrace{e^{i(\Phi_k-\Phi_1)}}_{N-1} \nu_{kL} \end{cases}$$

Number of independent phases that can be eliminated:2N-1 (not 2N!)

Number of physical phases: $\frac{N(N+1)}{2} - (2N-1) = \frac{(N-1)(N-2)}{2}$

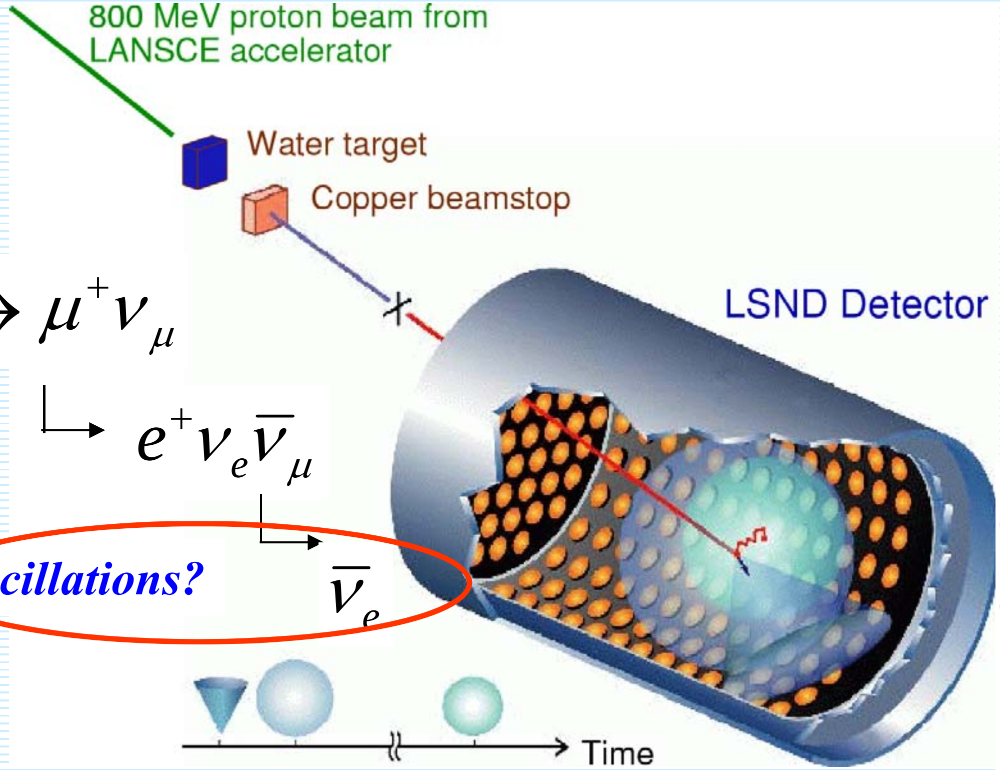
Remains global phase freedom of lepton fields => conservation of L

The LSND Experiment: Evidence sterile ν (3.8 σ)

LSND took data from 1993-98

- L = 30m and $20 < E_\nu < 53$ MeV

Saw an excess of ν_e : 87.9 ± 22.4 ± 6.0 events.



$$\pi^+ \rightarrow \mu^+ \nu_\mu$$

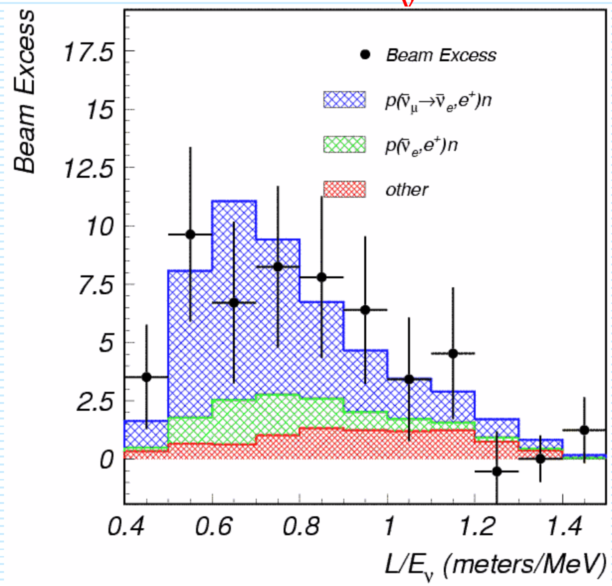
$$\hookrightarrow e^+ \nu_e \bar{\nu}_\mu$$

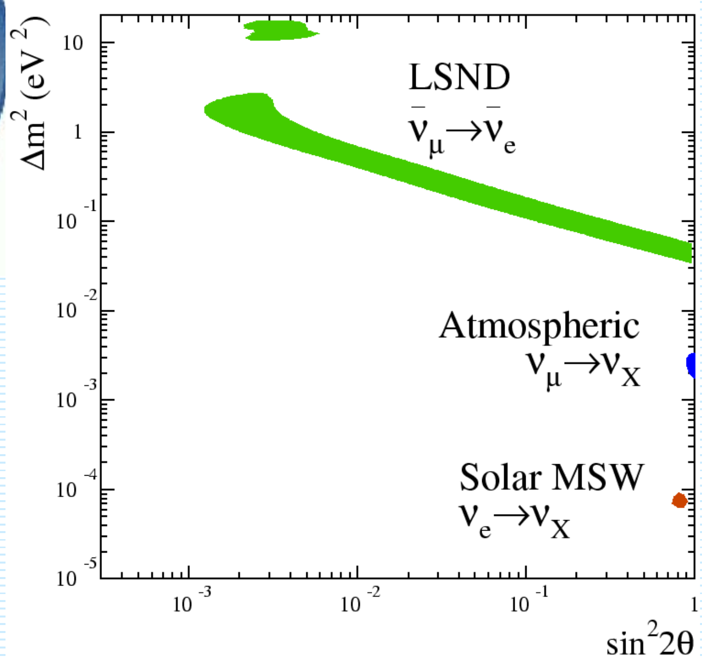
Oscillations? $\hookrightarrow \bar{\nu}_e$

$\bar{\nu}_e$

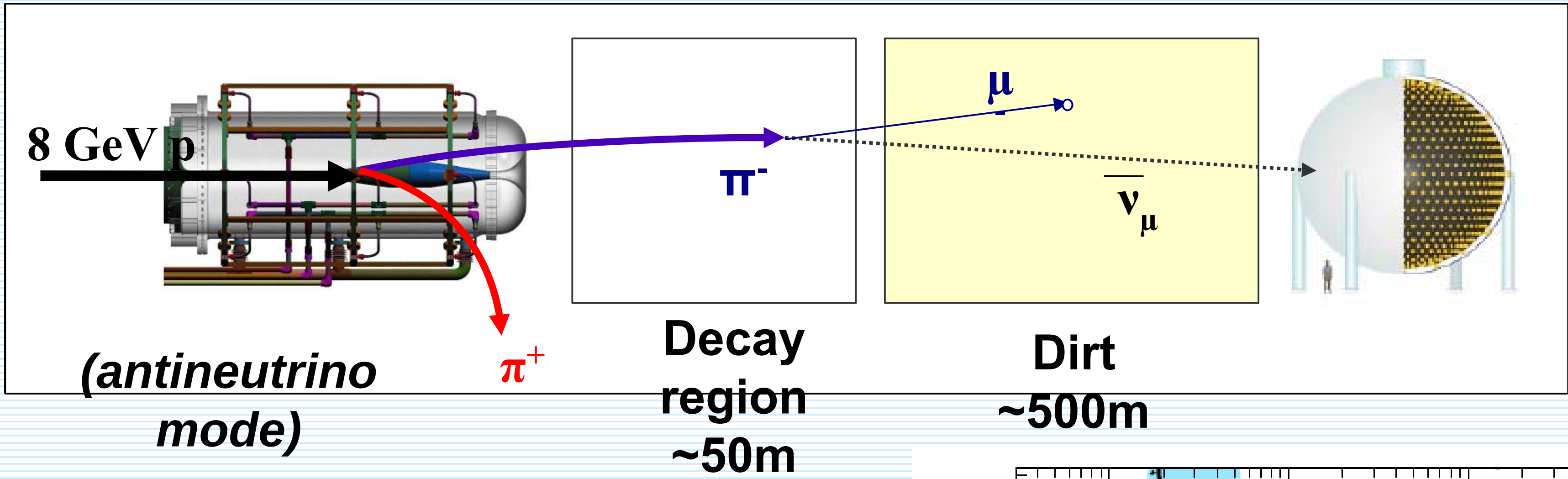
$\hookrightarrow$ **Signal: $p \rightarrow e^+ n$**

$n\,p \rightarrow d\,\gamma$(2.2MeV)

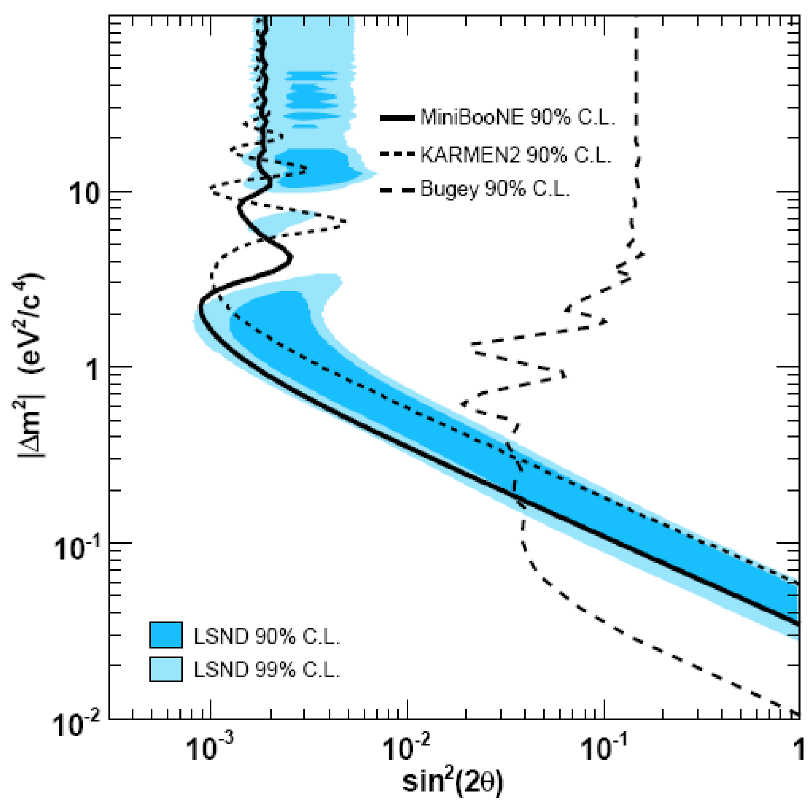




MiniBooNE was designed to test the LSND signal



- Similar **L/E** as LSND
  - MiniBooNE ~500m/~500MeV
  - LSND ~30m/~30MeV
- Horn focused neutrino beam (p+Be)
  - Horn polarity → neutrino or anti-neutrino mode
- 800 tons mineral oil Cherenkov detector
- Detector running since early 2003



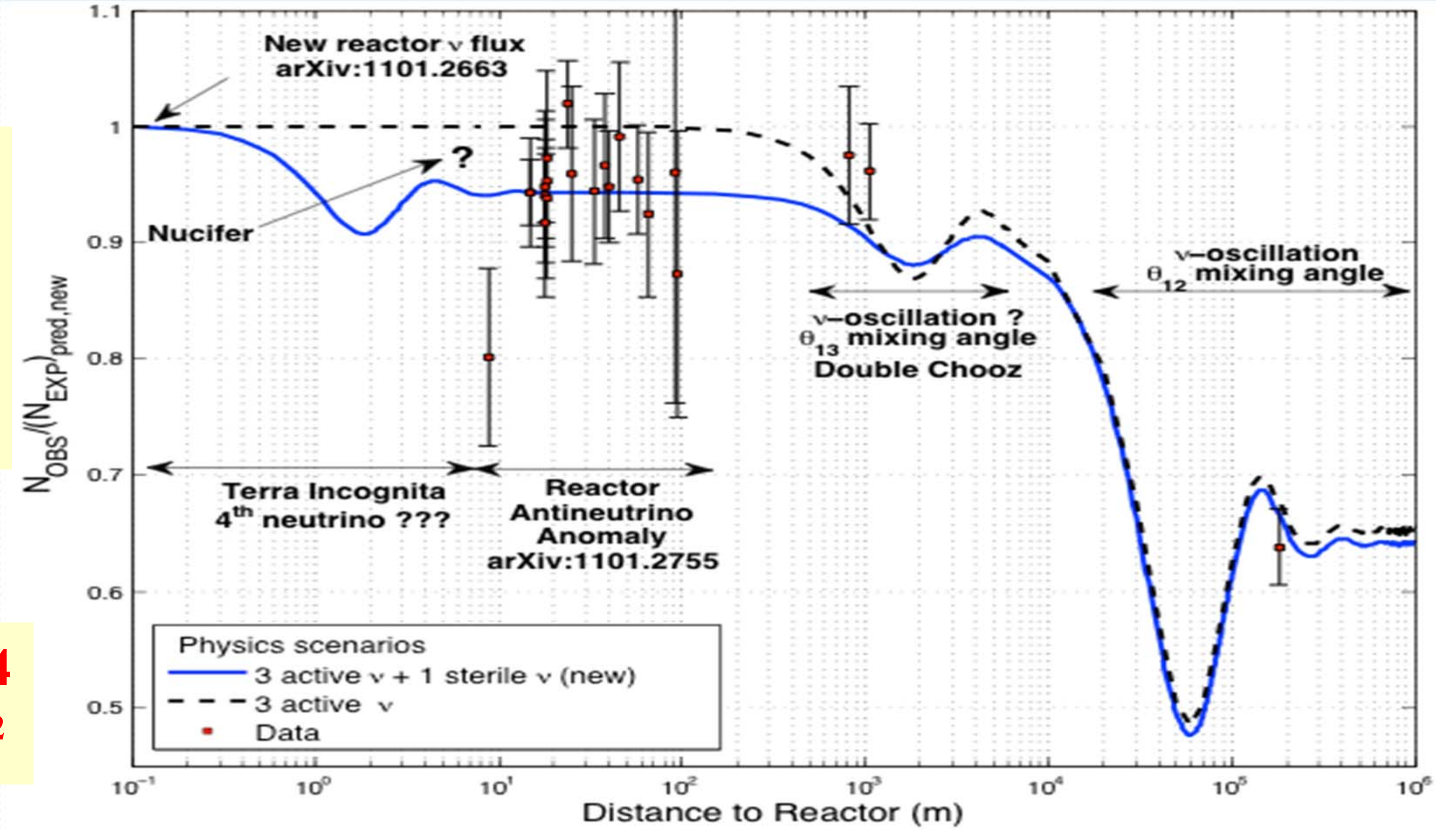
Excess of events observed at lower energy:
128.8 ± 20.4 ± 38.3 (3.0σ)

Reactor neutrinos anomaly (January 2011)

Double Chooz re-evaluated reactor antineutrino flux (PRD 83, 073006 (2011))

- previous procedure used a phenomenological model based **30** effective beta branches
- new analysis used detailed knowledge of the decays of **10,000 +** fission products

New calculations results in flux increase of 3.5% in reactor antineutrinos



$\sin^2(2\theta_{new}) \sim 0.14$

$\Delta m_{new}^2 > 1.5\ eV^2$

$$P_{ee} \approx 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m_{31}^2 L}{4E_\nu} + \left(\frac{\Delta m_{21}^2 L}{4E_\nu}\right)^2 \cos^4\theta_{13} \sin^2 2\theta_{12}$$

(3+1) mixing

6 angles
3+3 =6 phases

$$
\begin{aligned}
U &= R_{34}\tilde{R}_{24}\tilde{R}_{14} \\
&\quad R_{23}\tilde{R}_{13} \\
&\quad R_{12}P \\
P &= diag\left(e^{i\alpha_1/2}, e^{i\alpha_2/2}, e^{i(\alpha_3/2+\delta_{13})}, e^{i\delta_{14}}\right)
\end{aligned}
$$

$$
\begin{aligned}
m_{\beta\beta}^{(3+1)} = \quad & c_{12}^2 c_{13}^2 c_{14}^2 e^{i\alpha_1} m_1 \\
& + c_{13}^2 c_{14}^2 s_{12}^2 e^{i\alpha_2} m_2 \\
& + c_{14}^2 s_{13}^2 e^{i\alpha_3} m_3 \\
& + s_{14}^2 m_4
\end{aligned}
$$

4 masses
3 angles
3 phases

(3+2) mixing

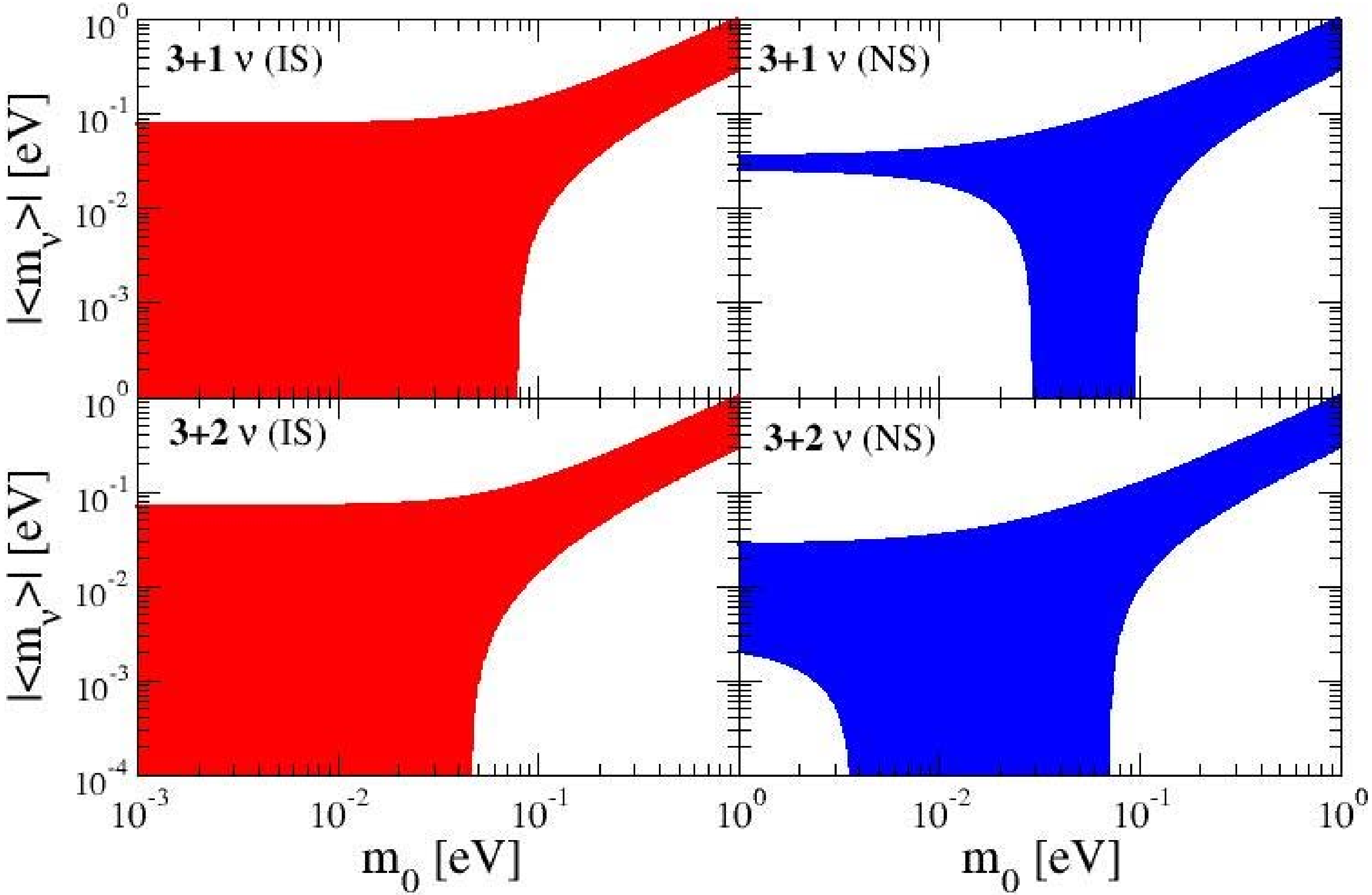
10 angles
6+4 =10 phases

$$
\begin{aligned}
U &= R_{45} \\
&\quad \tilde{R}_{35} R_{34} \\
&\quad \tilde{R}_{25}\tilde{R}_{24} R_{23} \\
&\quad \tilde{R}_{15}\tilde{R}_{14}\tilde{R}_{13} R_{12} P \\
P &= diag\left(e^{i\alpha_1/2}, e^{i\alpha_2/2}, e^{i(\alpha_3/2+\delta_{13})}, e^{i(\alpha_4/2+\delta_{14})}, e^{i\delta_{15}}\right)
\end{aligned}
$$

$$
\begin{aligned}
m_{\beta\beta}^{3+2} = \quad & c_{12}^2 c_{13}^2 c_{14}^2 c_{15}^2 e^{i\alpha_1} m_1 \\
& + c_{13}^2 c_{14}^2 c_{15}^2 s_{12}^2 e^{i\alpha_2} m_2 \\
& + c_{14}^2 c_{15}^2 s_{13}^2 e^{i\alpha_3} m_3 \\
& + c_{15}^2 s_{14}^2 e^{i\alpha_4} m_4 \\
& + s_{15}^2 m_5
\end{aligned}
$$

5 masses
4 angles
4 phases

$$\Delta m^2_{41} = 1.78\ eV^2, \quad U_{e4} = 0.151$$
$$\Delta m^2_{51} = 0.89\ eV^2, \quad U_{e5} = 0.124$$

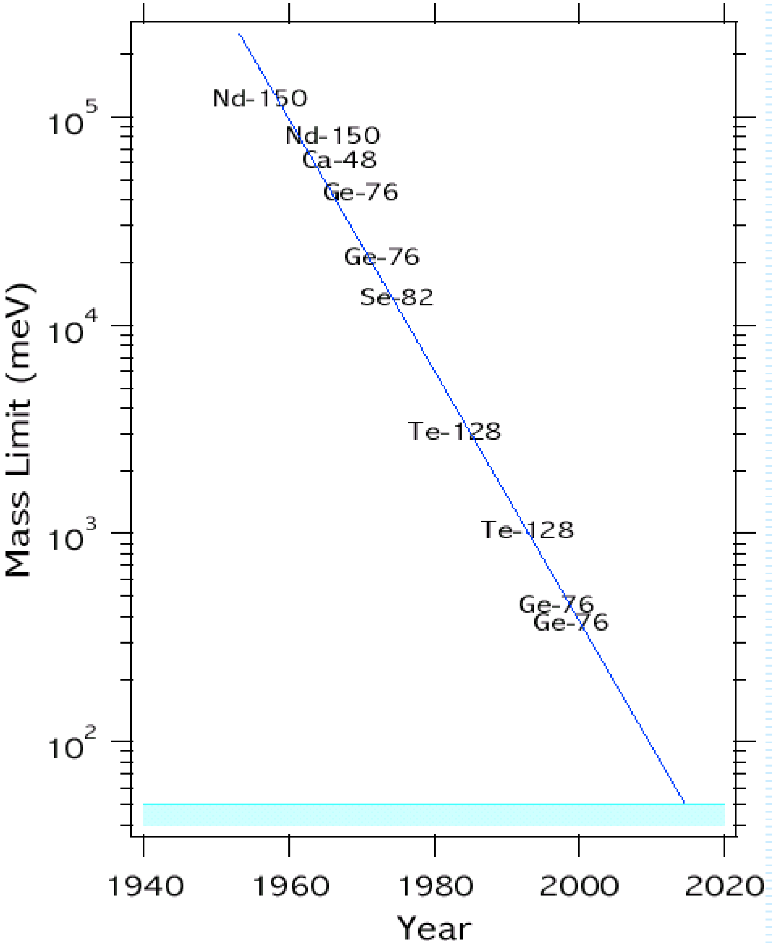


$0\nu\beta\beta$-decay NMEs

If (or when) the $0\nu\beta\beta$ decay is observed two theoretical problems must be resolved

S.R. Elliott, P. Vogel,
Ann.Rev.Nucl.Part.Sci. 52, 115 (2002)

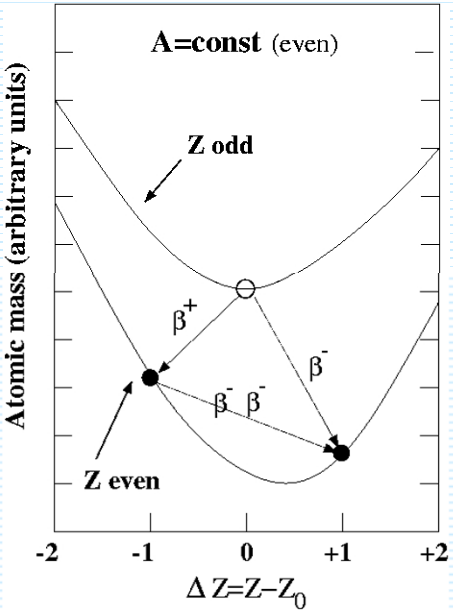
1) *How to relate the observed decay rate to the fundamental parameters, i.e., what is the value of the corresponding* ***nuclear matrix elements.***
2) *What is the* ***mechanism of the decay****, i.e., what kind of virtual particle is exchanged between the affected nucleons (quarks).*



Frank Avignone:

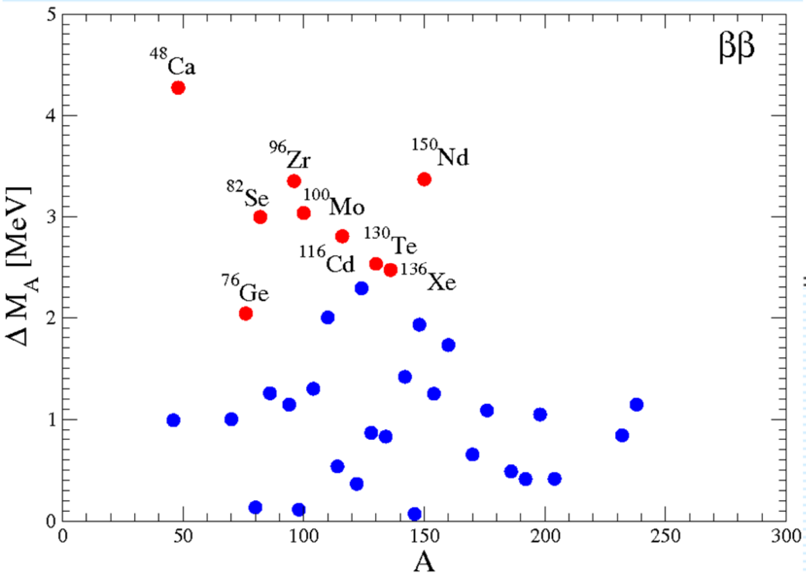
Nuclear Matrix Elements
are
as important as DATA

The double beta decay process can be observed due to nuclear pairing interaction that favors energetically the even-even nuclei over the odd-odd nuclei



$$\frac{1}{T^{0\nu}_{1/2}} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 G^{01}(E_0,Z)\left|M^{0\nu}\right|^2$$

transition	$G^{01}(E_0,Z)$ $\times 10^{14}y$	$Q_{\beta\beta}$ [MeV]	Abund. (%)	$\lvert M^{0\nu}\rvert^2$
$^{150}Nd \to {}^{150}Sm$	26.9	3.667	6	?
$^{48}Ca \to {}^{48}Ti$	8.04	4.271	0.2	?
$^{96}Zr \to {}^{96}Mo$	7.37	3.350	3	?
$^{116}Cd \to {}^{116}Sn$	6.24	2.802	7	?
$^{136}Xe \to {}^{136}Ba$	5.92	2.479	9	?
$^{100}Mo \to {}^{100}Ru$	5.74	3.034	10	?
$^{130}Te \to {}^{130}Xe$	5.55	2.533	34	?
$^{82}Se \to {}^{82}Kr$	3.53	2.995	9	?
$^{76}Ge \to {}^{76}Se$	0.79	2.040	8	?



The NMEs for 0νββ-decay must be evaluated using tools of nuclear theory

The $0\nu\beta\beta$-decay: A nuclear physics problem

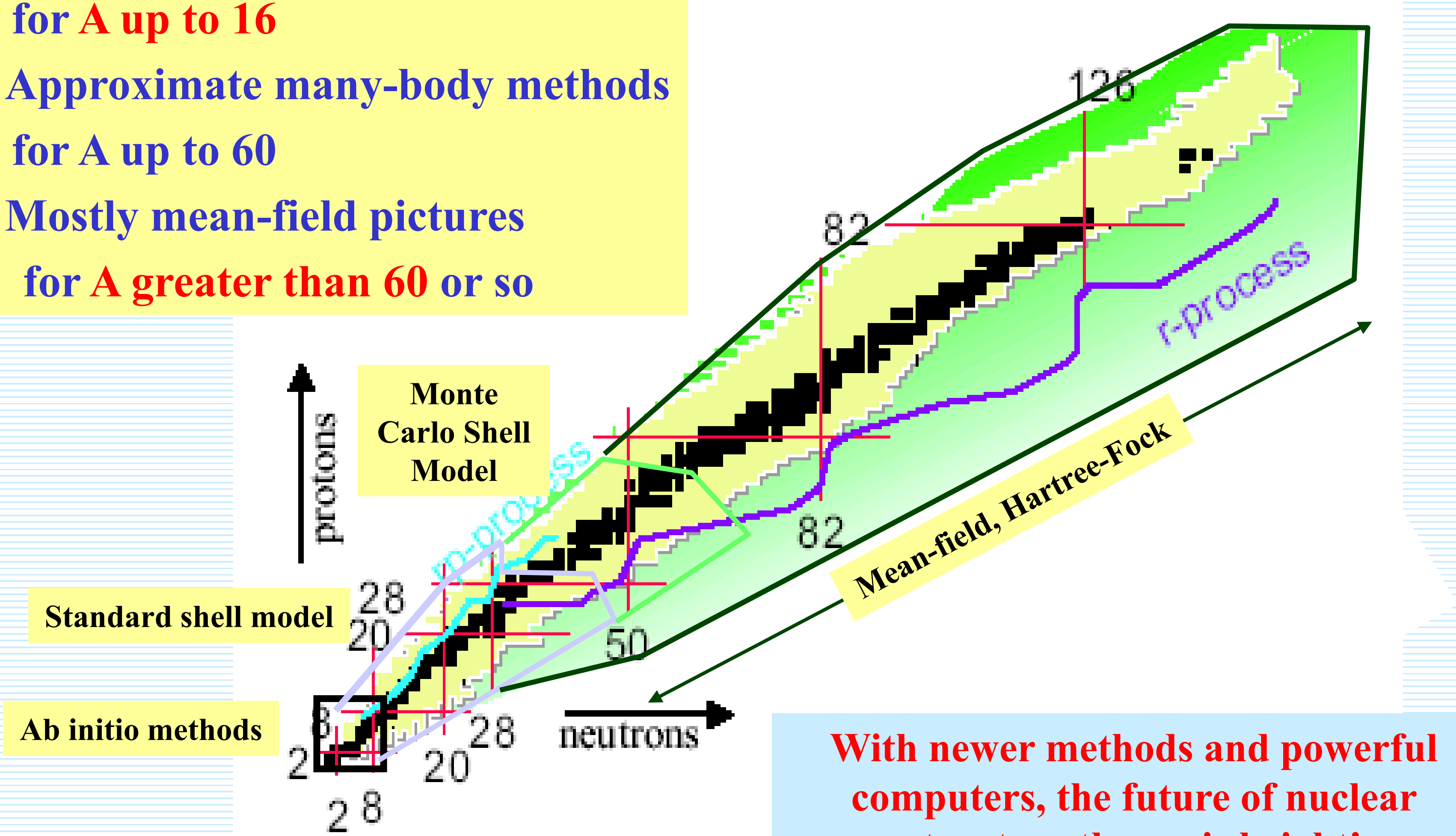
In double beta decay two neutrons bound in the ground state of an initial even-even nucleus are simultaneously transformed into two protons that are bound in the ground state or excited (0^+, 2^+) states of the final nucleus

It is necessary to evaluate, with a sufficient accuracy, wave functions of both nuclei, and evaluate the matrix element of the $0\nu\beta\beta$-decay operator connecting them

This can not be done exactly, some approximation and/or truncation is always needed. Moreover, there is no other analogues observable that can be used to judge directly the quality of the result.

Nuclear Structure

- Exact methods exist up to A=4
- Computationally exact methods for A up to 16
- Approximate many-body methods for A up to 60
- Mostly mean-field pictures for A greater than 60 or so



With newer methods and powerful computers, the future of nuclear structure theory is bright!

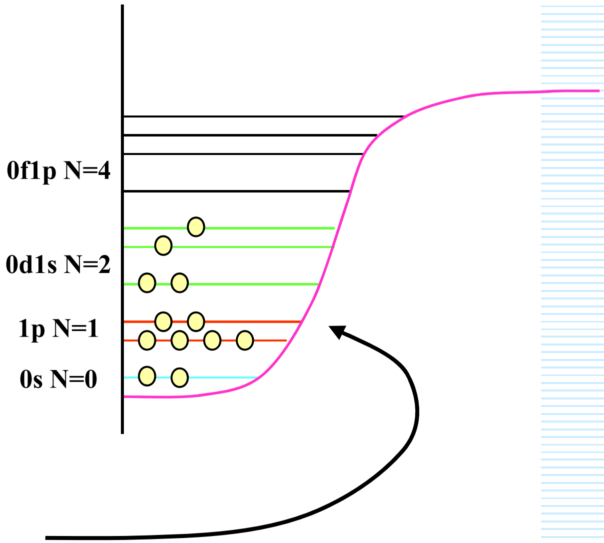
Many-body Hamiltonian

- **Start with the many-body Hamiltonian**

$$H = \sum_i \frac{\vec{p}_i^{\,2}}{2m} + \sum_{i<j} V_{NN}\left(\vec{r}_i - \vec{r}_j\right)$$

- **Introduce a mean-field *U* to yield basis**

$$H = \sum_i \left(\frac{\vec{p}_i^{\,2}}{2m} + U(r_i) \right) + \underbrace{\sum_{i<j} V_{NN}\left(\vec{r}_i - \vec{r}_j\right) - \sum_i U(r_i)}_{\text{Residual interaction}}$$



The success of any nuclear structure calculation depends on the choice of the mean-field basis and the residual interaction!

- **The mean field determines the shell structure**
- **In effect, nuclear-structure calculations rely on perturbation theory**

The 0νββ-decay NME (light ν exchange mech.)

The 0νββ-decay half-life

$$\frac{1}{T_{1/2}} = G^{0\nu}(E_0, Z)|M'^{0\nu}|^2|\langle m_{\beta\beta}\rangle|^2 ,$$

NME= sum of Fermi, Gamow-Teller and tensor contributions

$$M'^{0\nu} = \left(\frac{g_A}{1.25}\right)^2 \langle f| - \frac{M_F^{0\nu}}{g_A^2} + M_{GT}^{0\nu} + M_T^{0\nu}|i\rangle$$

Neutrino potential (about 1/r_{12})

$$H_K(r_{12}) = \frac{2}{\pi g_A^2} R \int_0^\infty f_K(qr_{12}) \frac{h_K(q^2) q dq}{q + E^m - (E_i + E_f)/2}$$

$$f_{F,GT}(qr_{12}) = j_0(qr_{12}), \qquad f_T(qr_{12}) = -j_2(qr_{12})$$

Form-factors: finite nucleon size

$$\begin{aligned} h_F &= g_V^2(q^2) \\ h_{GT} &= g_A^2 \left[1 - \frac{2}{3}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2} + \frac{1}{3}\left(\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\right)^2\right] \\ h_T &= g_A^2 \left[\frac{2}{3}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2} - \frac{1}{3}\left(\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\right)^2\right] \end{aligned}$$

Induced pseudoscalar coupling (pion exchange)

$$M_{K=F,GT,T} = \sum_{J^\pi, k_i, k_f, \mathcal{J}} \sum_{pnp'n'} (-1)^{j_n + j_{p'} + J + \mathcal{J}} \sqrt{2\mathcal{J}+1} \begin{Bmatrix} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{Bmatrix}$$
$$\langle p(1), p'(2); \mathcal{J} \parallel f(r_{12}) O_K f(r_{12}) \parallel n(1), n'(2); \mathcal{J}\rangle$$
$$\times \langle 0_f^+ || [c_{p'}^+ \tilde{c}_{n'}]_J || J^\pi k_f\rangle \langle J^\pi k_f | J^\pi k_i\rangle \langle J^\pi k_f || [c_p^+ \tilde{c}_n]_J || 0_i^+\rangle$$

Jastrow f. s.r.c.

J^π = $0^+, 1^+, 2^+ \ldots$ $0^-, 1^-, 2^- \ldots$

One two-body operators

$$\langle p|O(1)|n\rangle\langle p'|O(2)|n'\rangle = \langle p, p'|O'(1,2)|n, n'\rangle$$

Integration over angular part of ν momentum

$$\int e^{i\vec{q}\cdot(\vec{r}_1-\vec{r}_2)}d\Omega_q = \int e^{i\vec{q}\cdot\vec{r}}d\Omega_q =$$

$$\sqrt{4\pi}\ 4\pi \sum_{lm} i^l j_l(qr) Y_{lm}(\Omega_r) \int Y^*_{lm}(\Omega_q) Y_{00}(\Omega_q) d\Omega_q = 4\pi j_0(qr)$$

Neutrino potential

$$\begin{aligned} O_F(r_{12}, E^k_{J^\pi}) &= \tau^+(1)\tau^+(2) H_F(r_{12}, E^k_{J^\pi}) \,, \\ O_{GT}(r_{12}, E^k_{J^\pi}) &= \tau^+(1)\tau^+(2) H_{GT}(r_{12}, E^k_{J^\pi}) \sigma_{12} \,, \\ O_T(r_{12}, E^k_{J^\pi}) &= \tau^+(1)\tau^+(2) H_T(r_{12}, E^k_{J^\pi}) S_{12} \end{aligned}$$

$$H_K(r_{12}, E^k_{J^\pi}) = \frac{2}{\pi g_A^2} R \int_0^\infty f_K(qr_{12}) \frac{h_K(q^2) q dq}{q + E^k_{J^\pi} - (E_i + E_f)/2}$$

$$with \quad f_{F,GT}(qr_{12}) = j_0(qr_{12}), \quad f_T(qr_{12}) = -j_2(qr_{12})$$

$$\begin{aligned} \sigma_{12} &= \vec{\sigma}_1 \cdot \vec{\sigma}_2 \,, \\ S_{12} &= 3(\vec{\sigma}_1 \cdot \hat{r}_{12})(\vec{\sigma}_2 \cdot \hat{r}_{12}) - \sigma_{12} \end{aligned}$$

Calculation of two-body matrix elements

From j-j to LS coupling

$$\mathcal{M}^{2body} = \langle a(1), b(2); J'|O(1,2)|c(1)d(2); J'\rangle$$

$$|n_c l_c j_c, n_d l_d j_d; J'M'\rangle = \sum_{SL} \hat{S}^2 \hat{L}^2 \hat{j}_c \hat{j}_d \begin{Bmatrix} 1/2 & l_c & j_c \\ 1/2 & l_d & j_d \\ S & L & J' \end{Bmatrix} |n_c l_c, n_d l_d, SL; J'M'\rangle$$

Moshinsky transformation to relative coordinates

$$|n_c l_c n_d l_d; LM_L\rangle = \sum_{\substack{nl \\ \mathcal{N}\mathcal{L}}} \langle nl, \mathcal{N}\mathcal{L}, L|n_c l_c, n_d l_d, L\rangle |nl, \mathcal{N}\mathcal{L}; LM_L\rangle$$

Two-body m.e.

$$\begin{aligned} \mathcal{M}^{2body}_{F,GT} &= \hat{J}' \sum_{SL} \hat{S}\hat{L}\hat{j}_a\hat{j}_b\hat{j}_c\hat{j}_d \begin{Bmatrix} 1/2 & l_c & j_c \\ 1/2 & l_d & j_d \\ S & L & J' \end{Bmatrix} \begin{Bmatrix} 1/2 & l_a & j_a \\ 1/2 & l_b & j_b \\ S & L & J' \end{Bmatrix} \\ &\times \sum_{\substack{nl \\ \mathcal{N}\mathcal{L}}} \sum_{\substack{n'l' \\ \mathcal{N}'\mathcal{L}'}} \langle nl, \mathcal{N}\mathcal{L}, L|n_c l_c, n_d l_d, L\rangle \langle n'l', \mathcal{N}'\mathcal{L}', L|n_a l_a, n_b l_b, L\rangle \\ &\times \langle n'l', \mathcal{N}'\mathcal{L}'; L||j_0(q|\vec{r}_{i,j})|||nl, \mathcal{N}\mathcal{L}; L\rangle \langle s_a s_b; S|| \begin{pmatrix} 1 \\ \vec{\sigma}_1 \cdot \vec{\sigma}_2 \end{pmatrix} ||s_c s_d; S\rangle \end{aligned}$$

$$\langle n'l', \mathcal{N}'\mathcal{L}'; L||j_0(q|\vec{r}_{i,j}|)||nl, \mathcal{N}\mathcal{L}; L\rangle = \delta_{ll'}\delta_{\mathcal{N}\mathcal{N}'}\delta_{\mathcal{L}\mathcal{L}'} \langle n'l|j_0(q|\vec{r}_{i,j}|)|nl\rangle$$

$$\begin{aligned} \langle s_a s_b; S||\vec{\sigma}_1 \cdot \vec{\sigma}_2||s_c s_d; S\rangle &= \hat{S}(\delta_{S1} - 3\delta_{S0}), \\ \langle s_a s_b; S||1||s_c s_d; S\rangle &= \hat{S}(\delta_{S1} + \delta_{S0}) \end{aligned}$$

Nuclear structure approaches

(Renormalized) QRPA: Šimkovic, Faessler, Müther, Rodin, Stauf, PRC 79, 055501 (2009).

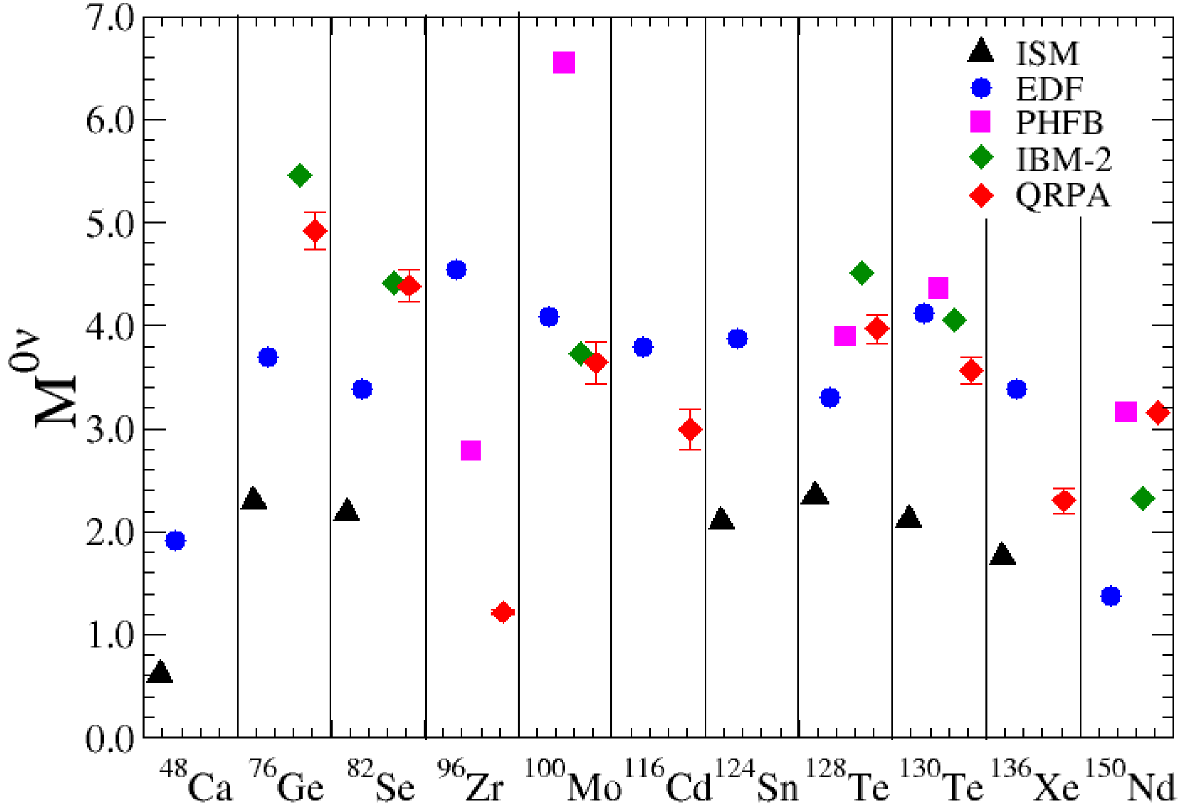
Interacting Boson Model: Barea, Iachello, PRC 79, 044301 (2009).

Large Scale Shell Model: Caurier, Menendez, Nowacki, Poves, PRL 100, 052503 (2008).

Projected Hartree-Fock-Bogoliubov: Rath, Chandra, et al. PRC 82, 064310 (2010).

Energy Dendity Functional appr.: Rodrígez, Martínez-Pinedo, arXiv:1008.5260 [nucl-th].

g_A=1.25, Jastrow s.r.c., r_0=1.20 fm



The 0νββ-decay NMEs (Status:2011)

Jyvaskula –La Plata QRPA NMEs are in good a agreement with Tuebingen-Bratislava-CALTECH NMEs

QRPA: Kortelainen, Suhonen, PRC 76, 024315 (2007)

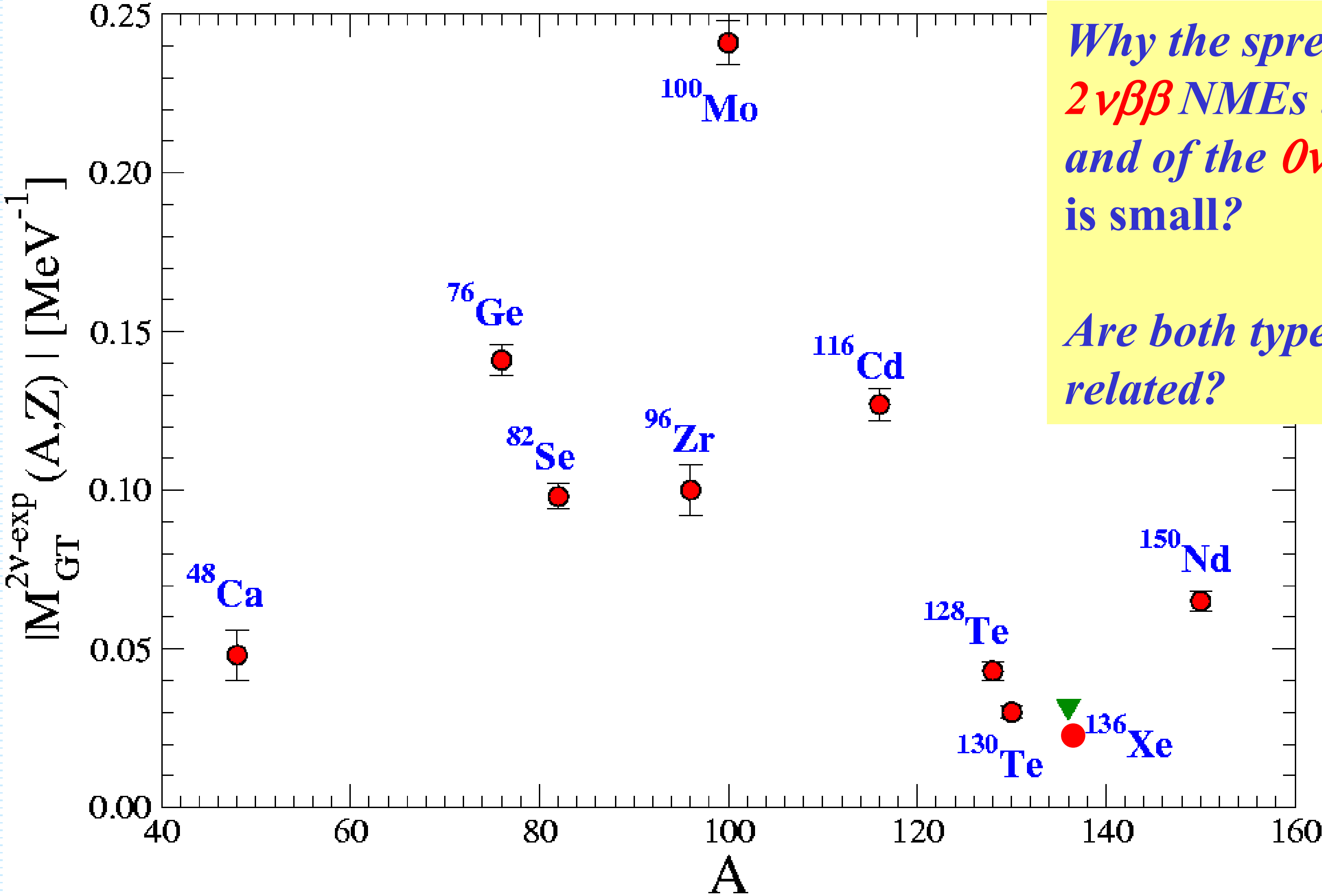
On the relation between 0νββ-decay and 2νββ-decay (GT) NMEs

F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

$$M^{0\nu} = M^{0\nu}_{GT}\left(1 + \frac{1}{g_A^2}\frac{M^{0\nu}_F}{M^{0\nu}_{GT}} + \frac{M^{0\nu}_T}{M^{0\nu}_{GT}}\right)$$

2νββ-decay NMEs

$$\frac{1}{T_{1/2}^{2\nu-exp}} = G^{2\nu}(E_0, Z)\ g_A^4\ |M_{GT}^{2\nu}|^2$$



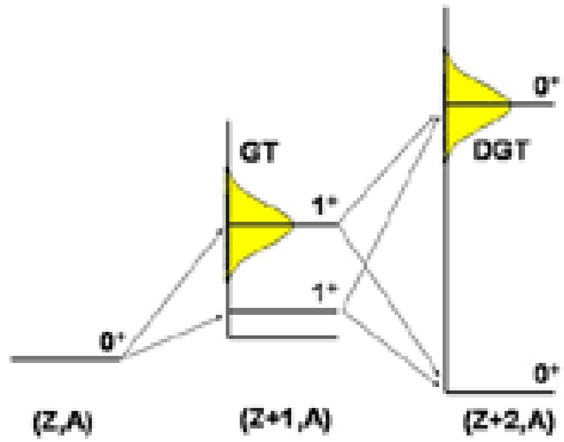
***Why the spread of the 2νββ NMEs is large and of the 0νββ* NMEs is small?**

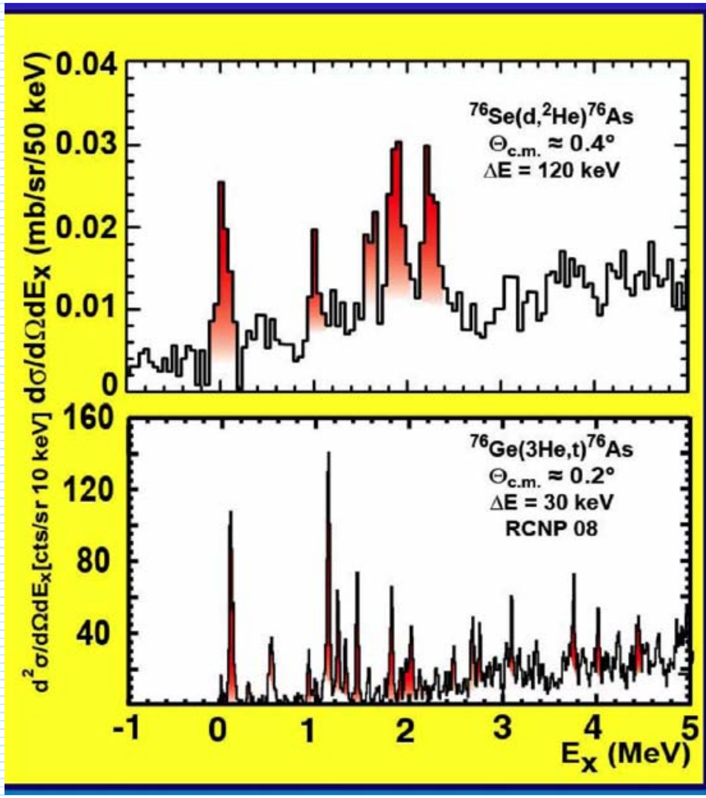
Are both type of NMEs related?

Differencies among 2νββ-decay NMEs: up to factor 10

The cross sections of **(t,^{3}He)** *and* **(d,^{2}He)** *reactions give* **B(GT$^\pm$)** *for β^+ and β^-, product of the amplitudes* **(B(GT)$^{1/2}$)** *entering the numerator of* **$M^{2\nu}_{GT}$**

$$M^{2\nu}_{GT} = \sum_m \frac{M^{(+)}_{GT}(m)\ M^{(-)}_{GT}(m)}{Q_{\beta\beta}/2 + m_e + E_x(1^+_m) - E_0}$$





2νββ-matrix element

0.16 ± 0.04 MeV^{-1}

with

G(2ν) = 3.4x 10^{-20} MeV^2 a^{-1}

2νββ - half-life

(1.1 ± 0.2) x 10^{21} a

recommended. exp. value:

(1.5 ± 0.1) x 10^{21} a

Closure 2νββ-decay NME

$$M^{2\nu}_{GT-cl} = \sum_m M^{(+)}_{GT}(m)\ M^{(-)}_{GT}(m)$$

SSD hypothesis

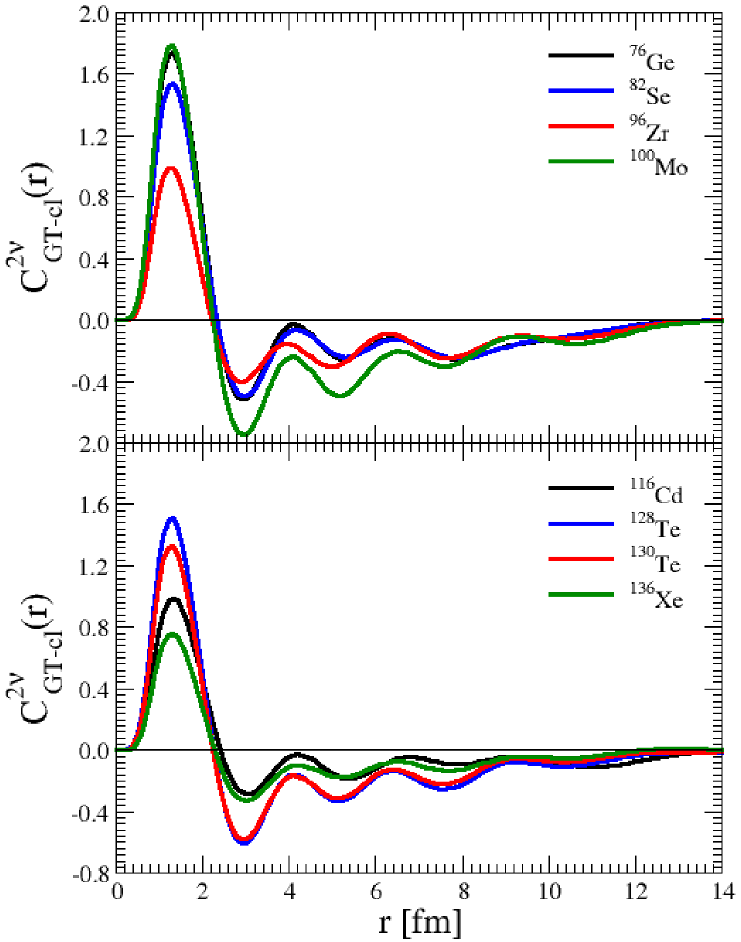
$$g_A^2\ M^{2\nu}_{GT-cl} = \frac{3\ D}{\sqrt{ft_{EC}ft_{\beta-}}}$$

Grewe, …Frekers at al, PRC 78, 044301 (2008)

Going to relative coordinates:

$$M^{2\nu}_{GT-cl} = \int_0^\infty C^{2\nu}_{GT-cl}(r)dr$$

r- relative distance of two nucleons



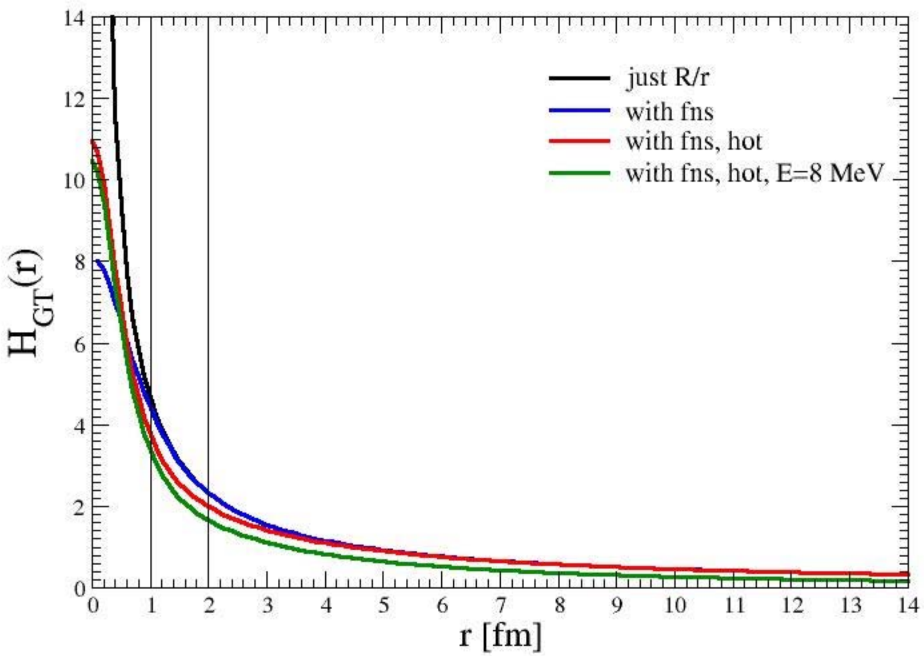
A connection between closure $2\nu\beta\beta$ and $0\nu\beta\beta$ GT NMEs

F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

$$M^{0\nu}_{GT} = \int_0^\infty H^{0\nu}_{GT}(r)C^{2\nu}_{GT-cl}(r)dr$$

Neutrino potential

$$H(r) = R\frac{2}{\pi}\int_0^\infty j_0(qr)\frac{q}{q+\overline{E}}f^2_{FNS}(q^2)g_{HOT}(q^2)dr$$



Neutrino potential prefer short distances

Closure 2νββ GT NME

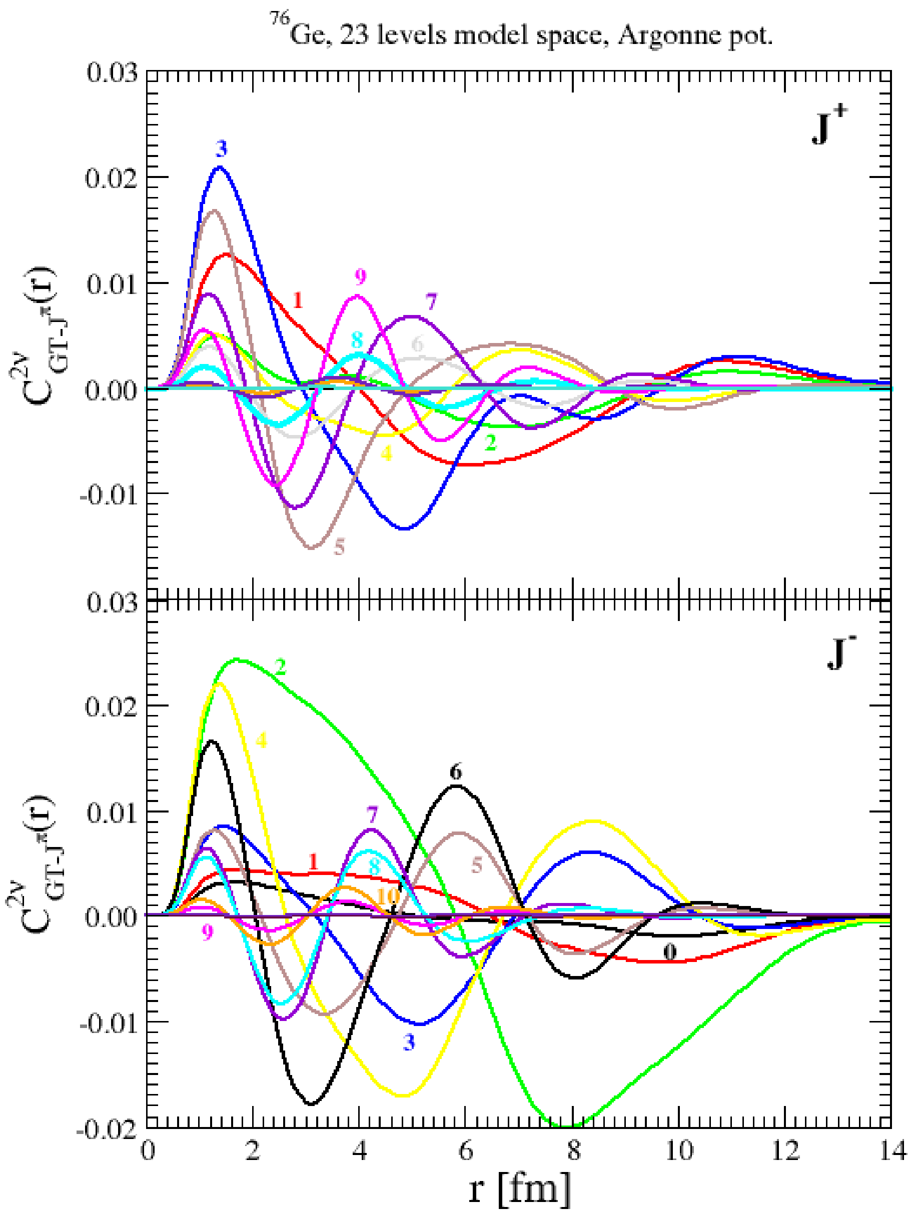
The only non-zero contribution from $J^{\pi}=1^{+}$

$$M^{2\nu}_{GT-cl} =$$

$$\sum_{J^\pi,m} < 0^+_f|\tau^+\vec{\sigma}|J^\pi,m > \cdot < J^\pi,m|\tau^+\vec{\sigma}|0^+_i >$$

$$=\sum_{m} < 0^+_f|\tau^+\vec{\sigma}|1^+,m > \cdot < 1^+,m|\tau^+\vec{\sigma}|0^+_i >$$

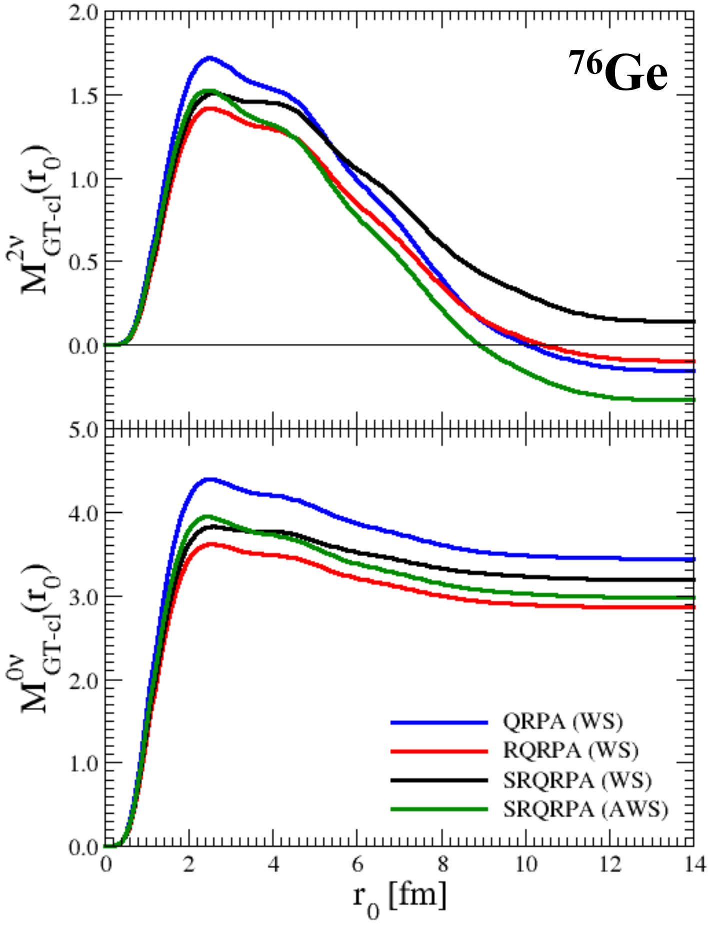
$$M^{2\nu}_{GT-cl} = \sum_{J^\pi} \int_0^\infty C^{2\nu}_{GT-J^\pi}(r)dr$$



$M^{0\nu}_{GT}$ depends weakly on g_A/g_{pp} and QRPA approach unlike $M^{2\nu}_{GT}$

$$M^{0\nu}_{GT}(r_0) = \int_0^{r_0} H^{0\nu}_{GT}(r) C^{2\nu}_{GT-cl}(r) dr$$

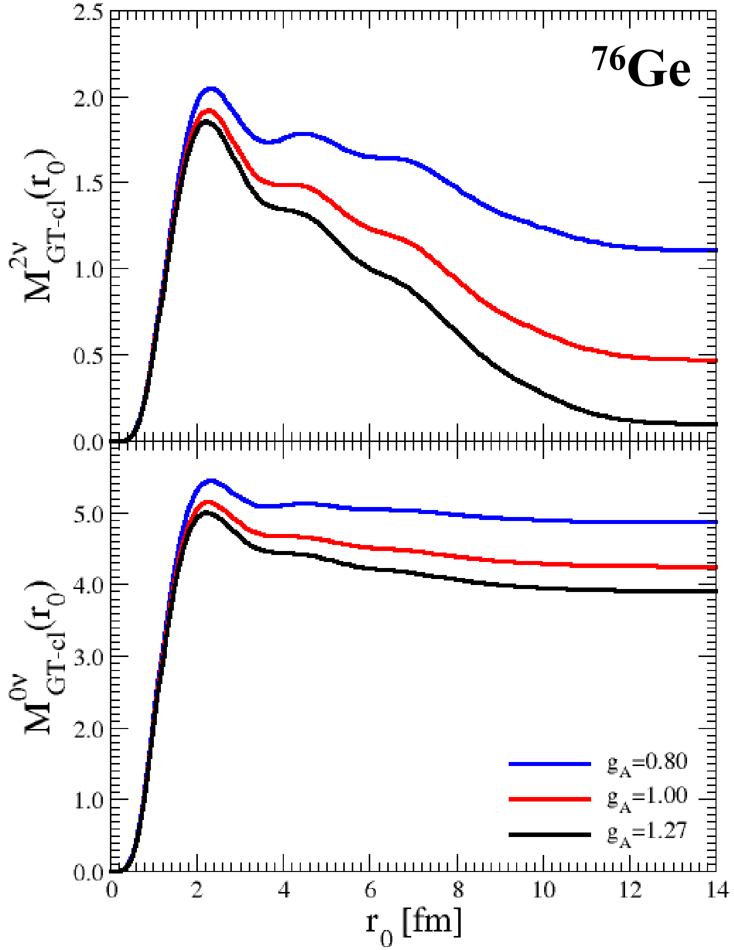
Nucleon ***Nuclear physics***



Different QRPA-like approches

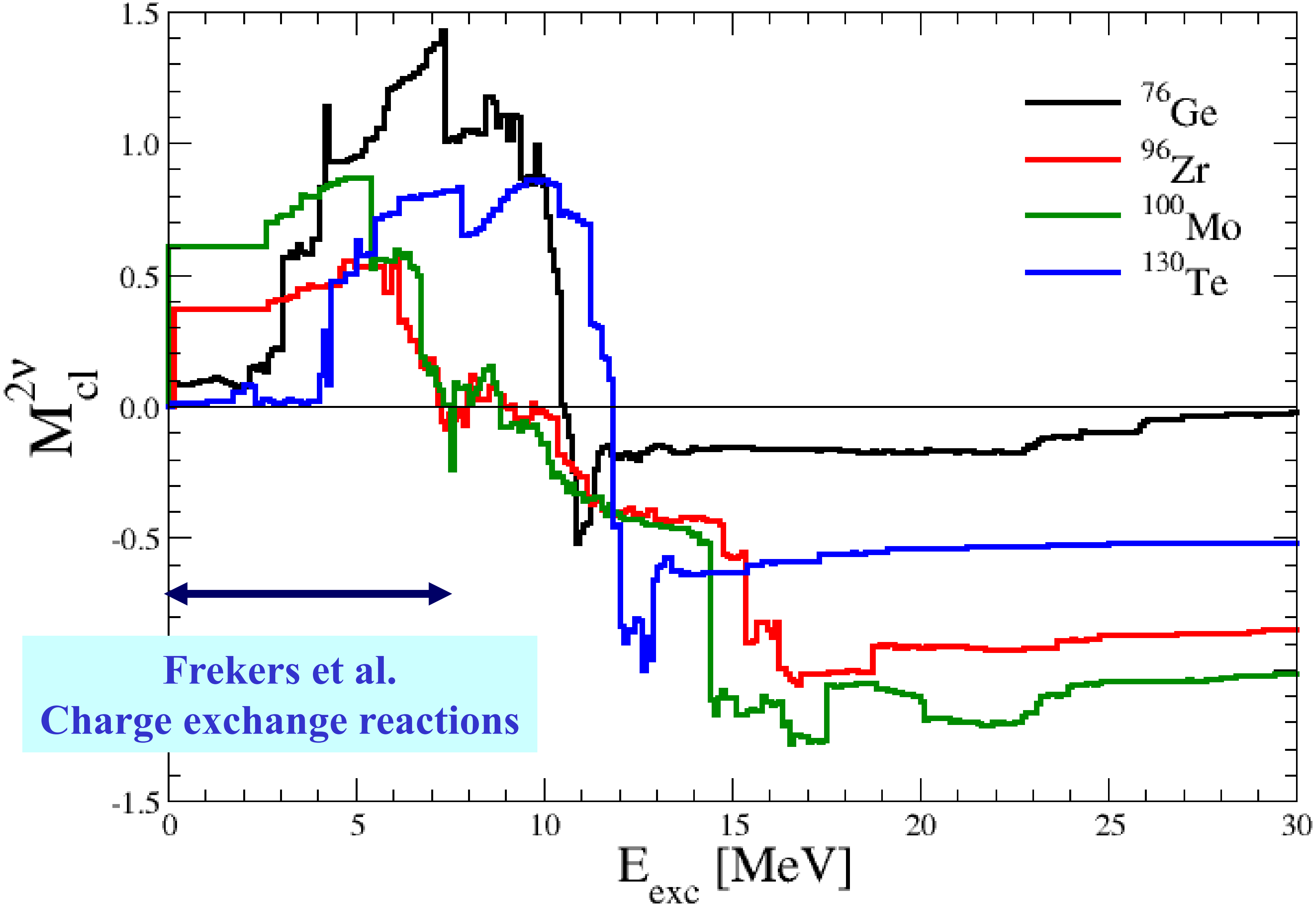
F.Š.,

Nucleon ***Nuclear physics***



Dependence on axial-vector coupling

There is no proportionality between 0νββ-decay and 2νββ-decay NME



Frekers et al.
Charge exchange reactions

Co-existence of few mechanisms of the 0$\nu\beta\beta$-decay

It may happen that in year 201? (or 2???) the 0$\nu\beta\beta$-decay will be detected for 2-3 or more isotopes …

F.Š., J.D. Vergados, A. Faessler, PRD 82, 113015 (2010)
A. Faessler, G. Fogli, E. Lisi, A. Rotunno, F. Š., Phys. Rev. D 83, 113015 (2011)
A. Faessler, A. Meroni, S.T. Petcov, F. Š., J.D. Vergados, Phys. Rev. D 83, 113003 (2011)

Probing the see-saw mechanism

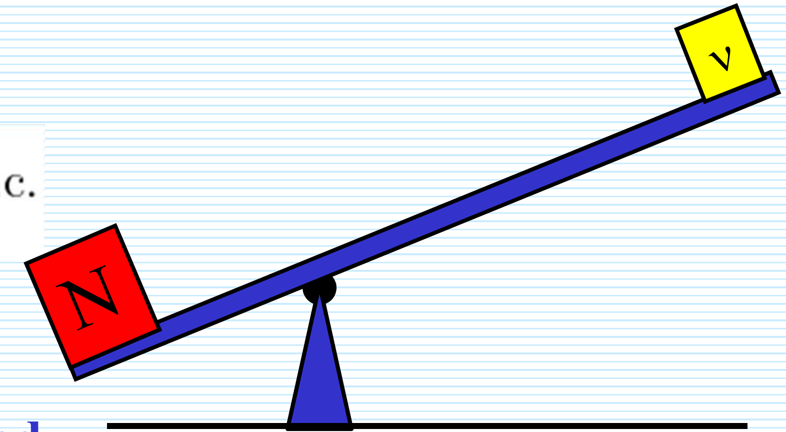
Bilenky, Faessler, Potzel, F.Š, Eur. Phys. J. C 71 (2011) 1754

There exist heavy Majorana neutral leptons N_i (singlet of SU(2)xU(1) group)

$$\mathcal{L} = -\sqrt{2}\sum_{i,l} Y_{li}\overline{L}_{lL}N_{iR}\tilde{H} + \mathrm{h.c.}. \qquad L_{lL} = \begin{pmatrix} \nu_{lL} \\ l_L \end{pmatrix} \qquad N_i = N_i^c = C\bar{N}_i^T$$

Effective interaction for processes with virtual N_i at electroweak scale

$$\mathcal{L}_{\mathrm{eff}} = -\frac{1}{\Lambda}\sum_{l',l,i}\overline{L_{l'L}}\tilde{H}\sum_i(Y_{l'i}\frac{\Lambda}{M_i}Y_{li})C\tilde{H}^T(\overline{L_{lL}})^T + \mathrm{h.c.}$$



After spontaneous violation of the electroweak symmetry the left-handed Majorana mass term is generated

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2}\sum_{l',l}\overline{\nu}_{l'L}\,M^L_{l'l}\,(\nu_{lL})^c + \mathrm{h.c.} = -\frac{1}{2}\sum_i m_i\bar{\nu}_i\nu_i,$$

$$M^L = Y\frac{v^2}{M}Y^T = U\;m\;U^T$$

Neutrino mixing

$$\nu_{lL} = \sum_i U_{li}\nu_{iL}$$

Inverted hierarchy

$$m_3 \ll m_1 < m_2$$

$$\Delta m_A^2(1-\sin^2 2\,\theta_{12}) \leq |m_{\beta\beta}|^2 \leq \Delta m_A^2$$

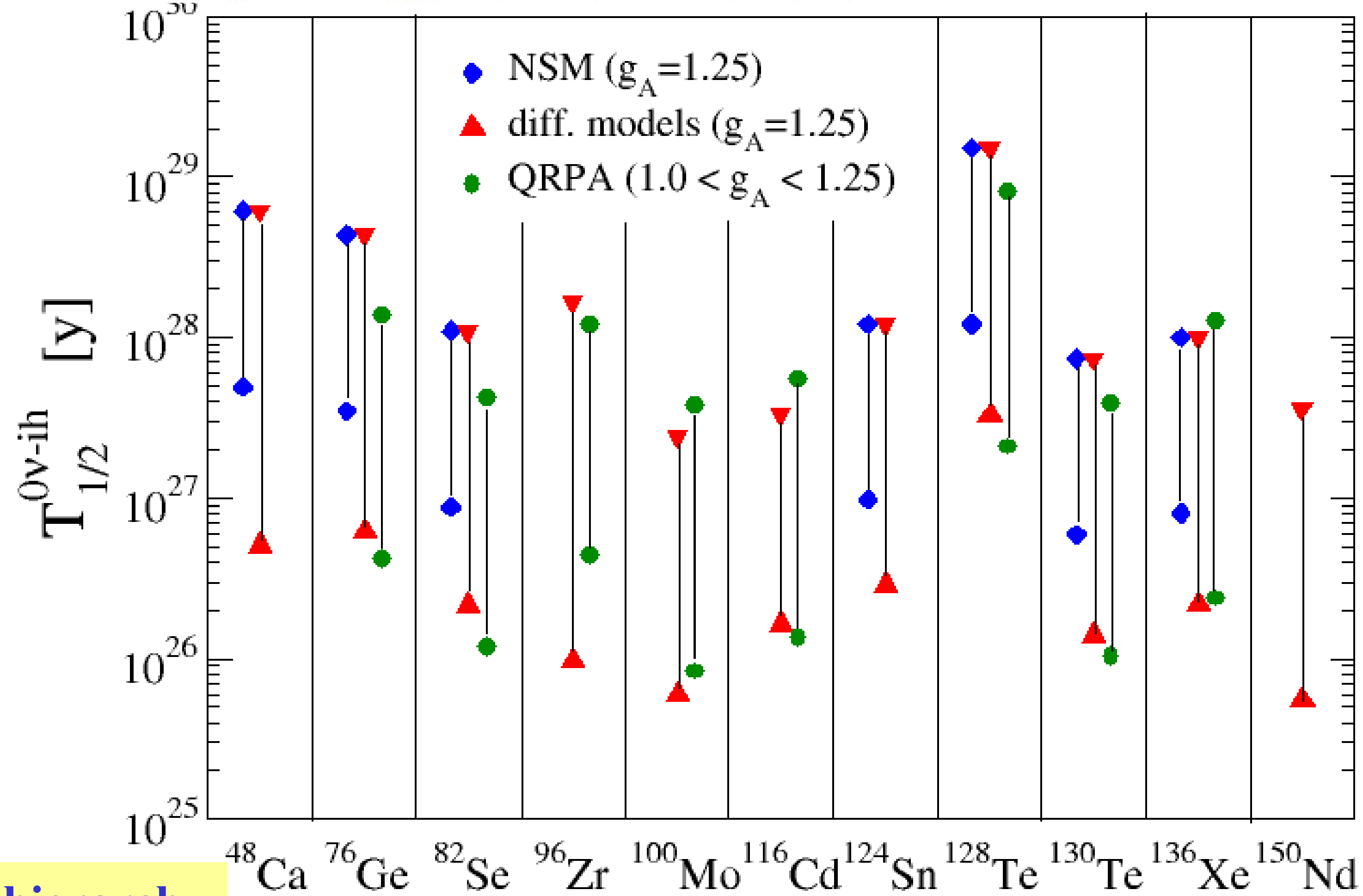
$$1.7\times 10^{-2}\ \mathrm{eV} \leq |m_{\beta\beta}| \leq 5.1\times 10^{-2}\ \mathrm{eV}.$$

$$T_{1/2}^{min}(A,Z) \leq T_{1/2}^{0\nu}(A,Z) \leq T_{1/2}^{max}(A,Z)$$

$$T_{1/2}^{min}(A,Z) = \frac{1}{\Delta m_A^2 |M(A,Z)|^2 G^{0\nu}(E_0,Z)},$$

$$T_{1/2}^{max}(A,Z) = \frac{1}{\Delta m_A^2 \cos^2 2\,\theta_{12} |M(A,Z)|^2 G^{0\nu}(E_0,Z)}$$

Probe of the see-saw mechanism



Inverted hierarchy

Bilenky, Faessler, Potzel, F.Š, Eur. Phys. J. C 71 (2011) 1754

Co-existence of 2, 3 or more interferring mechanisms of 0νββ-decay

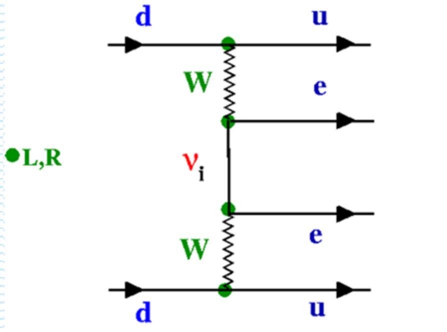
It is well-known that there exist many mechanisms that may contribute to the 0νββ. Let consider 3 mechanisms: i) light ν-mass mechanism, ii) heavy ν-mass mechanism, iii) R-parity breaking SUSY mechanism with gluino exchange and CP conservation

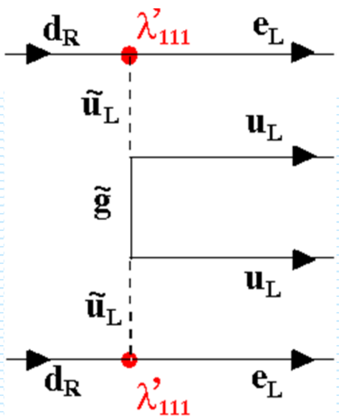
$$\frac{1}{T_{1/2}^{0\nu}} = G^{0\nu}(E_0, Z)\left|\frac{m_{\beta\beta}}{m_e} M_{\nu}^{0\nu} + \eta_N^L M_N^{0\nu} + \eta_{\lambda'_{111}} M_{\lambda'_{111}}^{0\nu} \ldots\right|^2$$

$$m_{\beta\beta} = \sum_k \left(U_{ek}^L\right)^2 \xi_k m_k$$

$$\eta_N^L = \sum_k^{heavy} U_{ek}^2 \frac{m_p}{M_k}$$

$$\eta_{\lambda'_{111}} = \frac{\pi\alpha_s}{6}\frac{\lambda'^2_{111}}{G_F^2 m_{\tilde{d}_R}^4}\frac{m_p}{m_{\tilde{g}}}\left[1+\left(\frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}}\right)^2\right]^2 .$$





Claim of evidence:

Klapdor-Kleingrothaus, Krivosheina, Mod. Phys. A 21, 1547 (2009)

$$T_{1/2}^{0\nu}(^{76}Ge) = 2.23^{+0.44}_{-0.31} \times 10^{25}\ \mathrm{y}$$

$$T_{1/2}^{0\nu}(^{100}\mathrm{Mo}) \geq 5.8 \times 10^{23}\ \mathrm{y}$$

$$T_{1/2}^{0\nu}(^{130}\mathrm{Te}) \geq 3.0 \times 10^{24}\ \mathrm{y}$$

$\xi_{Te} < 1.2$

$\xi_{Mo} < 2.6$

We introduce

$$\xi = \frac{|M_1^{\nu}|\sqrt{T_1\ G_1}}{|M_2^{\nu}|\sqrt{T_2\ G_2}}$$

ξ=0, non-observation ($T_2 \rightarrow \infty$)

ξ=1, solution for single active mech. is reproduced

4 sets of two linear eq.

$$\frac{\pm 1}{\sqrt{T_1\ G_1}} = \frac{m_{\beta\beta}}{m_e} M_1^{\nu} + \eta M_1^{\eta}$$

$$\frac{\pm 1}{\sqrt{T_2\ G_2}} = \frac{m_{\beta\beta}}{m_e} M_2^{\nu} + \eta M_2^{\eta}$$

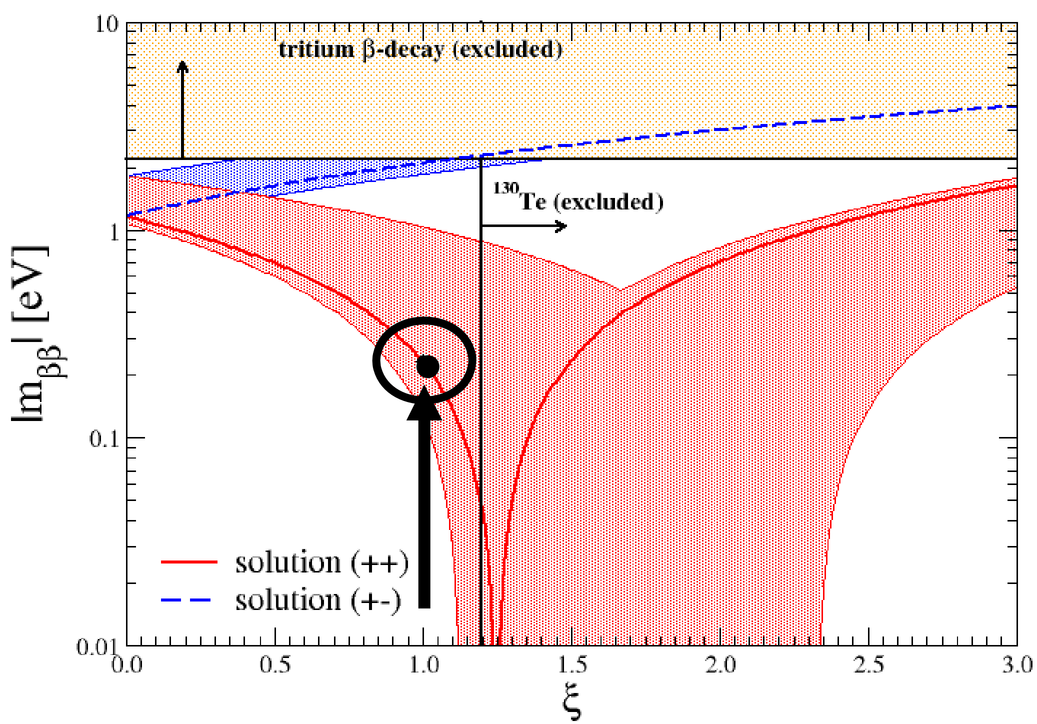
2 different solutions ***CP-conservation assumed***

$$|m_{\beta\beta}| = \left| \frac{m_e}{M_1^{\nu}\sqrt{T_1\ G_1}} \frac{M_1^{\nu}\ M_2^{\eta}}{(M_1^{\nu} M_2^{\eta} - M_2^{\nu} M_1^{\eta})} \pm \frac{m_e}{M_2^{\nu}\sqrt{T_2\ G_2}} \frac{M_2^{\nu}\ M_1^{\eta}}{(M_1^{\nu} M_2^{\eta} - M_2^{\nu} M_1^{\eta})} \right|$$

2 active mechanisms of the 0νββ-decay: Light and heavy ν-mass mechanism

Non-observation of the 0νββ-decay for some isotopes might be in agreement with non- zero $m_{\beta\beta}$

F.Š., J.D. Vergados, A. Faessler, PRD 82, 113015 (2010)



Non-observation for [130]Te

- *Single solution for* ***light ν-mass mech.***

9/14/2012

Two non-interfering mechanisms of the $0\nu\beta\beta$-decay (light LH and heavy RH neutrino exchange)

$$T^{0\nu}_{1/2}({}^{76}Ge) \geq 1.9 \times 10^{25} y, \qquad T^{0\nu}_{1/2}({}^{76}Ge) = 2.23^{+0.44}_{-0.31} \times 10^{25} y$$

$$5.8 \times 10^{23} y \leq T^{0\nu}_{1/2}({}^{100}Mo) \leq 5.8 \times 10^{24} y, \qquad 3.0 \times 10^{24} y \leq T^{0\nu}_{1/2}({}^{130}Te) \leq 3.0 \times 10^{25} y$$

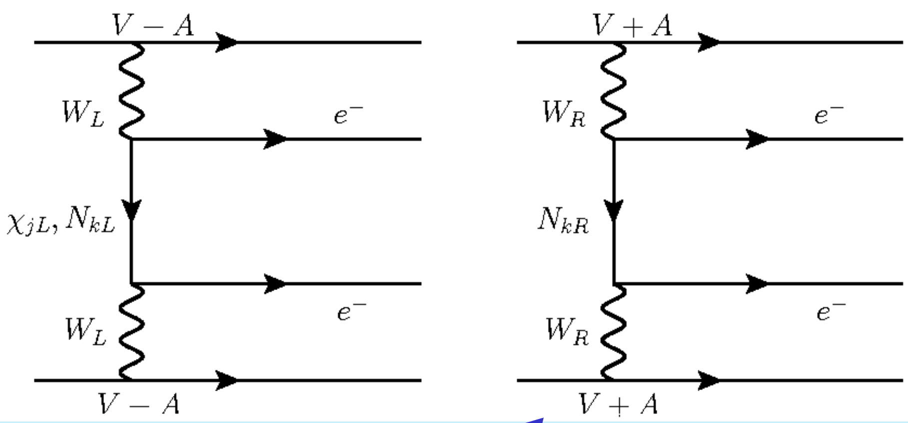
Half-life:

$$\frac{1}{T^{0\nu}_{1/2,i} G^{0\nu}_i(E,Z)} \cong |\eta_\nu|^2 |M'^{0\nu}_{i,\nu}|^2 + |\eta_R|^2 |M'^{0\nu}_{i,N}|^2$$

$$\eta_\nu = \frac{m_{\beta\beta}}{m_e}$$

Set of equations:

$$\begin{aligned} \frac{1}{T_1 G_1} &= |\eta_\nu|^2 |M'^{0\nu}_{1,\nu}|^2 + |\eta_R|^2 |M'^{0\nu}_{1,N}|^2 \\ \frac{1}{T_2 G_2} &= |\eta_\nu|^2 |M'^{0\nu}_{2,\nu}|^2 + |\eta_R|^2 |M'^{0\nu}_{2,N}|^2 \end{aligned}$$



Solutions:

$$\begin{aligned} |\eta_\nu|^2 &= \frac{|M'^{0\nu}_{2,N}|^2/T_1G_1 - |M'^{0\nu}_{1,N}|^2/T_2G_2}{|M'^{0\nu}_{1,\nu}|^2|M'^{0\nu}_{2,N}|^2 - |M'^{0\nu}_{1,N}|^2|M'^{0\nu}_{2,\nu}|^2} \\ |\eta_R|^2 &= \frac{|M'^{0\nu}_{1,\nu}|^2/T_2G_2 - |M'^{0\nu}_{2,\nu}|^2/T_1G_1}{|M'^{0\nu}_{1,\nu}|^2|M'^{0\nu}_{2,N}|^2 - |M'^{0\nu}_{1,N}|^2|M'^{0\nu}_{2,\nu}|^2} \end{aligned}$$

$$\eta^R_N = \left(\frac{M_W}{M_{WR}}\right)^4 \sum_k^{heavy} V^2_{ek} \frac{m_p}{M_k}$$

ic

A. Faessler, A. Meroni, S.T. Petcov, F. Š., J.D. Vergados,
B. Phys. Rev. D 83, 113003 (2011)

Two non-interfering mechanisms of the 0νββ-decay (light LH and heavy RH neutrino exchange)

The positivity condition:

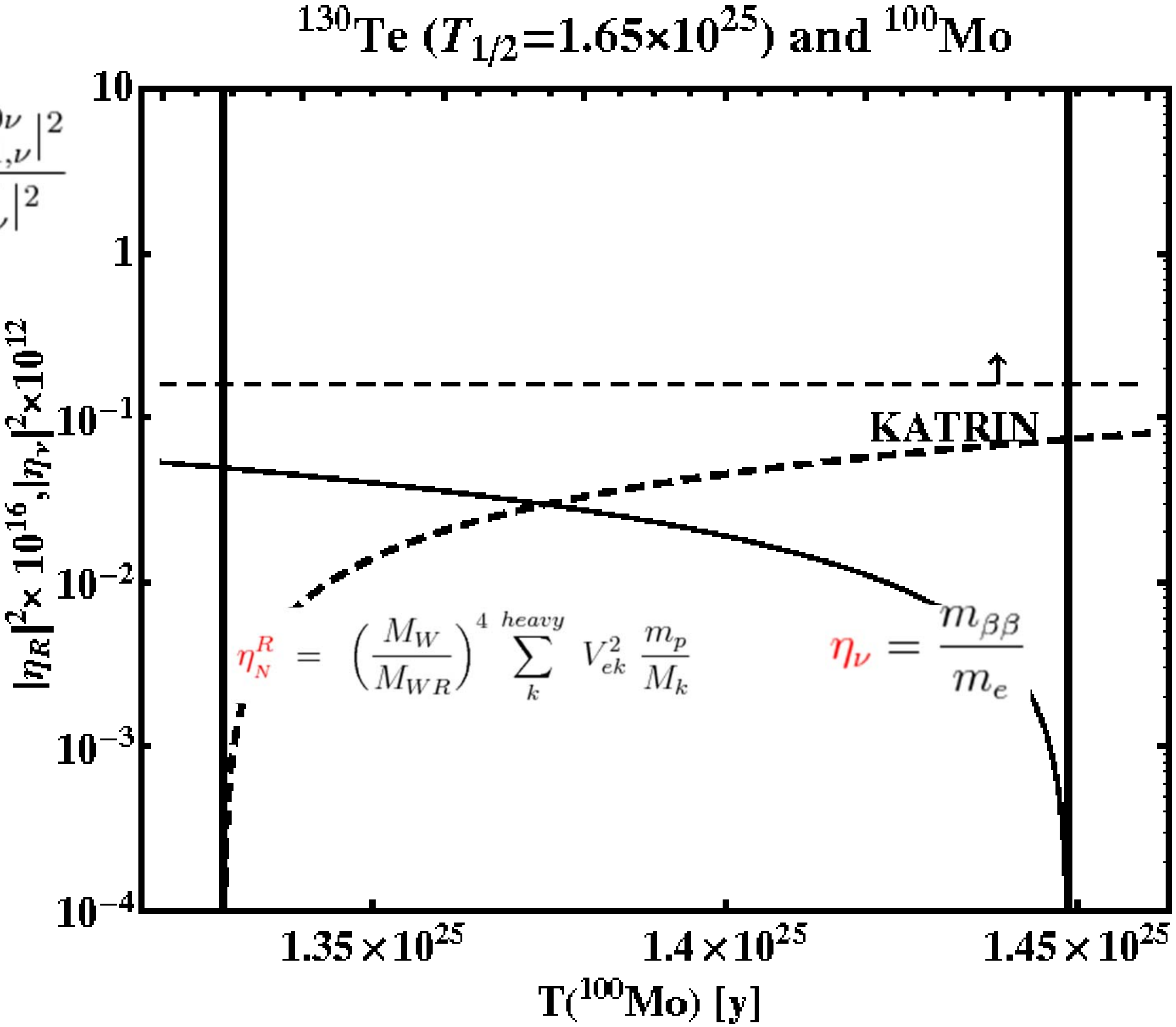
$$\frac{T_1 G_1 |M'^{0\nu}_{1,N}|^2}{G_2 |M'^{0\nu}_{2,N}|^2} \le T_2 \le \frac{T_1 G_1 |M'^{0\nu}_{1,\nu}|^2}{G_2 |M'^{0\nu}_{2,\nu}|^2}$$

Very narrow ranges!

$$0.15 \le \frac{T^{0\nu}_{1/2}(^{100}Mo)}{T^{0\nu}_{1/2}(^{76}Ge)} \le 0.18$$

$$0.17 \le \frac{T^{0\nu}_{1/2}(^{130}Te)}{T^{0\nu}_{1/2}(^{76}Ge)} \le 0.22$$

$$1.14 \le \frac{T^{0\nu}_{1/2}(^{130}Te)}{T^{0\nu}_{1/2}(^{100}Mo)} \le 1.24$$

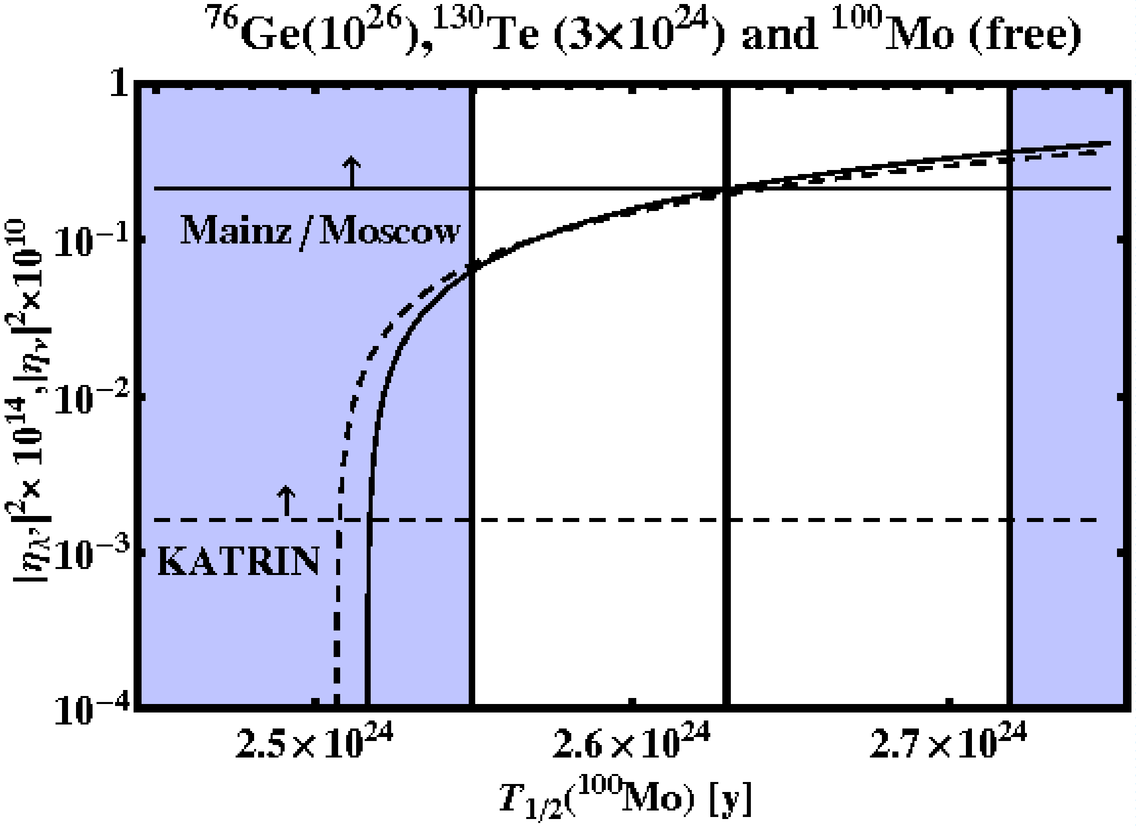


Two interfering mechanisms of the 0νββ-decay
(Light neutrino and gluino exchange)

$$\frac{1}{T^{0\nu}_{1/2,i} G^{0\nu}_i(E,Z)} = |\eta_\nu|^2 |M'^{0\nu}_{i,\nu}|^2 + |\eta_{\lambda'}|^2 |M'^{0\nu}_{i,\lambda'}|^2 + 2\cos\alpha |M'^{0\nu}_{i,\lambda'}||M'^{0\nu}_{i,\nu}||\eta_\nu||\eta_{\lambda'}|$$

$$|\eta_\nu|^2 = \frac{D_1}{D}, \quad |\eta_{\lambda'}|^2 = \frac{D_2}{D}$$
$$z \equiv 2\cos\alpha |\eta_\nu||\eta_{\lambda'}| = \frac{D_3}{D}$$

Given (i.e. having data on) T_1, T_2 + using the condition on the interference term $z = 2\cos\alpha|\eta_\nu||\eta_{\lambda'}|$, Determines an interval of allowed values of T_3.



Neutrinoless Double-Electron Capture

$$(A,Z) \rightarrow (A,Z\text{-}2)^{**}$$

Additional modes of the 0νECEC-decay:

$$e_b + e_b + (A,Z) \rightarrow (A,Z\text{-}2) + \gamma$$
$$+ 2\gamma$$
$$+ e^+e^-$$
$$+ M$$

Resonance enhancement of neutrinoless double electron capture
M.I. Krivoruchenko, F. Šimkovic, D. Frekers, and A. Faessler,
Nucl. Phys. A 859, 140-171 (2011)

- **New physical phenomenon, oscillations of atoms, was proposed. A connection to process of resonant neutrinoless double electron capture (0νεε) estsblished.**
- **The process of the 0νεε has been revisited for those cases where the two participating atoms are nearly degenerate in mass. New 0νεε transitions with parity violation to ground and excited states of final atom/nucleus were found. Selection rules for the 0νεε transitions were established. The explicit form of corresponding NMEs was derived.**
- **Available data of atomic masses, as well as nuclear and atomic excitations were used to select the most likely candidates for resonant 0νεε transitions. Assuming an effective Majorana neutrino mass of 1 eV, some half-lives has been predicted to be as low as 10^{22} years in the unitary limit. According to obtained estimates, in the case of ^{152}Gd the sensitivity can be comparable to the favored 0νββ decays of nuclei.**
- **More accurate atomic mass measurements in the context of the 0νεε were initialized, which have been partially accomplished using the modern high-precision ion traps. In addition, new 0νεε experiments were initialized (TGV, R. Bernabei group at Gran Sasso, Muenster-Bratislava)**

Oscillations of atoms

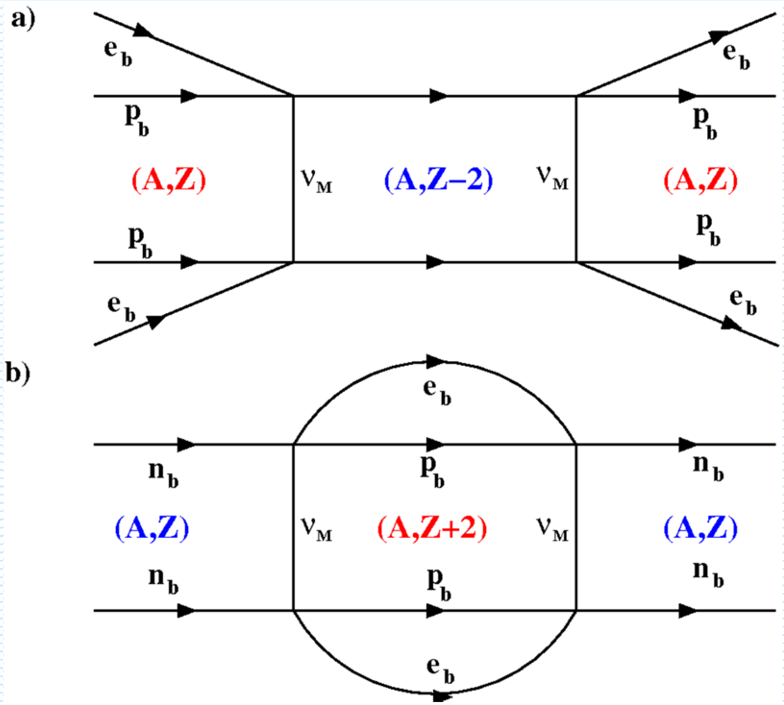
$$H_{eff}^{atom} = \begin{pmatrix} M_i & V^{LNV} \\ V^{LNV} & M_f - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillation of atoms
(lepton number violation)

F.Š., M. Krivoruchenko, Phys.Part.Nucl.Lett. 6 (2009) 485

In analogy with oscillations of
n-anti{n} (baryon number violation)

$$H_{eff}^{n\bar{n}} = \begin{pmatrix} M & V^{BNV} \\ V^{BNV} & M - \frac{i}{2}\Gamma \end{pmatrix}$$



OSCILLATIONS & RESONANT TRANSITIONS BETWEEN GROUND-STATE AND EXCITED ATOMS

To the lowest order in V:

$$\lambda_+ = M_i + \Delta M - \frac{i}{2}\Gamma_1, \quad \leftarrow \quad i = (A,Z)$$

where

$$\lambda_- = M_f - \Delta M - \frac{i}{2}(\Gamma - \Gamma_1), \quad \leftarrow \quad f = (A,Z-2)^{**}$$

$$\Delta M = \frac{V^2(M_i - M_f)}{(M_i - M_f)^2 + \frac{1}{4}\Gamma^2},$$

$$\Gamma_1 = \frac{V^2\Gamma}{(M_i - M_f)^2 + \frac{1}{4}\Gamma^2} << \Gamma.$$

Breit-Wigner factor

UNITARY LIMIT: $M_{A,Z} = M^{**}_{A,Z-2}$

$$\Gamma_1^{\max} = \frac{4V^2}{\Gamma}.$$

LIKELY RESONANT 0νεε TRANSITIONS

Decay width of the parent atom:

$$\Gamma_1 = \frac{V^2\Gamma}{(M_i - M_f)^2 + \frac{1}{4}\Gamma^2}.$$

Calculations:

1. Mass difference → Coulomb energy of electron holes.
2. Decay width Γ → widths of the excited electron shells, Auger & radiative transitions.
3. V → L_T violating potential, electron wave functions & nuclear matrix elements.

DECAY WIDTHS OF TWO-ELECTRON HOLES

Decays are dominated by

1. radiative transitions with the emission of X-rays,
2. the emission of Auger electrons.

X-rays are dominant in the decays of atoms with Z > 35.

The width of a two-hole state αβ is represented by

$$\Gamma_{\alpha\beta} = \underbrace{\Gamma_{\alpha} + \Gamma_{\beta}}_{\text{electron shell}} + \Gamma^{*}.$$

nuclear de-excitation

Γ_{α} are taken from: J. L. Campbell and T. Papp, At. Data and Nuclear Data Tables, *77* (2001) 1.

Capture of $s_{1/2}$ and $p_{1/2}$ atomic electrons is prefered

$$\Psi_{\alpha m_\alpha}(\vec{x}) = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} f_\alpha(r)\ \chi_{m_\alpha} \\ -ig_\alpha(r)\ (\vec{\sigma}\cdot\hat{r})\ \chi_{m_\alpha} \end{pmatrix} \qquad (\alpha = n_\alpha s_{1/2})$$

$$\Psi_{\alpha m_\alpha}(\vec{x}) = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} -if_\alpha(r)\ (\vec{\sigma}\cdot\hat{r})\ \chi_{m_\alpha} \\ -g_\alpha(r)\ \chi_{m_\alpha} \end{pmatrix} \qquad (\alpha = n_\alpha p_{1/2})$$

$J^\pi = 0^+, 0^-, 1^+, 1^-$

Shell		^{78}Se	^{112}Cd	^{124}Te	^{130}Xe	^{156}Gd
$1s_{1/2}$	$<\mathbf{f}>$	3.45×10^3	6.80×10^3	8.83×10^3	1.09×10^4	1.33×10^4
	$<g>$	-4.34×10^2	-1.23×10^3	-1.81×10^3	-2.47×10^3	-3.30×10^3
$2s_{1/2}$	$<f>$	1.25×10^3	2.54×10^3	3.35×10^3	4.19×10^3	5.20×10^3
	$<g>$	-1.58×10^2	-4.59×10^2	-6.87×10^2	-9.48×10^2	-1.29×10^3
$3s_{1/2}$	$<f>$	6.83×10^2	1.39×10^3	1.83×10^3	2.29×10^3	2.85×10^3
	$<g>$	-8.60×10^1	-2.51×10^2	-3.76×10^2	-5.18×10^2	-7.05×10^2
$4s_{1/2}$	$<f>$	4.43×10^2	8.99×10^2	1.19×10^3	1.48×10^3	1.84×10^3
	$<g>$	-5.58×10^1	-1.63×10^2	-2.43×10^2	-3.36×10^2	-4.57×10^2
$2p_{1/2}$	$<f>$	-1.72×10^1	-7.22×10^1	-1.23×10^2	-1.87×10^2	-2.78×10^2
	$<g>$	-1.37×10^2	-3.99×10^2	-5.97×10^2	-8.25×10^2	-1.12×10^3
$2p_{3/2}$	$<f>$	8.06×10^{-1}	2.38×10^0	3.48×10^0	4.62×10^0	6.31×10^0
	$<g>$	-5.02×10^{-2}	-2.10×10^{-1}	-3.46×10^{-1}	-5.03×10^{-1}	-7.47×10^{-1}

1) **Solution of Dirac equation in the screened Coulomb field (upper line)**
2) **Solution based on the Dirac-Hartree-Fock method (lower line)**

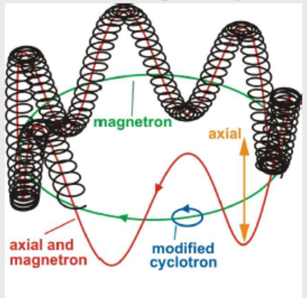
Transition	$^{158}_{64}\mathrm{Gd}$	$^{166}_{68}\mathrm{Er}$
$\langle f(1s_{1/2})\rangle$	1.33×10^4	1.57×10^4
	1.27×10^4	1.50×10^4
$\langle f(2s_{1/2})\rangle$	5.20×10^3	6.20×10^3
	4.59×10^3	5.46×10^3
$\langle f(3s_{1/2})\rangle$	2.84×10^3	3.39×10^3
	2.15×10^3	2.60×10^3
$\langle g(2p_{1/2})\rangle$	-1.12×10^3	-1.43×10^3
	-9.53×10^2	-1.22×10^3

Improved Q-value measurements
Klaus Blaum (MPI Heidelberg)

nucl. tr.	Q_{old}	$E = B + E_\gamma$	Orbit.	$\Delta = Q(old) - E$	Q_{new}	$\Delta = Q(new) - E$
$^{112}Sn \rightarrow ^{112}Cd$	1919.5(4.8)	1901.7	KL_1	17.8(4.8)	1919.82(16)	18.12(16)
		1924.4	KK	-4.9(4.8)		-4.56(16)
$^{152}Gd \rightarrow ^{152}Sm$	54.6(3.5)	54.79+0	KL_1	-0.19(3.50)	55.70(18)	0.91(18)
$^{164}Er \rightarrow ^{164}Dy$	23.3(3.9)	18.09	l_1L_1	5.21(3.90)		

^{152}Gd→^{152}Sm (Eliseev, et al., F.Š, M. Krivoruchenko, PRL 106, 052504 (2011))
(F.Š., Krivoruchenko, Faessler, PPNP 66, 446 (2011))

Penning Trap



ν_+ - modified cyclotron
ν_- - magnetron
ν_z - axial

$$\Gamma_{\varepsilon\varepsilon} = |V_{\varepsilon\varepsilon}|^2 \frac{\Gamma}{\Delta^2 + \Gamma^2/4} = |V_{\varepsilon\varepsilon}|^2 R$$

$$V_{\varepsilon\varepsilon} = m_{\beta\beta} \frac{\sqrt{2} g_A^2 G_\beta^2}{(4\pi)^2 R_{nucl}} \overline{f}_a \overline{f}_b M^{0\nu}$$

$$T_{1/2}^{0\nu} = 4 \times 10^{26} \left(\frac{1 \text{ eV}}{m_{\beta\beta}}\right)^2 \text{ years.}$$

Remeasured Q-value: ^{112}Sn, ^{74}Se, ^{136}Ce, ^{96}Ru, ^{152}Gd, ^{162}Er, ^{168}Yb, ^{106}Cd, ^{156}Dy, ^{180}W

need to be remeasured: ^{124}Xe, ^{130}Ba, ^{184}Os, ^{190}Pt

Nuclear matrix elements for $0\nu\varepsilon\varepsilon$

Ground state to ground state nuclear transitions

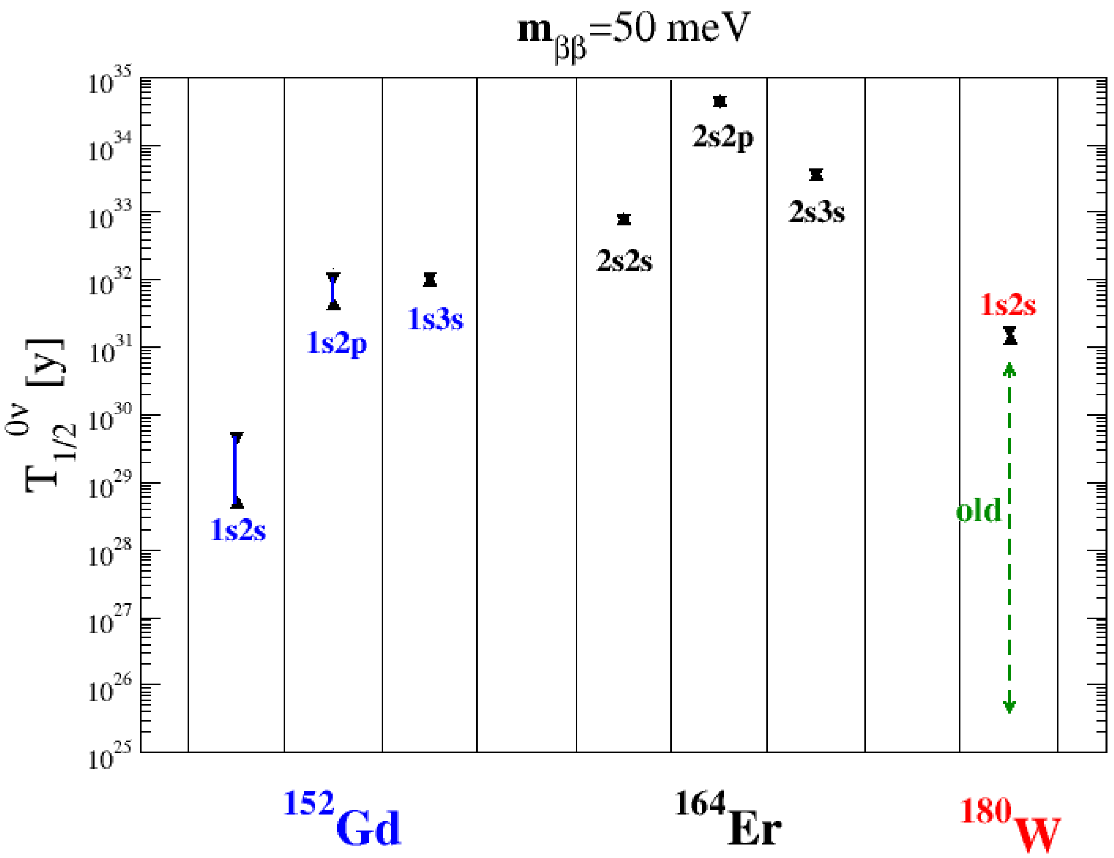
Initial (final)	β_{Q_p}	$\beta_{B(E2)}$	$\langle BCS_i \vert BCS_f \rangle$
^{152}Gd (^{152}Sm)	(+0.29)	0.212 (0.306)	0.44
^{164}Er (^{164}Dy)	0.36 (+0.32)	0.333 (0.348)	0.73
^{180}W (^{180}Hf)	0.27 (+0.27)	0.252 (0.273)	0.75

Nucleus	$M^{2\nu}_{GT}$ $[MeV^{-1}]$	$M^{0\nu}$ sph. QRPA	$M^{0\nu}$ def. QRPA ($\beta_2 = 0$)	$M^{0\nu}$ def. QRPA
^{152}Gd	0.10	7.59	7.50	3.23
	0.00	7.21		2.67
^{164}Er	0.10	6.12	7.20	2.64
	0.00	5.94		2.27
^{180}W	0.10	5.79	6.22	2.05
	0.00	5.56		1.79

Suppression of the NME depends not only on the relative deformation but also their absolute values

$0\nu\varepsilon\varepsilon$ half-lives

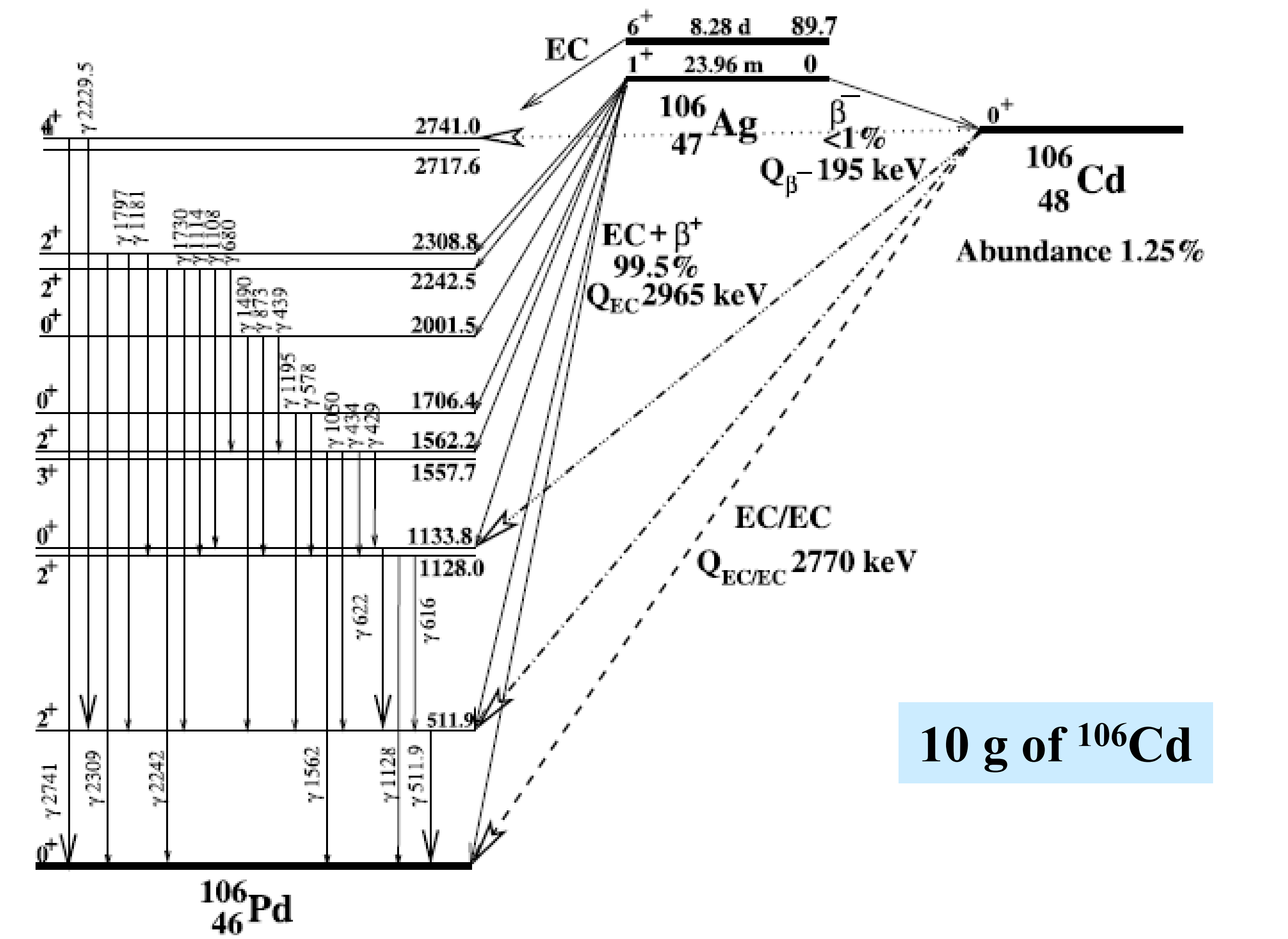
$m_{\beta\beta}$=50 meV



Nucleus	$(n2jl)_a$	$(n2jl)_b$	E_a	E_b	E_C	Γ_{ab} (keV)	Δ (keV)	$T^{\min}_{1/2}$ (y)	$T^{\max}_{1/2}$ (y)
^{152}Gd	110	210	46.83	7.74	0.34	2.3×10^{-2}	-0.83 ± 0.18	4.7×10^{28}	4.8×10^{29}
	110	211	46.83	7.31	0.32	2.3×10^{-2}	-1.27 ± 0.18	4.2×10^{31}	1.1×10^{32}
	110	310	46.83	1.72	0.11	3.2×10^{-2}	-7.07 ± 0.18	9.4×10^{31}	1.1×10^{32}
^{164}Er	210	210	9.05	9.05	0.22	8.6×10^{-3}	-6.82 ± 0.12	7.5×10^{32}	8.4×10^{32}
	210	211	9.05	8.58	0.23	8.3×10^{-3}	-7.28 ± 0.12	4.2×10^{34}	4.6×10^{34}
	210	310	9.05	2.05	0.11	1.8×10^{-2}	-13.92 ± 0.12	3.5×10^{33}	3.9×10^{33}
^{180}W	110	110	63.35	63.35	1.26	7.2×10^{-2}	-11.24 ± 0.27	1.3×10^{31}	1.8×10^{31}

$0\nu\varepsilon\varepsilon$ experiments in Modane and Grand Sasso

New level
2737 keV
(J^π=?)



10 g of ^{106}Cd

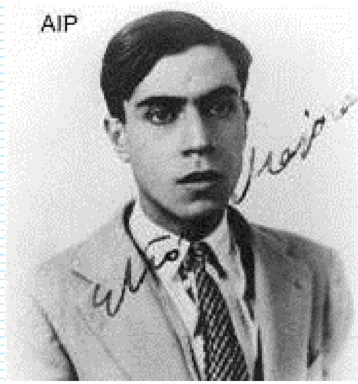
$T_{1/2}^{2\nu\varepsilon\varepsilon}$ (^{106}Cd) > 3.6 10^{20} y

TGV Coll., Rukhadze et al., NPA 852, 197 (2011)

$T_{1/2}^{0\nu\varepsilon\varepsilon}$ (^{106}Cd) > 1.1 10^{20} y

What is the nature of neutrinos?

Study of the 0νββ-decay
is one of the highest priority issues
in particle and nuclear physics



$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$

Perturbation theory

$$\frac{1}{T^{0\nu}_{1/2}} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 G^{01}(E_0, Z) \left|M^{0\nu}\right|^2$$

- **2νββ-decay background can be a problem**
- **Uncertainty in NMEs factor ~2, 3**
- **$0^+ \rightarrow 0^+, 2^+$ transitions**
- **Large Q-value**
- **^{76}Ge, ^{82}Se, ^{100}Mo, ^{130}Te, ^{136}Xe …**
- **Many exp. in construction, potential for observation in the case of inverted hierarchy (2020)**

$e^- + e^- + (A,Z) \rightarrow (A,Z\text{-}2)^{**}$

Breit-Wigner form

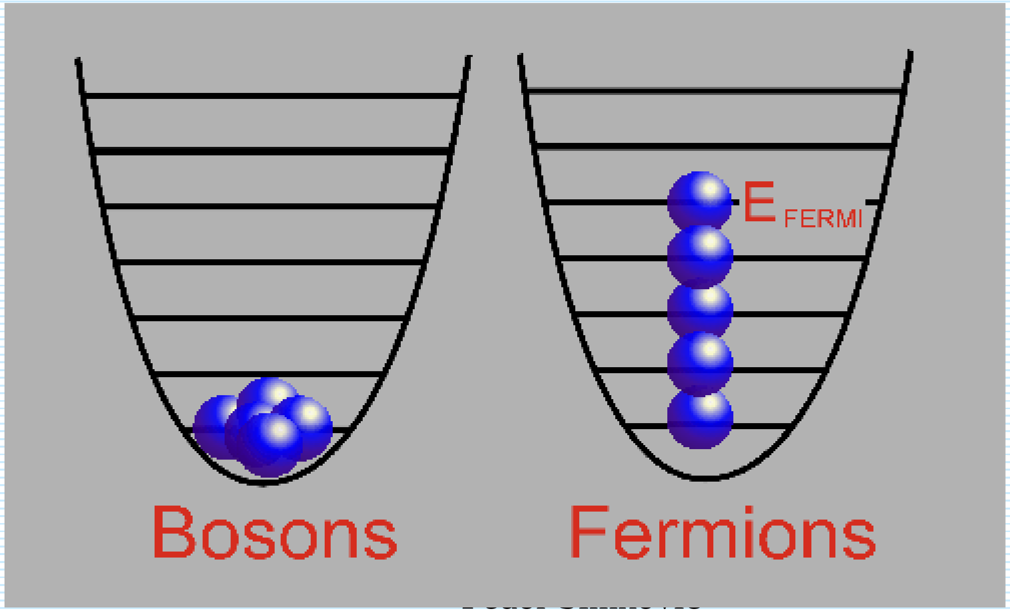
$$\Gamma^{0\nu ECEC}(J^\pi) = \frac{|V_{\alpha\beta}(J^\pi)|^2}{(M_i - M_f)^2 + \Gamma^2_{\alpha\beta}/4}\,\Gamma_{\alpha\beta}$$

- **2νεε-decay strongly suppressed**
- **NMEs need to be calculated**
- **$0^+ \rightarrow 0^+, 0^-, 1^+, 1^-$ transitions**
- **Small Q-value**
- **Q-value needs to be measured at least with 100 eV accuracy**
- **^{152}Gd, looking for additional**
- **small experiments yet**

(Partly)bosonic or fermionic neutrinos?

Bosons:
In the ground state (T=0) all bosons occupy lowest energy state.

Fermions:
No two fermions can occupy the same state, so in the ground state (T=0), fermions stack from The lowest energy level to higher Energy levels, leaving no holes.



Mixed statistics for neutrinos

Definnition of mixed state

$$
\begin{aligned}
|\nu> &= \hat{a}^\dagger|0> \\
&\equiv \cos\delta \;\; \hat{f}^\dagger|0> \;+\; \sin\delta \;\; \hat{b}^\dagger|0> \\
&= \cos\delta \;\; |f> \;+\; \sin\delta \;\; |b>
\end{aligned}
$$

with commutation Relations

$$
\begin{aligned}
\hat{f}\hat{b} &= e^{i\phi}\hat{b}\hat{f} \qquad \hat{f}^\dagger\hat{b}^\dagger = e^{i\phi}\hat{b}^\dagger\hat{f}^\dagger \\
\hat{f}\hat{b}^\dagger &= e^{-i\phi}\hat{b}^\dagger\hat{f} \qquad \hat{f}^\dagger\hat{b} = e^{-i\phi}\hat{b}\hat{f}^\dagger
\end{aligned}
$$

Amplitude for 2νββ

$$
\begin{aligned}
A^{2\nu} &= [\cos\delta^4 + \cos\delta^2 \sin\delta^2(1-\cos\phi)]A^f \;+\; [\cos\delta^4 + \cos\delta^2\sin\delta^2(1+\cos\phi)]A^b \\
&= \cos\chi^2 A^f \;+\; \sin\chi^2 A^b
\end{aligned}
$$

Decay rate

$$
\begin{aligned}
W^{2\nu} &= \cos\chi^4 \; W^f \;+\; \sin\chi^4 \; W^b \\
&= (1-b^2) \; W^f \;+\; b^2 \; W^b
\end{aligned}
$$

Partly bosonic neutrino requires knowing NME or log ft values for HSD or SSD

(calculations coming up soon)

Looking for a signature of bosonic ν

2νββ–decay half-lives $(0^+ \to 0^+_{g.s.},\ 0^+ \to 0^+_1,\ 0^+ \to 2^+_1)$

- HSD – NME needed
- SSD – log ft_{EC}, log ft_β needed

$$\frac{T^{2\nu-SSD}_{1/2}(2^+_f)}{T^{2\nu-SSD}_{1/2}(0^+_f)} \begin{array}{lll} = & 2.41 \times 10^4 & \text{fermionic } \nu \\ = & 403 & \text{bosonic } \nu \end{array} \qquad \begin{array}{rcl} T^{2\nu}_{1/2}(2^+) & = & 1.73 \times 10^{23} years \\ & = & 2.74 \times 10^{21} years \\ T^{2\nu-exp}_{1/2}(2^+) & > & 1.6 \times 10^{21} years \end{array}$$

Normalized differential characteristics

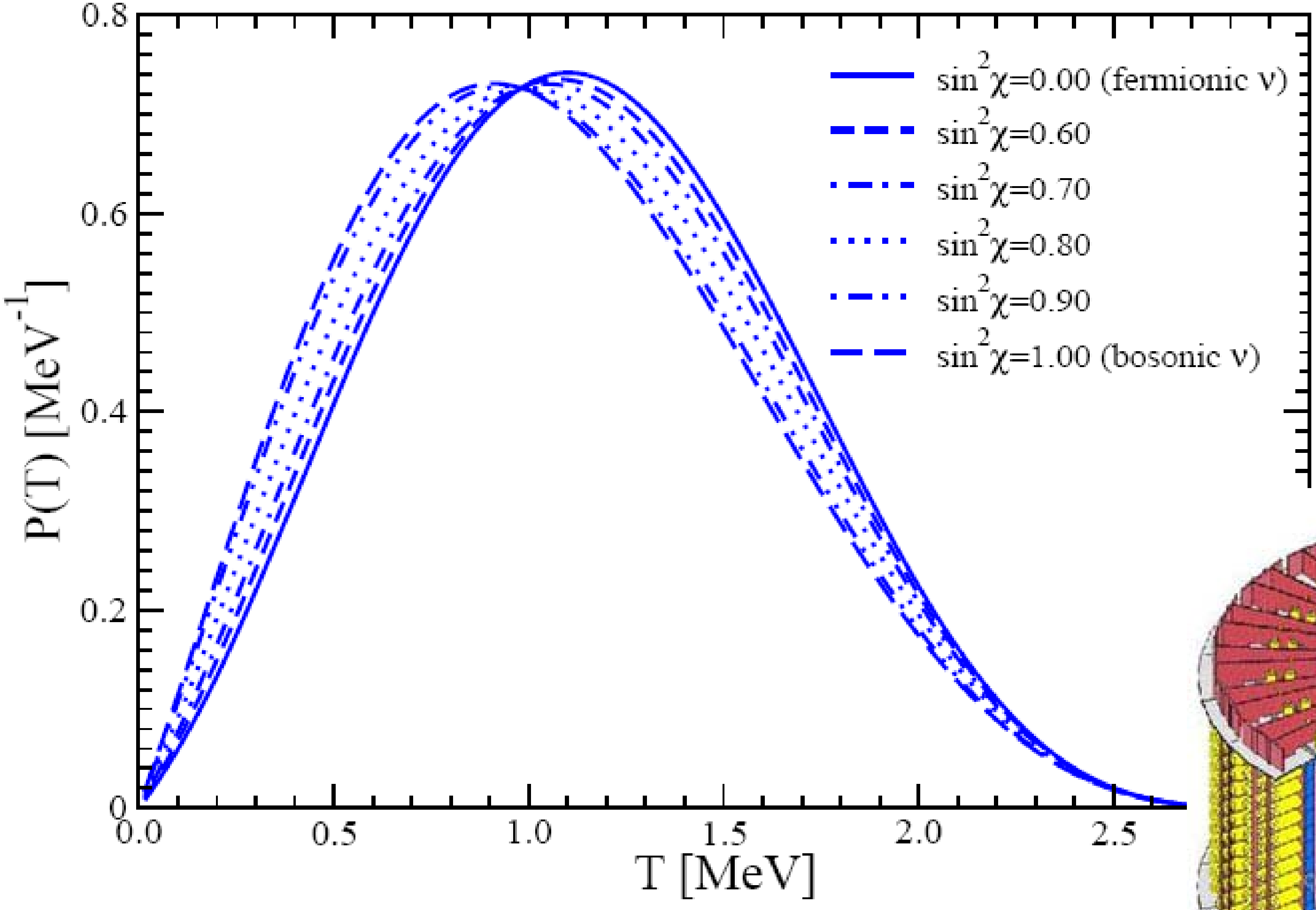
- **The single electron energy distribution**
- **The distribution of the total energy of two electrons**
- **Angular correlations of two electrons**

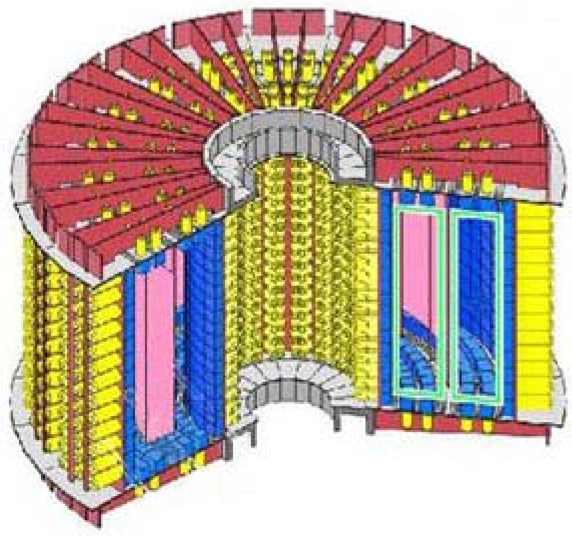
(free of NME and log ft)

Mixed ν excluded for $\sin^2\chi < 0.6$ (NEMO3 data)

$$W^{2\nu} = \cos\chi^4\, W^f + \sin\chi^4\, W^b = (1-b^2)\, W^f + b^2\, W^b$$

$^{100}Mo \rightarrow {}^{100}Ru$ (SSD)





Barabash, Dolgov, Dvornický, F. Š., Smirnov, NPB 783, 90 (2007)

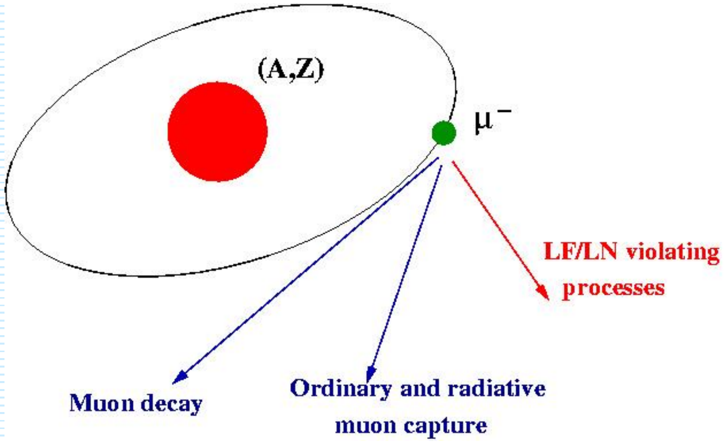
Analogues of neutrinoless double beta decay

$$\mu^- + (A,Z) \rightarrow (A,Z\text{-}2) + e^+$$
$$\mu^- + (A,Z) \rightarrow (A,Z\text{-}2) + \mu^+$$
$$e^- + e^- \rightarrow W^- + W^-$$
$$K^+ \rightarrow p^- + \mu^+ + \mu^+$$

$$m_{bb} \longrightarrow \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{\mu e} & M_{\mu\mu} & M_{\mu\tau} \\ M_{\tau e} & M_{\tau\mu} & M_{\tau\tau} \end{pmatrix}$$

Muon-positron conversion

$$\mu_b^- + (A,Z) \rightarrow (A,Z-2) + e^+$$

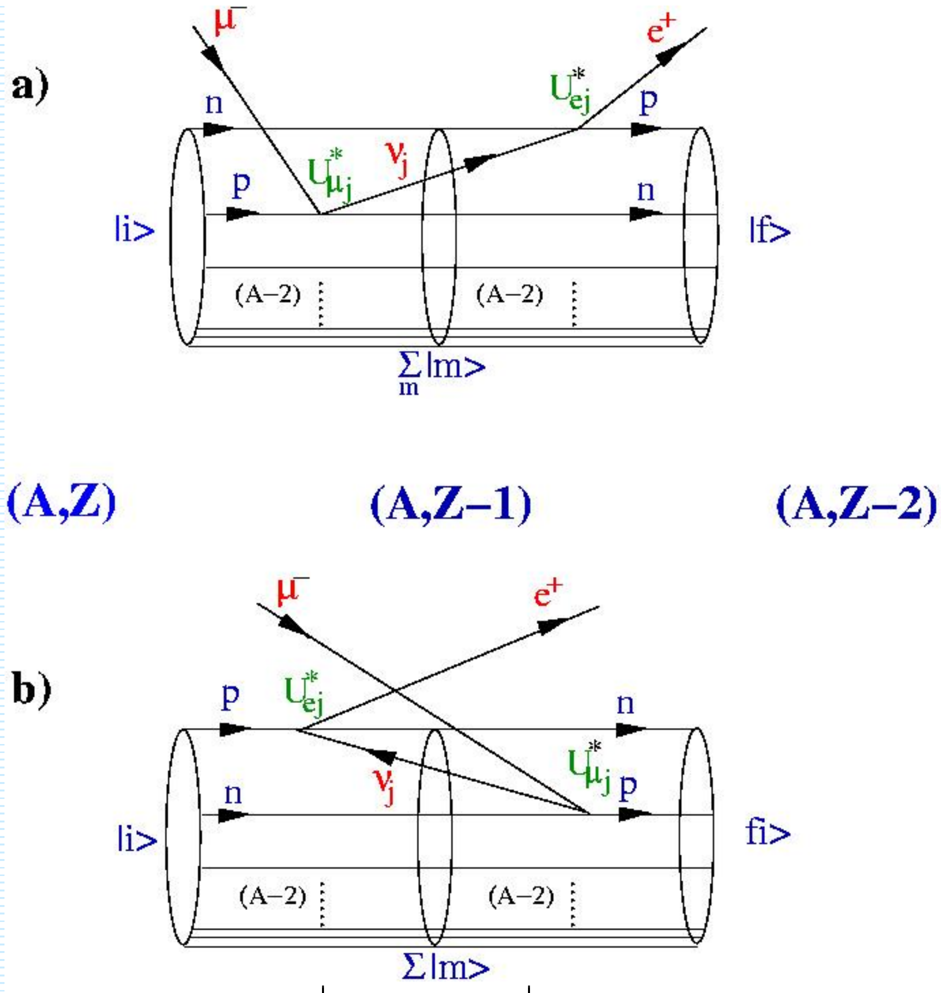


$$\frac{\Gamma_{\mu e^+}}{\Gamma_{\beta\beta}} = 176 \left| \frac{m_{\mu e}}{m_{\beta\beta}} \right|^2$$

$$\frac{\Gamma_{\mu e^+}}{\Gamma_{\mu}} = 1.3\ 10^{-25} \left| \frac{m_{\mu e}}{m_e} \right|^2$$

Domin, Kovalenko, Faessler, Šimkovic, PRD 70, 065501 (2004)

$$\sum_m \frac{|m><m|}{q - E_{\mu^-} + E_m - E_i + i\epsilon_m}$$



$$\sum_m \frac{|m><m|}{q + E_{e^+} + E_m - E_i + i\epsilon_m}$$

Magnetic moments of Majorana ν's

(in light of neutrino oscillations and R-parity breaking MSSM)

Gozdz, Kaminski , F. Š., Faessler, Phys. Rev. D. 74, 055007 (2006)

Dirac and Majorana
ν Magnetic Moment

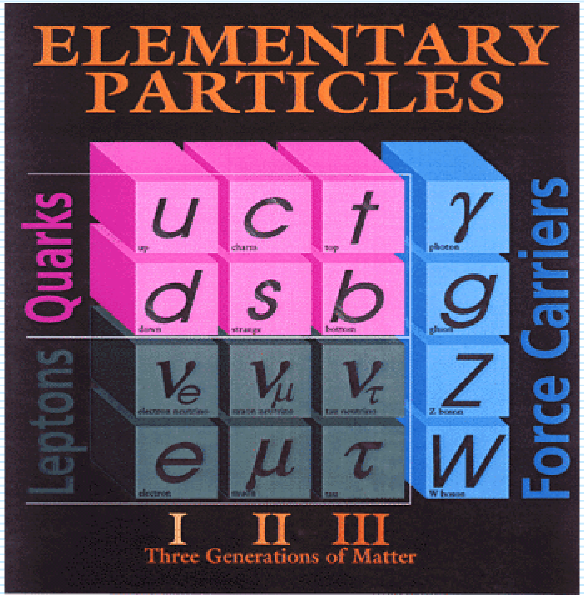
$$H^{D}_{\rm eff} = \frac{1}{2}\mu^{D}_{\nu_{ij}}\bar{\nu}_{iL}\sigma^{\alpha\beta}\nu_{jR}F_{\beta\alpha} + {\rm h.c.}$$

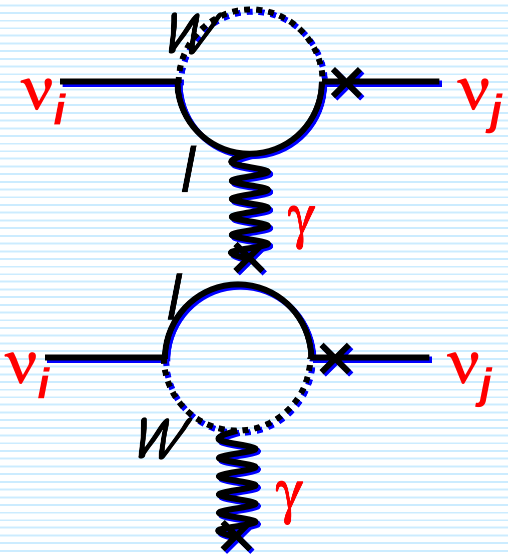
$$H^{M}_{\rm eff} = \frac{1}{2}\mu^{M}_{ij}\bar{\nu}_{iL}\sigma^{\alpha\beta}\nu^{c}_{jL}F_{\beta\alpha} + {\rm h.c.}$$

	Flavor changing MM	Flavor unchanging MM
Dirac	☺	☺
Δ L= 2 **Majorana**	☺	×

- **Dirac:** **can have both diagonal and non-diagonal element**
- **Majorana:** **cannot have diagonal elememts,**
  means spin flip causes flavor changing.

ν magnetic moment in non-minimal SM (+ RH ν)





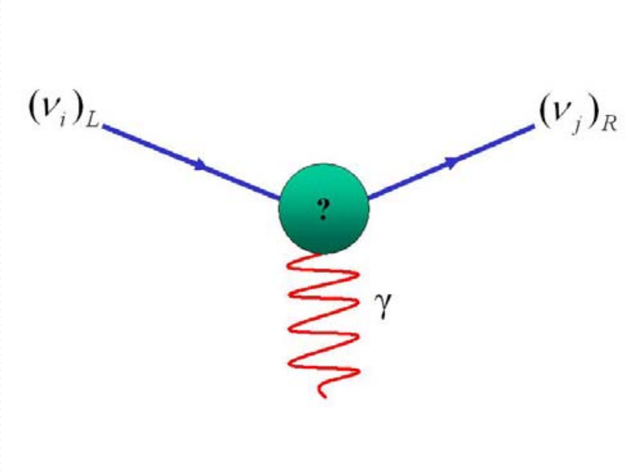
$$SU(3)_{strong} \text{ x } SU(2)_{weak} \text{ x } U_{em}$$

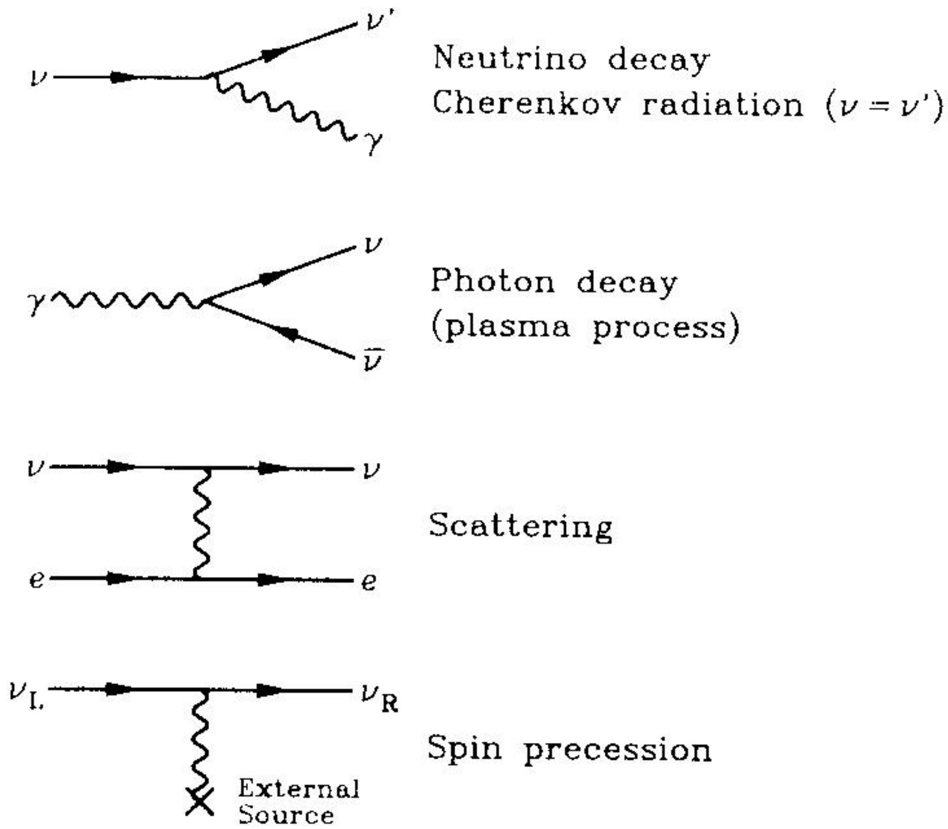
Left-handed	Right-handed
ν_{eL} u_L e_L d_L	e_R u_R d_R ν_{eR}
$\nu_{\mu L}$ c_L μ_L s_L	μ_R c_R s_R $\nu_{\mu R}$
$\nu_{\tau L}$ t_L τ_L b_L	τ_R t_R b_R $\nu_{\tau R}$

added

$$\mu_\nu = \frac{3}{4\pi^2}\frac{G_F m_e m_\nu}{\sqrt{2}}\mu_B \geq 4 \times 10^{-20}\ \mu_B$$

Measuring ν magnetic moment

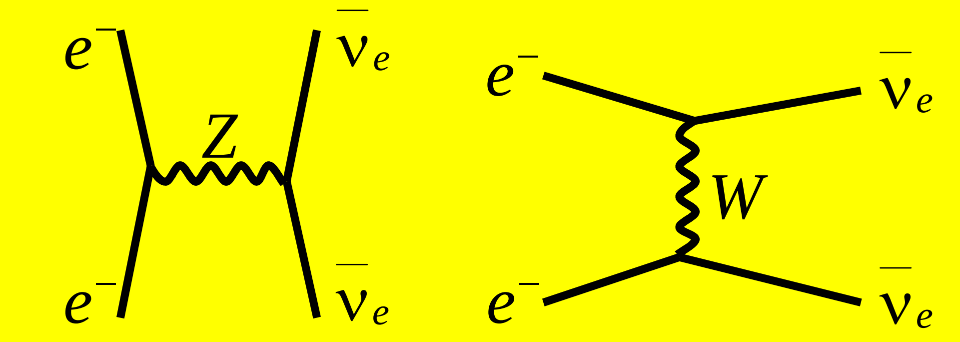




The most restrictive constraint come from astrophysics:
From cooling rate of globular cluster stars ⇒

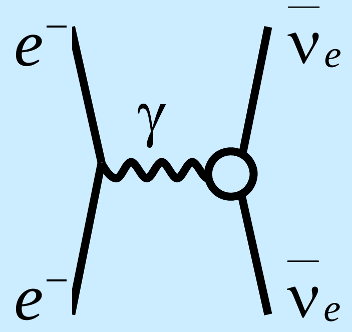
$$\mu^D_{\nu_{ij}} \leq 3 \times 10^{-12}\ \mu_B$$

ν scattering experiments



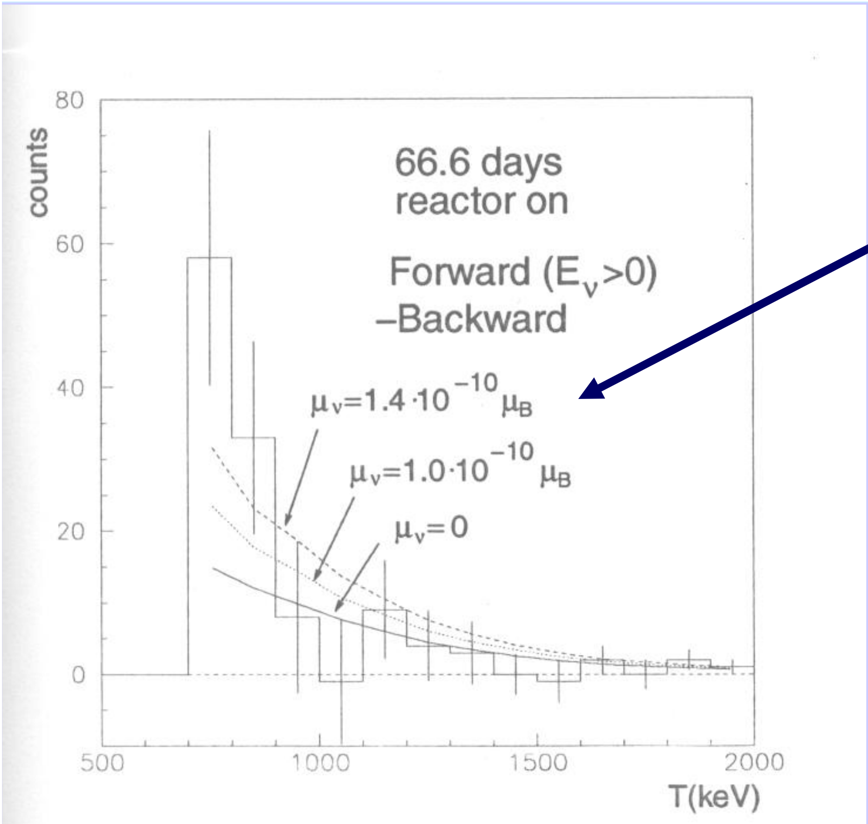
Weak Interactions

+



Magnetic Moment

Sensitivity to electron recoil energy



Reactor ν (Rovno, Bugey, Irvine, Krasnoyarsk, Kalinin), solar ν

$$\mu^D_{\nu_{ej}} \leq 0.9 \times 10^{-10}\ \mu_B,$$
$$\mu^D_{\nu_{\mu j}} \leq 6.8 \times 10^{-10}\ \mu_B,$$
$$\mu^D_{\nu_{\tau j}} \leq 3.9 \times 10^{-7}\ \mu_B$$

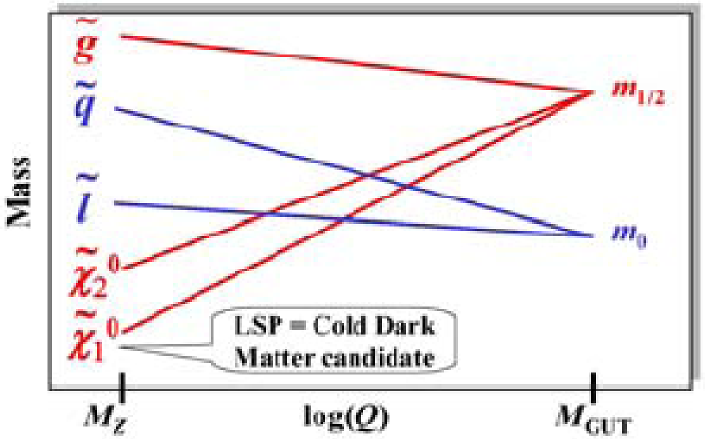
$\mu^D_{\nu e} \leq 3.2\ 10^{-11} \mu_B$ GEMMA exp.

Minimal Supergravity Model (mSUGRA)

SUSY model with two Higgs fields in the framework of unification

All SUSY masses are unified at the grand unified scale

$m_{1/2}$ for gaugino masses
m_0 for squarks and sleptons



$m_{1/2}$ = gaugino mass parameter
$m_0(M_2)$ = scalar mass parameter for squarks and sleptons
A_0 = Common Yukawa coupling
(A_b-bottom sector
A_t-top sector)
$\tan\beta = \langle H_1\rangle/\langle H_2\rangle$
μ = Higgsino mass parameter

GUT constrained low energy spectrum found by solving RGE

- **finite Yukawa coupling at GUT scale**
- **requirements for masses at low energies**
- **FCNC phenomenology ($b \rightarrow sg$ processes)**

Magnetic moment and ν mass

SUSY input				The SUSY conversion coefficient		
A_0 $[GeV]$	m_0 $[GeV]$	$m_{1/2}$ $[GeV]$	$\tan\beta$	$f^q_{\rm SUSY}$ $[eV^{-1}]$	$f^{q-CKM}_{\rm SUSY}$ $[eV^{-1}]$	$f^\ell_{\rm SUSY}$ $[eV^{-1}]$
100	150	150	5	$(0.3, 1.0)\times 10^{-16}$	$(2.8, 8.8)\times 10^{-17}$	$(0.5, 1.5)\times 10^{-15}$
			19	$(0.3, 1.2)\times 10^{-16}$	$(2.8, 9.8)\times 10^{-17}$	$(0.5, 1.6)\times 10^{-15}$
500	1000	1000	5	$(1.1, 2.8)\times 10^{-18}$	$(1.0, 2.4)\times 10^{-18}$	$(1.7, 4.1)\times 10^{-17}$
			19	$(1.1, 3.1)\times 10^{-18}$	$(1.0, 2.7)\times 10^{-18}$	$(1.7, 4.3)\times 10^{-17}$

Loop integrals

$$w^\ell_{jk} = \frac{\sin 2\phi^k}{2}\left(\frac{y_2^{jk}\ln y_2^{jk} - y_2^{jk} + 1}{(1-y_2^{jk})^2} - (y_2 \to y_1)\right)$$

$$v^q_{jk} = \frac{\sin 2\theta^k}{2}\left(\frac{\ln x_2^{jk}}{1-x_2^{jk}} - \frac{\ln x_1^{jk}}{1-x_1^{jk}}\right)$$

$$x_1^{jk} \equiv m^2_{dj}/m^2_{\tilde{d}_1^k}, \qquad x_2^{jk} \equiv m^2_{dj}/m^2_{\tilde{d}_2^k}$$

Magnetic moment expressed with elements of ν mass matrix

$$\mu^q_{\nu_{ii'}} \simeq (1-\delta_{ii'})\frac{4}{3}\mu_B m_{e^1}\mathcal{M}^q_{ii'}\left[\frac{\sum_a V_{ja}V_{la}w^q_{ak}/m_{q^a}}{\sum_a V_{ja}V_{la}v^q_{ak}m_{q^a}}\right]_{\max}$$

$$\equiv (1-\delta_{ii'})\mathcal{M}^q_{ii'}f^q_{\rm SUSY}$$

Calculated ν magnetic moments

SUSY input in GeV				Transition magnetic moment $\mu_{\nu_{ij}}$ in μ_B		
A_0	m_0	$m_{1/2}$	ij	$0\nu\beta\beta$ constraints	inverted hierarchy	normal hierarchy
				lepton-slepton loop mechanism		
100	150	150	$e\mu, e\tau$	$\leq (0.62, 2.10) \times 10^{-15}$	$(0.01, 5.00) \times 10^{-17}$	$(0.13, 1.60) \times 10^{-17}$
			$\mu\tau$	$\leq (0.50, 1.70) \times 10^{-15}$	$(0.45, 3.80) \times 10^{-17}$	$(0.87, 4.10) \times 10^{-17}$
500	1000	1000	$e\mu, e\tau$	$\leq (2.20, 5.50) \times 10^{-17}$	$(0.04, 13.0) \times 10^{-19}$	$(0.47, 4.20) \times 10^{-19}$
			$\mu\tau$	$\leq (1.80, 4.50) \times 10^{-17}$	$(0.16, 1.00) \times 10^{-18}$	$(0.32, 1.10) \times 10^{-18}$
				quark-squark loop mechanism (without d-quarks mixing)		
100	150	150	$e\mu, e\tau$	$\leq (0.41, 1.50) \times 10^{-16}$	$(0.08, 36.0) \times 10^{-19}$	$(0.08, 1.10) \times 10^{-18}$
			$\mu\tau$	$\leq (0.33, 1.20) \times 10^{-16}$	$(0.30, 2.80) \times 10^{-18}$	$(0.58, 3.00) \times 10^{-18}$
500	1000	1000	$e\mu, e\tau$	$\leq (1.40, 4.00) \times 10^{-18}$	$(0.03, 9.60) \times 10^{-20}$	$(0.30, 3.00) \times 10^{-20}$
			$\mu\tau$	$\leq (1.20, 3.20) \times 10^{-18}$	$(1.00, 7.30) \times 10^{-20}$	$(2.00, 7.80) \times 10^{-20}$
				quark-squark loop mechanism (with d-quarks mixing)		
100	150	150	$e\mu, e\tau$	$\leq (0.36, 1.30) \times 10^{-16}$	$(0.07, 31.0) \times 10^{-19}$	$(0.76, 9.70) \times 10^{-19}$
			$\mu\tau$	$\leq (0.29, 1.00) \times 10^{-16}$	$(0.27, 2.30) \times 10^{-18}$	$(0.51, 2.50) \times 10^{-18}$
500	1000	1000	$e\mu, e\tau$	$\leq (1.30, 3.40) \times 10^{-18}$	$(0.03, 8.30) \times 10^{-20}$	$(0.28, 2.60) \times 10^{-20}$
			$\mu\tau$	$\leq (1.10, 2.80) \times 10^{-18}$	$(0.97, 6.40) \times 10^{-20}$	$(1.90, 6.80) \times 10^{-20}$

Main uncertainty:
MSSM parameter space

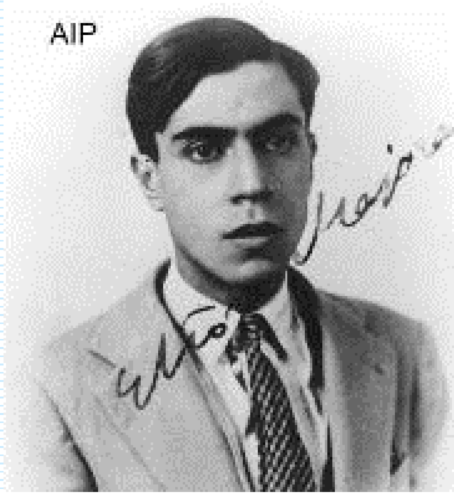
Bounds

Predictions !

What is the nature of neutrinos?

$\nu \Rightarrow$

theory



Only the 0νββ-decay can answer this fundamental question

^{76}Ge

GERDA

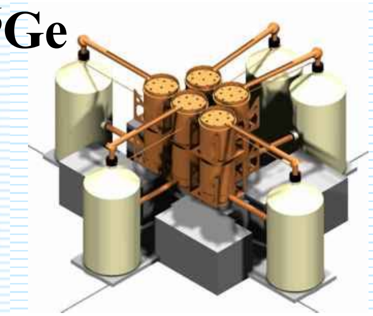
^{130}Te

CUORE

^{82}Se

^{150}Nd

^{76}Ge



^{106}Cd

^{116}Cd

^{100}Mo

^{150}Nd

^{136}Xe

ic

+ (?)

Double electron capture?

**Like most people,
physicists enjoy a good mystery.**

**When you start investigating a mystery
you rarely know where it is going**

Mathematics is Egyptian

Neutrino physics is Babylonian

The truth is covered in ν–experiments.

Thanks to neutrinos we understand Sun, Supernova, Earth (nuclear reactions)