

# *Electromagnetic properties of the neutrino*

*Vth International  
Pontecorvo School  
of Neutrino Physics  
Alushta, 08/09/2012*

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# 2012

- *the Year of the Higgs Boson*  
(... probably ...)
- *status of Standard Model*  
(... lecture of Igor Boiko ...)



*is the only  
known*

*particle*

*Beyond*

*Standard*

*Model*



*is quite*

*invisible*

*particle*

✓ exhibits unexpected properties (*puzzles*)

W. Pauli, 1930

E. Fermi,  
1933

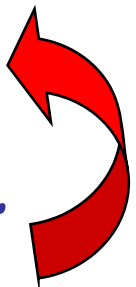
- neutral 'neutron'  $\Rightarrow$  ✓
- $m_\nu = 0$  (now  $m_\nu \neq 0$  but ? after 80 yrs !)

● probably  $\mu_\nu \neq 0$  ! ?

...recent claim for  
new experimental  
bound on  $\mu_\nu$   
(with atomic ionization  
effect) continue  
chain of puzzles...

- Pauli himself wrote to Baade:

"Today I did something a physicist should never do.  
I predicted something which will never be observed  
experimentally...".



*H. Bethe, R. Peierls, «The 'neutrino'»  
Nature 133 (1934) 532,*

- *«There is no practically possible way of observing the neutrino»*

*... puzzles ...*

- *...up to now absolute value ?*

$$m_\nu \neq 0$$

*after 80 years left !*

*... however ...*



## Crucial role of neutrino

**ν** is a “tiny” particle :

● **very light**  $m_{\nu f} \ll m_f, \quad f = e, \mu, \tau$

● **electrically neutral**  $q_\nu = 0$   $q_\nu < 4 \times 10^{-17} e$

● **with very small magnetic moment**  $\mu_\nu ?$

$\sigma_{\nu e N} \sim 10^{-39} \text{ cm}^2$   $\nu$ -N scattering  
 $\sigma_{\bar{\nu} e p} \sim 10^{-40} \text{ cm}^2$  inverse  $\beta$ -decay  
 $\sigma_{\nu e e} \sim 10^{-43} \text{ cm}^2$   $\nu$ -e scattering

● **weak interactions are**  $\bar{\nu} + p \rightarrow e^+ + n$

**indeed weak**  $\sigma \sim 10^{-43} \text{ cm}^2$   $L \sim 10^{15} \text{ km}$

$E_\nu \sim 3 \text{ MeV}$  ... free path in water...

at the final stages of development of particular elementary particle physics framework

horizons of new physics



*manifests itself most vividly  
under the influence of  
extreme external conditions:*

- *strong external electromagnetic fields*
- and*
- *dense background matter*



## *Outline*

- ✓ *electromagnetic  
properties  
(review)*



1

*Carlo Giunti, Alexander Studenikin :*  
*“Neutrino electromagnetic properties”*  
*Phys.Atom.Nucl. 73, 2089-2125 (2009)*  
*arXiv:0812.3646 v5, Apr 12, 2010*

2

*A.Studenikin :* *“Neutrino magnetic moment: a window to new physics”*  
*Nucl.Phys.B (Proc.Supl.) 188, 220 (2009)*

3

*C. Giunti, A. Studenikin :* *“Electromagnetic properties of neutrinos”*  
*J.Phys.: Conf.Series. 203 (2010) 012100*  
*arXiv:1006.1502 June 8, 2010*

4

*C.Broggini, C.Giunti, A.Studenikin :*  
*“Electromagnetic properties of neutrinos”*,  
*in: “Neutrino Physics”(Adv. in High Ener. Phys. )*  
*arXiv: 1207.3980 July 17, 2012*



5

*C.Giunti, A.Studenikin :* *“Theory and phenomenology  
of neutrino electromagnetic properties”*  
*Rev.Mod.Phys. (in preparation)*

# Outline (short list)

- ✓ electromagnetic properties - theory
- ✓ magnetic moment - experiment
- constraints on ✓ electromagnetic properties
- ✓ electromagnetic interactions  
( $\nu$ - $\gamma$  processes)

## 0. Introduction

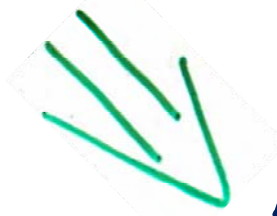
1. ✓ magnetic moment in experiments
2. New experimental result on  $\mu_\nu$
3. ✓ electromagnetic properties - theory
  - 3.1 ✓ vertex function
  - 3.2  $\mu_\nu$  (arbitrary masses)
  - 3.3 relationship between  $m_\nu$  and  $\mu_\nu$
  - 3.4 ✓ vertex function in case of flavour mixing
  - 3.5 ✓ dipole moments in case of mixing
  - 3.6  $\mu_\nu$  in left-right symmetry models
  - 3.7 astrophysical bounds on  $\mu_\nu$
  - 3.8 ✓ millicharge (**Red Giants** cooling etc)
  - 3.9 ✓ charge radius and anapole moment
  - 3.10 ✓ electromagnetic properties in **matter** and **e.m.f.**
4. Effects of ✓ electromagnetic properties
  - 3.11 ✓ radiative decay, *Ch* radiation and *Spin Light* of ✓ in matter
  - 3.12 ✓ radiative  $2^* \gamma$ -decay
  - 3.13 ✓ spin-flavour oscillations
5. Direct-Indirect influence of **e.m.f.** on ✓
6. Conclusion

## Outline (II)

- *Neutrino magnetic moments*

- *results of recent experimental searches for upper bound on  $\mu_\nu$*

- *our corresponding theoretical studies of  $\nu$ - $e$  scattering*



*present best indeed  
laboratory limit*

*on  $\mu_\nu$  (GEMMA Coll.)*

K.Kouzakov, A.Studenikin,

- “Magnetic neutrino scattering on atomic electrons revisited” ●  
**Phys.Lett. B 105 (2011) 061801**, arXiv: 1011.5847
- “Electromagnetic neutrino-atom collisions: The role of electron binding”  
**Nucl.Phys.B (Proc.Suppl.) 217 (2011) 353**  
arXiv: 1108.2872, 14 Aug 2011

K.Kouzakov, A.Studenikin, M.Voloshin,

- “Neutrino-impact ionization of atoms in search for neutrino magnetic moment”, **Phys.Rev.D 83 (2011) 113001**  
arXiv: 1101.4878, 25 Jan 2011
- “On neutrino-atom scattering in searches for neutrino magnetic moments” **Nucl.Phys.B (Proc.Suppl.) 2011 (Proc. of Neutrino 2010 Conf.)**  
arXiv: 1102.0643, 3 Feb 2011
- “Testing neutrino magnetic moment in ionization of atoms by neutrino impact”, **JETP Lett. 93 (2011) 699**  
arXiv: 1105.5543, 27 May 2011

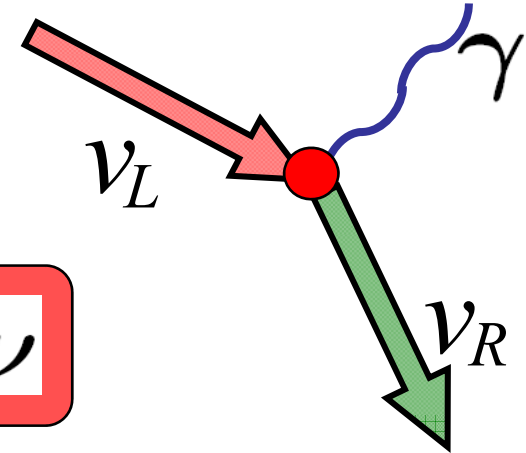
M.Voloshin,

- “Neutrino scattering on atomic electrons in search for neutrino magnetic moment”  
**Phys.Rev.Lett. 105 (2010) 201801**, arXiv: 1008.2171

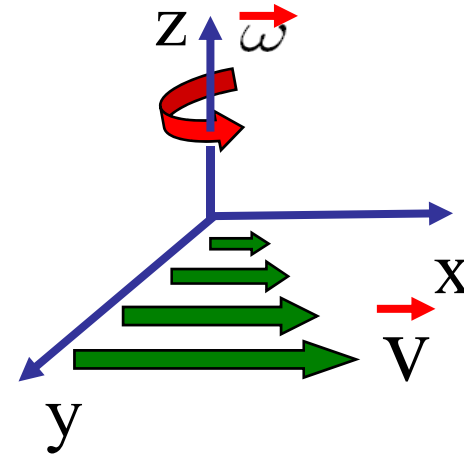
Outline (III)

2 problems:

1. Spin Light of  $\nu$   
in matter



2.  $\nu$  energy quantization  
in rotating matter



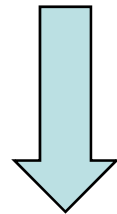
$\nu$  quantum states in matter  
New approach to particles in matter

# Method of exact solutions

Modified *Dirac equations* for  $\nu$  (and  $e$ )  
(containing the correspondent effective matter potentials)

+

*exact solutions* (particles wave functions)



a basis for investigation of different phenomena  
which can proceed when *neutrinos* (and *electrons*)  
move in *dense media*  
(*astrophysical* and *cosmological* environments).



«method of exact solutions»

# Interaction of particles in external electromagnetic fields ( **Furry representation** in quantum electrodynamics )

Potential of electromagnetic field

$$A_\mu(x) = A_\mu^q(x) + A_\mu^{ext}(x),$$

evolution operator

$$U_F(t_1, t_2) = T \exp \left[ -i \int_{t_1}^{t_2} j^\mu(x) A_\mu^q(x) dx \right],$$

quantized part  
of potential

charged particles **current**

$$j_\mu(x) = \frac{e}{2} [\bar{\Psi}_F \gamma_\mu, \Psi_F],$$

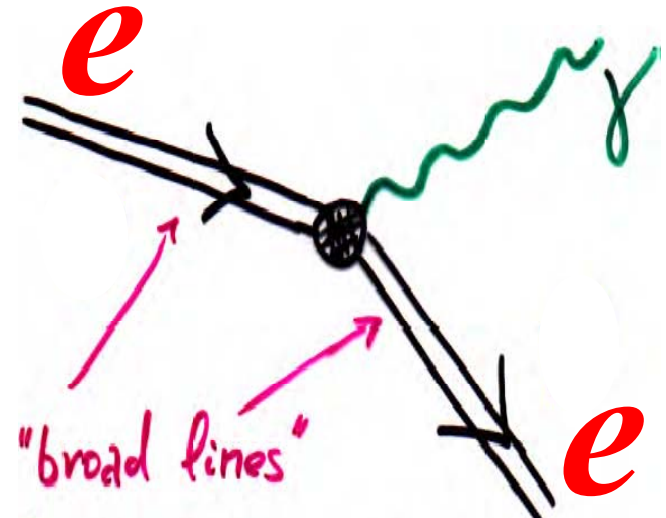
**Dirac equation** in external classical (non-quantized) field  $A_\mu^{ext}(x)$

$$\left\{ \gamma^\mu \left( i\partial_\mu - eA_\mu^{ext}(x) \right) - m_e \right\} \Psi_F(x) = 0$$



...beyond perturbation series expansion,  
**strong fields and non linear effects...**

$B_\perp$   
 $e \rightarrow e + \gamma$   
synchrotron radiation



## Outline (IV) and $e$

in matter treated within  
«*method of exact solutions*»  
(of quantum wave equations)

A.Studenikin, A.Ternov,  
“Neutrino quantum states in  
matter”,  
*Phys.Lett.B* 608 (2005) 107;

“Generalized Dirac-Pauli equation  
and neutrino quantum states in  
matter” [hep-ph/0410296](#),

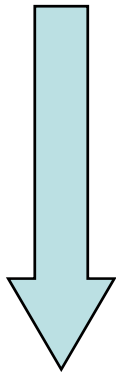
A.Grigoriev, A.Studenikin,  
A.Ternov,  
*Phys.Lett.B* 608 622 (2005)19

- *energy quantization  
in rotating matter...*

- A.Studenikin, “Method of wave equations  
exact solutions in studies of neutrino and  
electron interactions in dense matter”,
- *J.Phys.A:Math.Theor.* 41 (2008) 16402  
Neutrinos and electrons in background  
matter: a new approach”,
  - *Ann. Fond. de Broglie* 31 (2006) 289,
  - *J.Phys.A: Math.Gen.*39 (2006) 6769
  - I.Balantsev, Yu.Popov, A.Studenikin, “On a  
problem of relativistic particles motion in a  
strong magnetic field and dense matter”,  
*J.Phys.A: Math.Theor.* 44 (2011) 255301
  - A.Studenikin, I.Tokarev, “Millicharged  
neutrino with anomalous magnetic  
moment in rotating magnetized matter”,  
[arXiv:1209.3245](#) September 3, 2012

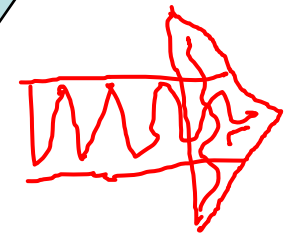
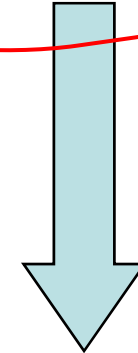
# Outline ( $\nu$ )

**Direct**



and  
influence  
of electromagnetic fields  
on  $\nu$

**Indirect**



through non-trivial  
neutrino electromagnetic  
properties (magnetic moment):

due to e.m. field influence on  
charged particles coupled  
to neutrinos

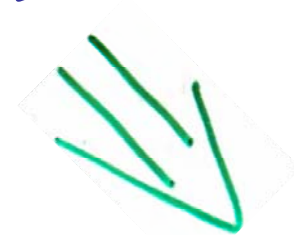
- ★ neutrino spin
- ★ spin-flavour oscillations...
- ★ different  $\nu\bar{\nu}$  processes

- ★ neutron beta-decay in **B**
- ★ change of  $\nu$  oscillation pattern due to matter polarization under influence of external e.m. fields ...



- *A review on  
neutrino electromagnetic  
properties*

*(including magnetic  
moments  $\mu_\nu$  )*



*...Why*

*electromagnetic  
properties of*



*provide a kind of  
window / bridge  
to*

*NEW Physics ?*

*... simple answer ...*

*... in spite of*

- *results of terrestrial laboratory experiments on  $\nu$  EM properties and  $\mu_\nu$*

*as well as*

- *data from astrophysics and cosmology*

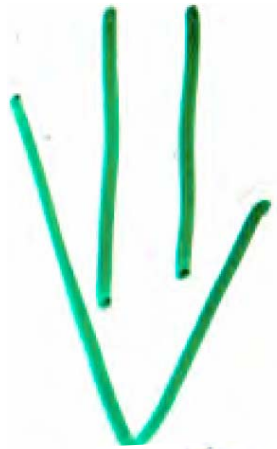
*are in agreement with "ZERO"  $\nu$  EM properties*

*... However, in course of recent development of knowledge on  $\nu$  mixing and oscillations,*

*... simple answer ...*

$$m_\nu \neq 0$$

Neutrino mass



$$m_\nu \neq 0 !$$

Neutrino magnetic moment

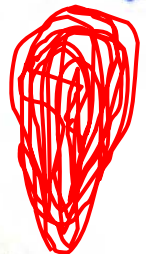

$$\mu_\nu \neq 0$$

 { Lee } 1977  
          { Shrock }  
          { Fujikawa } 1980

... Massive neutrino electromagnetic properties ...





In the Standard Model :  $m_\nu = 0$ ,  
there is no  $\nu_R \Rightarrow$   
 $\nu$  magnetic moment  $\mu_\nu = 0$ .  
Thus,  $\mu_\nu \neq 0 \leftarrow$  beyond the SM.   


*... a tool for studying physics beyond the Standard Model...*

$$m_\nu \neq 0$$

Theory (Standard Model with  $\nu_R$ )

$$\mu_\nu = \frac{3eG_F}{8\sqrt{2}\pi^2} m_\nu \sim 3 \cdot 10^{-19} \mu_B \left( \frac{m_\nu}{1\text{eV}} \right), \quad \mu_B = \frac{e}{2m_e}$$

Lee Shrock, 1977; Fujikawa Shrock, 1980

$$a_e = \frac{\alpha_{QED}}{2\pi} \sim 10^{-3}$$

*anomalous  
magnetic  
moment of  
electron*

... much greater values are desired  
for astrophysical or cosmology

visualization of  $\mu_\nu$

# Astrophysical bounds

$$\mu_\nu \leq 3 \cdot 10^{-12} \mu_B$$

(Red Giant Lumin.)  
etc.

G. Raffelt, D. Dearborn,  
J. Silk, 1989.

Theory (Standard Model with  $\nu_R$ )

$$\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e} \sim 3 \cdot 10^{-19} \mu_B \left( \frac{m_{\nu_e}}{1\text{eV}} \right), \quad \mu_B = \frac{e}{2m_e}$$

Lee Shrock, 1977; Fujikawa Shrock, 1980

*...the present status  
(preliminary conclusion) ...*

*to have visible  $\mu \neq 0$*

*is not an easy task for*

*theoreticians*

*and experimentalists*

*... puzzling*

  $\nu$  *electromagnetic  
properties  
and  $\mu_\nu$*   
*something that is tiny but not zero*

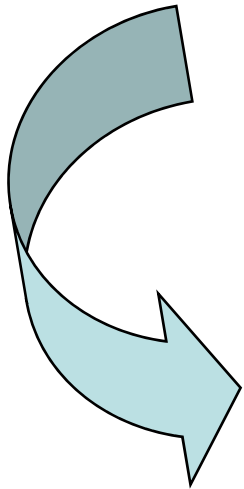
$m_\nu \neq 0 \Rightarrow$

*weak  
gravitational  
and  
electromagnetic  
interactions*



*electromagnetic*

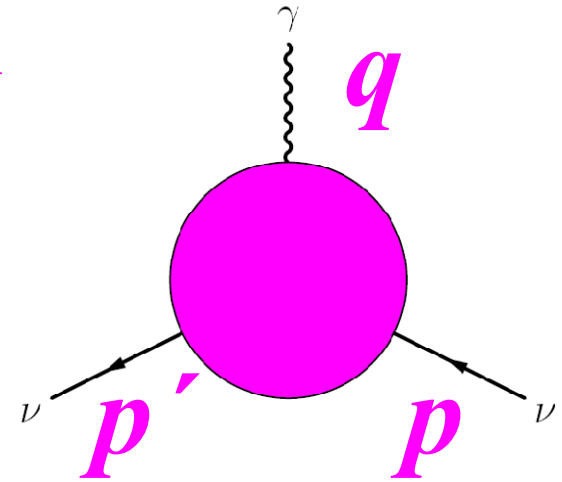
*properties*



*... theoretical introduction...*

# ✓ electromagnetic vertex function

$$\langle \psi(p') | J_\mu^{EM} | \psi(p) \rangle = \bar{u}(p') \Lambda_\mu(q, l) u(p)$$



Matrix element of *electromagnetic current* is a Lorentz vector

## Lorentz covariance (1)

$\Lambda_\mu(q, l)$  should be constructed using

*matrices*  $\hat{1}, \gamma_5, \gamma_\mu, \gamma_5 \gamma_\mu, \sigma_{\mu\nu},$

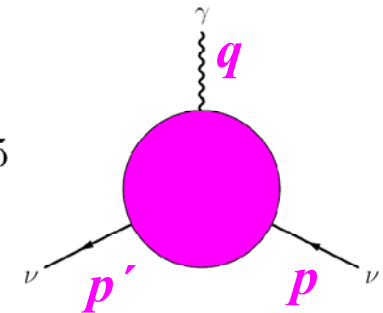
*tensors*  $g_{\mu\nu}, \epsilon_{\mu\nu\sigma\gamma}$

*vectors*  $q_\mu$  and  $l_\mu$

$$q_\mu = p'_\mu - p_\mu, \quad l_\mu = p'_\mu + p_\mu$$

**Vertex function**  $\Lambda_\mu(q, l) \longrightarrow$  there are three sets of operators:

- $\hat{\mathbf{1}}q_\mu, \hat{\mathbf{1}}l_\mu, \gamma_5 q_\mu, \gamma_5 l_\mu$
- $\not{q}q_\mu, \not{l}q_\mu, \gamma_5 q_\mu, \gamma_5 \not{q}q_\mu, \gamma_5 \not{l}q_\mu, \sigma_{\alpha\beta} q^\alpha l^\beta q_\mu, (q_\mu \leftrightarrow l_\mu)$
- $\gamma_\mu, \gamma_5 \gamma_\mu, \sigma_{\mu\nu} q^\nu, \sigma_{\mu\nu} l^\nu.$
- $\epsilon_{\mu\nu\sigma\gamma} \sigma^{\alpha\beta} q^\nu, \epsilon_{\mu\nu\sigma\gamma} \sigma^{\alpha\beta} l^\nu, \epsilon_{\mu\nu\sigma\gamma} \sigma^{\nu\beta} q_\beta q^\sigma l^\gamma,$   
 $\epsilon_{\mu\nu\sigma\gamma} \sigma^{\nu\beta} l_\beta q^\sigma l^\gamma, \epsilon_{\mu\nu\sigma\gamma} \gamma^\nu q^\sigma l^\gamma \hat{\mathbf{1}}, \epsilon_{\mu\nu\sigma\gamma} \gamma^\nu q^\sigma l^\gamma \gamma_5$



✓ **vertex function** (using Gordon-like identities)

$$\Lambda_\mu(q, l) = f_1(q^2)q_\mu + f_2(q^2)q_\mu \gamma_5 + f_3(q^2)\gamma_\mu + f_4(q^2)\gamma_\mu \gamma_5 + f_5(q^2)\sigma_{\mu\nu} q^\nu + f_6(q^2)\epsilon_{\mu\nu\rho\gamma} \sigma^{\rho\gamma} q^\nu,$$

the only dependence on  $q^2$  remains because  $p^2 = p'^2 = m^2, l^2 = 4m^2 - q^2$



## *Gordon-like identities*

$$\bar{u}(\mathbf{p}_1)\gamma^\mu u(\mathbf{p}_2) = \frac{1}{2m}\bar{u}(\mathbf{p}_1)[l^\mu + i\sigma^{\mu\nu}q_\nu]u(\mathbf{p}_2)$$

$$\bar{u}(\mathbf{p}_1)\gamma^\mu\gamma_5 u(\mathbf{p}_2) = \frac{1}{2m}\bar{u}(\mathbf{p}_1)[\gamma_5 q^\mu + i\gamma_5\sigma^{\mu\nu}l_\nu]u(\mathbf{p}_2)$$

$$\bar{u}(\mathbf{p}_1)i\sigma^{\mu\nu}l_\nu u(\mathbf{p}_2) = -\bar{u}(\mathbf{p}_1)q^\nu u(\mathbf{p}_2)$$

$$\bar{u}(\mathbf{p}_1)i\sigma^{\mu\nu}q_\nu u(\mathbf{p}_2) = \bar{u}(\mathbf{p}_1)[2m\gamma^\mu l^\mu]u(\mathbf{p}_2)$$

$$\bar{u}(\mathbf{p}_1)i\sigma^{\mu\nu}\gamma_5 q_\nu u(\mathbf{p}_2) = -\bar{u}(\mathbf{p}_1)l^\mu\gamma_5 u(\mathbf{p}_2)$$

$$\bar{u}(\mathbf{p}_1)[\epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\beta q_\mu l_\nu]u(\mathbf{p}_2) = \bar{u}(\mathbf{p}_1)\{-i[q^\alpha \not{\lambda} - l^\alpha \not{\not{q}}] + i(q^2 - 4m^2)\gamma^\alpha + 2im(l^\alpha + q^\alpha)\}u(\mathbf{p}_2)$$

$$\bar{u}(\mathbf{p}_1)[\epsilon^{\alpha\mu\nu\beta}\gamma_\beta q_\mu l_\nu]u(\mathbf{p}_2) = \bar{u}(\mathbf{p}_1)\{i[q^\alpha \not{\lambda} - l^\alpha \not{\not{q}}]\gamma_5 + iq^2\gamma_5\gamma^\alpha - 2im(l^\alpha + q^\alpha)\gamma_5\}u(\mathbf{p}_2)$$

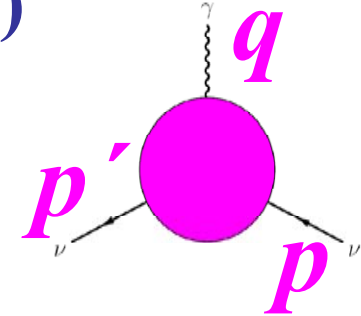
$$\bar{u}(\mathbf{p}_1)[\epsilon^{\mu\nu\alpha\beta}q_\alpha l_\beta\gamma_\nu\gamma_5]u(\mathbf{p}_2) = \frac{i}{2m}\bar{u}(\mathbf{p}_1)[\epsilon^{\mu\nu\alpha\beta}q_\alpha l_\beta\sigma_{\nu\rho}q^\rho]u(\mathbf{p}_2)$$

$$\bar{u}(\mathbf{p}_1)[\epsilon^{\mu\nu\alpha\beta}q_\alpha l_\beta\sigma_{\nu\rho}l^\rho]u(\mathbf{p}_2) = 0$$

# Electromagnetic gauge invariance (2)

(requirement of current conservation)

$$\partial_\mu j^\mu = 0$$



$$f_1(q^2)q^2 + f_2(q^2)q^2\gamma_5 + 2mf_4(q^2)\gamma_5 = 0,$$

$$f_1(q^2) = 0, \quad f_2(q^2)q^2 + 2mf_4(q^2) = 0$$

✓ vertex function

$$\Lambda_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2)i\sigma_{\mu\nu}q^\nu + f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)(q^2\gamma_\mu - q_\mu\not{q})\gamma_5$$

charge

dipole electric and magnetic

anapole

... consistent with  
Lorentz-covariance (1)

4 Form Factors

+  
electromagnetic gauge invariance (2)



Matrix element of **electromagnetic current** between neutrino states

$$\langle \nu(p') | J_\mu^{EM} | \nu(p) \rangle = \bar{u}(p') \Lambda_\mu(q) u(p)$$

where **vertex function** generally contains **4 form factors**

$$\Lambda_\mu(q) = f_Q(q^2) \gamma_\mu + f_M(q^2) i \sigma_{\mu\nu} q^\nu - f_E(q^2) \sigma_{\mu\nu} q^\nu \gamma_5$$

1. electric

dipole

2. magnetic

3. electric

$$+ f_A(q^2) (q^2 \gamma_\mu - q_\mu \not{q}) \gamma_5$$

4. anapole



● Hermiticity and discrete symmetries of EM current  $J_\mu^{EM}$  put constraints on **form factors**

**Dirac** ✓

- 1) CP invariance + hermiticity  $\implies f_E = 0$ ,
- 2) at zero momentum transfer only electric charge  $f_Q(0)$  and magnetic moment  $f_M(0)$  contribute to  $H_{int} \sim J_\mu^{EM} A^\mu$ ,
- 3) hermiticity itself  $\implies$  three form factors are real:  $Im f_Q = Im f_M = Im f_A = 0$ .



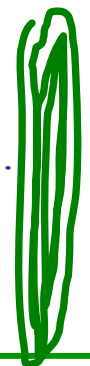
**Majoran** ✓

- 1) from CPT invariance (regardless CP or ~~CP~~).

$$f_Q = f_M = f_E = 0$$



...as early as 1939, W.Pauli...



EM properties  $\implies$  a way to distinguish **Dirac** and **Majorana** ✓

In general case **matrix element** of  $J_\mu^{EM}$  can be considered between **different initial**  $\psi_i(p)$  **and final**  $\psi_j(p')$  **states of different masses**  $p^2 = m_i^2, p'^2 = m_j^2$ :

$$\langle \psi_j(p') | J_\mu^{EM} | \psi_i(p) \rangle = \bar{u}_j(p') \Lambda_\mu(q) u_i(p) \quad \dots \text{beyond beyond SM...}$$

and

$$\Lambda_\mu(q) = \left( f_Q(q^2)_{ij} + f_A(q^2)_{ij} \gamma_5 \right) (q^2 \gamma_\mu - q_\mu \not{q}) + f_M(q^2)_{ij} i \sigma_{\mu\nu} q^\nu + f_E(q^2)_{ij} \sigma_{\mu\nu} q^\nu \gamma_5$$



**form factors** are matrices in  $\checkmark$  mass eigenstates space.

**Dirac**  $\checkmark$

*(off-diagonal case  $i \neq j$ )*

**Majorana**  $\checkmark$

1) **hermiticity itself** does not apply restrictions on form factors,

1) **CP invariance + hermiticity**

2) **CP invariance + hermiticity**

$$\mu_{ij}^M = 2\mu_{ij}^D \quad \text{and} \quad \epsilon_{ij}^M = 0 \quad \text{or}$$

$$\mu_{ij}^M = 0 \quad \text{and} \quad \epsilon_{ij}^M = 2\epsilon_{ij}^D$$

$f_Q(q^2), f_M(q^2), f_E(q^2), f_A(q^2)$

are relatively real (no relative phases).

**... quite different EM properties ...**

... importance of  $\mu_\nu$  studies...

If diagonal  $\mu_\nu \neq 0$

were confirmed



then  $\checkmark$  Dirac



... for  $\checkmark$  Majorana  
non-diagonal = transitional  
 $\mu_\nu \neq 0$

... progress  
in experimental  
studies of  $\mu_\nu$



**...two remarks ...**

# 1

## Difference between electromagnetic vertex function of massive and massless $\nu$

*Dirac Form factor (the only one...)*



$m_\nu = 0$ :

$$\bar{u}(p') \Lambda_\mu(q) u(p) = f_D(q^2) \bar{u}(p') \gamma_\mu (1 + \gamma_5) u(p)$$

*electric charge  $f_Q(q^2)$  and anapole  $f_A(q^2)$  FF are related to DF (and to each other):*

$$f_Q(q^2) = f_D(q^2), \quad f_A(q^2) = f_D(q^2)/q^2$$



*In case  $m \neq 0$  there is no such simple relation (because term  $q_\mu \not{q} \gamma_5$  in anapole FF cannot be neglected).*

# 2

## ✓ form factors in gauge models

$$\langle \psi_j(p') | J_\mu^{EM} | \psi_i(p) \rangle = \bar{u}_j(p') \Lambda_\mu(q) u_i(p)$$

● **Form Factors at zero momentum transfer ( $q^2 = 0$ ) are elements of scattering matrix** in any consistent theoretical model **FF in matrix element**  $\Rightarrow$  **(gauge independent and finite.)**

Therefore

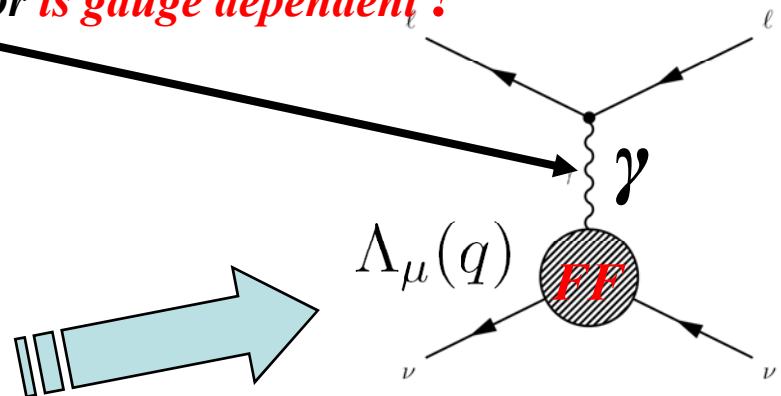
**FF at  $q^2 = 0$  determine static properties of ✓ that can be probed (measured) in direct interaction with external em fields.**

**This is the case for  $f_Q(q^2), f_M(q^2), f_E(q^2)$  in minimally extended SM ( $f_A(q^2)$  is an exceptional case)**

● **In non-Abelian gauge models, FF at  $q^2 \neq 0$  can be not invariant under gauge transformation**

**because (in general) off-shell photon propagator is gauge dependent !**

- ... One-photon approximation is not enough to get physical quantity...
- ... FF in matrix element cannot be directly measured in experiment with em field ...
- ... FF can contribute to higher order processes accessible for experimental observation.







**magnetic moment ?**

**Dipole magnetic**  $f_M(q^2)$  and **electric**  $f_E(q^2)$

are most well studied and theoretically understood  
among **form factors**

...because even in the limit  $q^2 \rightarrow 0$  they may have  
nonvanishing values

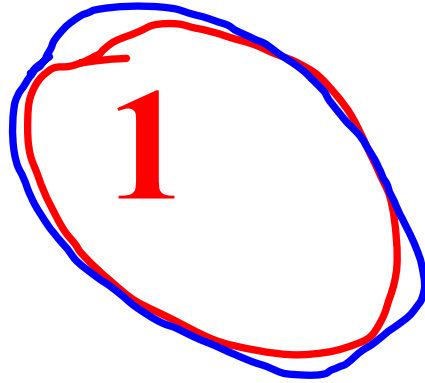
$$\mu_\nu = f_M(0)$$

$\nu$  magnetic moment



$$\epsilon_\nu = f_E(0)$$

$\nu$  electric moment ???



# *magnetic moment in experiments*

**Samuel Ting**

*( wrote on the wall at Department of Theoretical  
Physics of Moscow State University ) :*

*“Physics is an experimental science”*

# Studies of $\nu$ - $e$ scattering - most sensitive method of experimental investigation of $\mu_\nu$

Cross-section:

$$\frac{d\sigma}{dT}(\nu + e \rightarrow \nu + e) = \left(\frac{d\sigma}{dT}\right)_{\text{SM}} + \left(\frac{d\sigma}{dT}\right)_{\mu_\nu}$$

where the Standard Model contribution

$$\left(\frac{d\sigma}{dT}\right)_{\text{SM}} = \frac{G_F^2 m_e}{2\pi} \left[ (g_V + g_A)^2 + (g_V - g_A)^2 \left(1 - \frac{T}{E_\nu}\right)^2 + (g_A^2 - g_V^2) \frac{m_e T}{E_\nu^2} \right],$$

$$\left(\frac{d\sigma}{dT}\right)_{\mu_\nu} = \frac{\pi \alpha_{em}^2}{m_e^2} \left[ \frac{1 - T/E_\nu}{T} \right] \mu_\nu^2$$

$T$  is the electron recoil energy

$$0 \leq T \leq \frac{2E_\nu^2}{2E_\nu + m_e},$$

$$g_V = \begin{cases} 2 \sin^2 \theta_W + \frac{1}{2} & \text{for } \nu_e, \\ 2 \sin^2 \theta_W - \frac{1}{2} & \text{for } \nu_\mu, \nu_\tau, \end{cases} \quad g_A = \begin{cases} \frac{1}{2} & \text{for } \nu_e, \\ -\frac{1}{2} & \text{for } \nu_\mu, \nu_\tau \end{cases} \quad \begin{matrix} \text{for anti-neutrinos} \\ g_A \rightarrow -g_A \end{matrix}$$

to incorporate **charge radius**:  $g_V \rightarrow g_V + \frac{2}{3} M_W^2 \langle r^2 \rangle \sin^2 \theta_W$

- $$\frac{d\sigma}{dT}(\nu + e \rightarrow \nu + e) = \left(\frac{d\sigma}{dT}\right)_{\text{SM}} + \left(\frac{d\sigma}{dT}\right)_{\mu\nu}$$

$\nu$ - $\gamma$  coupling ... valid for  $\nu$  scattering on free  $e$

- $$\left(\frac{d\sigma}{dT}\right)_{\mu\nu} = \frac{\pi\alpha_{em}^2}{m_e^2} \left[ \frac{1 - T/E_\nu}{T} \right] \mu_\nu^2$$

with change of helicity, contrary to SM

$T$  is the electron recoil energy:  $0 \leq T \leq \frac{2E_\nu^2}{2E_\nu + m_e}$

If neutrino has electric dipole moment,  
or electric or magnetic transition moments,  
 these quantities would also contribute to scattering cross section

$$\mu_\nu^2 = \sum_{j = \nu_e, \nu_\mu, \nu_\tau} |\mu_{ij} - \epsilon_{ij}|^2, \quad i \text{ refers to initial neutrino flavour}$$

Possibility of distractive interference between **magnetic** and **electric** transition moments of **Dirac** neutrino  
 (**Majorana** neutrino has only magnetic or electric transition moment, but not both if CP is conserved)

# Effective $\nu_e$ magnetic moment measured in $\nu$ - $e$ scattering experiments ?

$$\mu_e^2$$

## Two steps:

- 1) consider  $\nu_e$  as superposition of mass eigenstates ( $i=1,2,3$ ) at some distance  $L$  from the source, and then sum up magnetic moment contributions to  $\nu$ - $e$  scattering amplitude (of each of mass components) induced by their magnetic moments

$$A_j \sim \sum_i U_{ei} e^{-iE_i L} \mu_{ji}$$

*J. Beacom,  
P. Vogel, 1999*

- 2) amplitudes combine incoherently in total cross section

$$\sigma \sim \mu_e^2 = \sum_j \left| \sum_i U_{ei} e^{-iE_i L} \mu_{ji} \right|^2$$

*C. Giunti,  
A. Studenikin,  
2009*

**NB!** Summation over  $j=1,2,3$  is outside the square because of incoherence of different final mass states contributions to cross section.

# Effective $\nu$ magnetic moment in experiments

(for neutrino produced as  $\nu_l$  with energy  $E_\nu$   
and after traveling a distance  $L$ )

$$\mu_\nu^2(\nu_l, L, E_\nu) = \sum_j \left| \sum_i U_{li} e^{-iE_i L} \mu_{ji} \right|^2$$

where

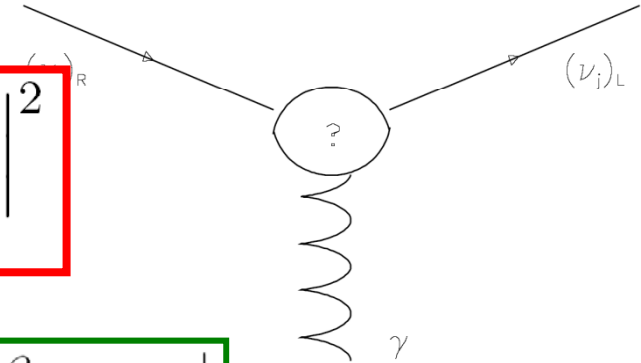
neutrino mixing matrix

$$\mu_{ij} \equiv |\beta_{ij} - \epsilon_{ij}|$$

magnetic and electric moments

Observable  $\mu_\nu$  is an effective parameter that depends on neutrino flavour composition at the detector.

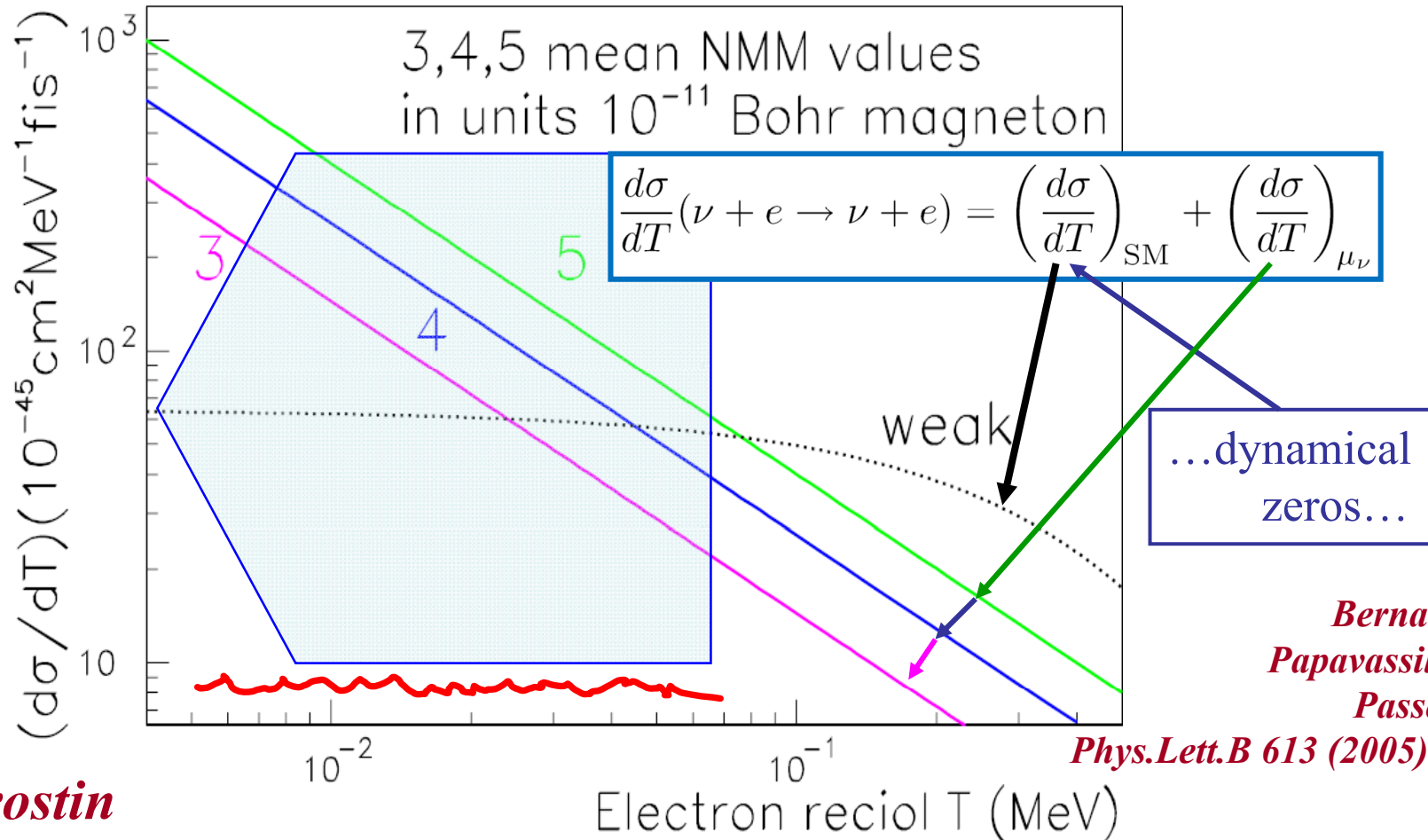
Implications of  $\mu_\nu$  limits from different experiments (reactor, solar  $^8\text{B}$  and  $^7\text{Be}$ ) are different.



Magnetic moment contribution is dominated at low electron recoil energies

and  $\left(\frac{d\sigma}{dT}\right)_{\mu\nu} > \left(\frac{d\sigma}{dT}\right)_{SM}$  when  $\frac{T}{m_e} < \frac{\pi^2 \alpha_{em}}{G_F^2 m_e^4} \mu_\nu^2$

... the **lower** the smallest measurable electron recoil energy is, ●  
 the **smaller** values of  $\mu_\nu^2$  can be probed in scattering experiments ...



from  
*A. Starostin*

*Bernabeu,  
Papavassiliou,  
Passera,  
Phys.Lett.B 613 (2005) 162*



# First and future $\nu$ - $e$ scattering experiments

- $\mu_\nu \leq 2 \div 4 \times 10^{-10} \mu_B$   
Savannah River (1976), *first observation*  
*Vogel, Engel, 1989* of  $\nu$ - $e$  ✓  
Kurchatov, Krasnoyarsk (1992),  
Rovno (1993) reactors
- $\mu_\nu \leq 1.1 \times 10^{-10} \mu_B$   
SuperKamiokande (2004)
- $\mu_\nu \leq \text{few} \times 10^{-11} \mu_B$   
Beta-beams  
*...in the future...*  
McLaughlin, Volpe, 2004

**MUNU** experiment at Bugey reactor (2005)

$$\mu_{\nu} \leq 9 \times 10^{-11} \mu_B$$

**TEXONO** collaboration at Kuo-Sheng power plant (2006)

$$\mu_{\nu} \leq 7 \times 10^{-11} \mu_B$$

**GEMMA** (2007)

$$\mu_{\nu} \leq 5.8 \times 10^{-11} \mu_B$$

**GEMMA I 2005 - 2007**

? **BOREXINO** (2008)

$$\mu_{\nu} \leq 5.4 \times 10^{-11} \mu_B$$

*...was considered as the world best constraint...*

$$\mu_{\nu} \leq 8.5 \times 10^{-11} \mu_B \quad (\nu_{\tau}, \nu_{\mu})$$

*Montanino,  
Picariello,  
Pulido, PRD 2008  
based on first release of  
BOREXINO data*

## GEMMA (2005-2008)

### Germanium Experiment on measurement of Magnetic Moment of Antineutrino

JINR (Dubna) + ITEP (Moscow) at Kalinin Nuclear Power Plant



$$\mu_{\nu} < 3.2 \times 10^{-11} \mu_B$$



...till 13 January 2010 and again since 23 August 2010  
best limit on  $\checkmark$  magnetic moment

A.Beda et al, Phys.Part.Nucl.Lett. 7 (2010) 406

result known since 2009:

A.Beda, E.Demidova, A.Starostin et al,  
arXiv:09.06.1926, June 10, 2009,

A.Beda, V.Brudanin, E.Demidova et al,  
in: "Particle Physics on the Eve of LHC",

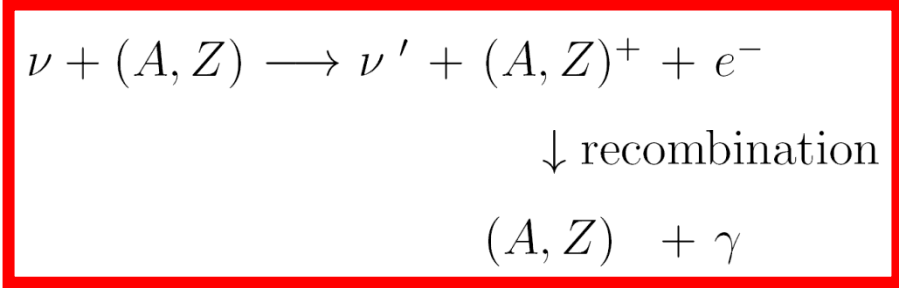
ed. A.Studenikin, World Scientific (Singapore),

p.112, 2009 (13th Lomonosov Conference) [www.icas.ru](http://www.icas.ru)

... quite recent *claim*  
 that  $\nu$ - $e$  cross section  
 should be increased by



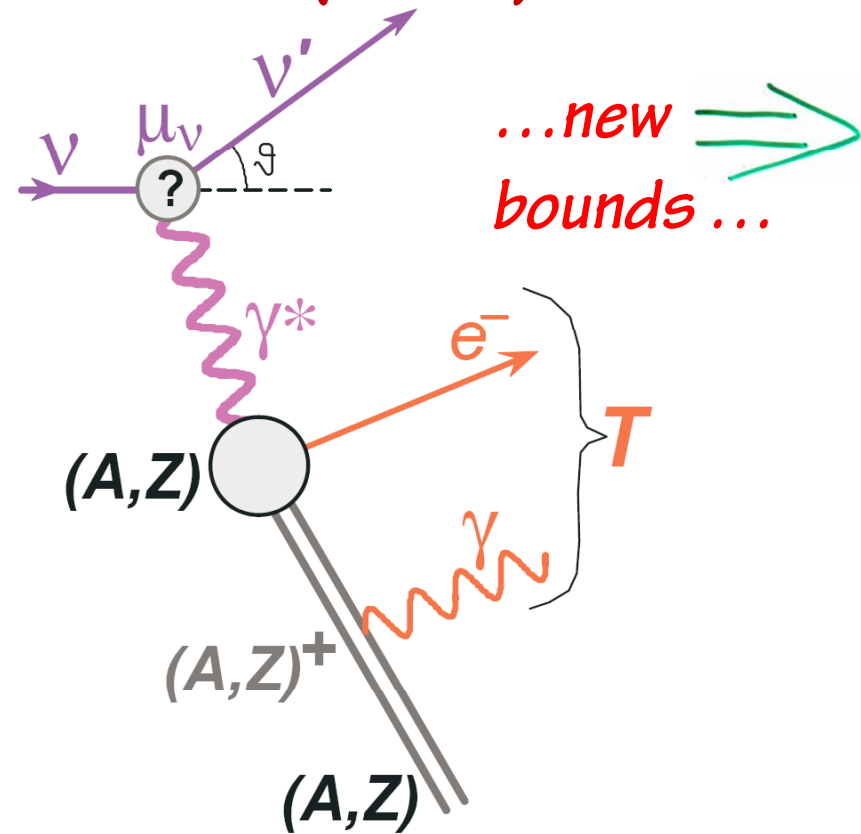
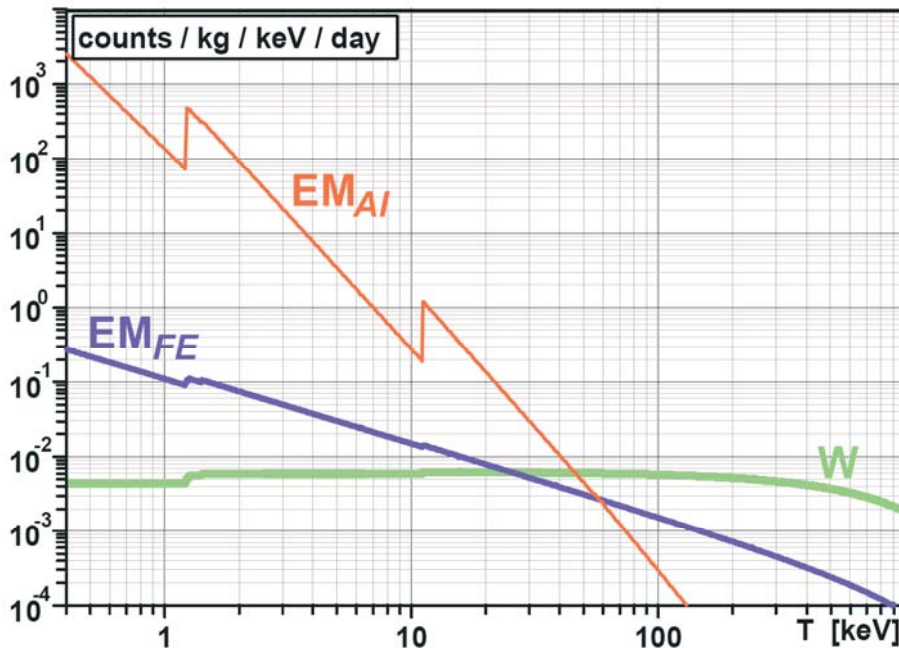
*Atomic Ionization effect:*



H.Wong et al. (TEXONO Coll.), arXiv:  
 1001.2074,  
 13 Jan 2010,  
 reported at

Neutrino 2010 Conference  
 (Athens, June 2010),

PRL 105 (2010) 061801



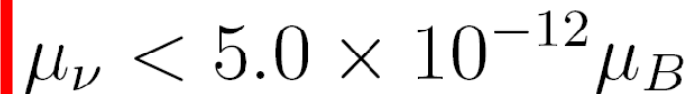
...much better limits on  $\nu$  effective magnetic moment ...


$$\mu_\nu < 1.3 \times 10^{-11} \mu_B$$

... *atomic ionization* effect accounted for ...


H.Wong et al.,  
(TEXONO Coll.),  
arXiv: 1001.2074,  
13 Jan 2010,  
PRL 105 (2010)  
061801

Neutrino 2010 Conference, Athens

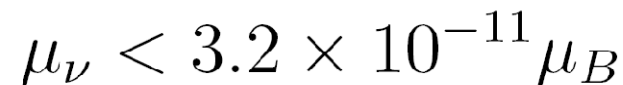

$$\mu_\nu < 5.0 \times 10^{-12} \mu_B$$

... *atomic ionization* effect accounted for ...

... *however* ...



A.Beda et al.  
(GEMMA Coll.),  
arXiv: 1005.2736,  
16 May 2010


$$\mu_\nu < 3.2 \times 10^{-11} \mu_B$$

...  $\nu$ -*e* scattering on free electrons ...

(without *atomic ionization*)

K.Kouzakov, A.Studenikin,

- “Magnetic neutrino scattering on atomic electrons revisited” ●  
Phys.Lett. B 105 (2011) 061801, arXiv: 1011.5847
- “Electromagnetic neutrino-atom collisions: The role of electron binding”  
to appear in Nucl.Phys.B (Proc.Suppl.) 217 (2011) 353  
arXiv: 1108.2872, 14 Aug 2011

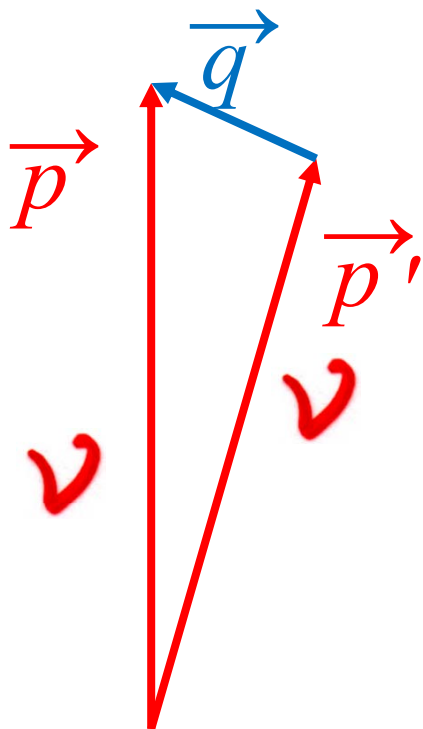
K.Kouzakov, A.Studenikin, M.Voloshin,

- “Neutrino-impact ionization of atoms in search for neutrino magnetic moment”, arXiv: 1101.4878, 25 Jan 2011  
Phys.Rev.D 83 (2011) 113001
- “On neutrino-atom scattering in searches for neutrino magnetic moments” arXiv: 1102.0643, 3 Feb 2011  
Nucl.Phys.B (Proc.Suppl.) 2011 (Proc. of Neutrino 2010 Conference)
- “Testing neutrino magnetic moment in ionization of atoms by neutrino impact”, arXiv: 1105.5543, 27 May 2011  
JETP Lett. 93 (2011) 699

M.Voloshin,

- “Neutrino scattering on atomic electrons in search for neutrino magnetic moment”  
Phys.Rev.Lett. 105 (2010) 201801, arXiv: 1008.2171

• **Neutrino-impact ionization of atoms**  
**in search for  $\mu_\nu$**



scattering on atoms (Ge) at low energy transfer

$T \sim$  few keV and lower so that  $\frac{T}{E_\nu} \ll 1$  for most of reactor  $\checkmark$

Ge atom recoil energy  $< \frac{2E_\nu^2}{M_{\text{Ge}}} \ll T$   $M_{\text{Ge}} \rightarrow \infty$

$\checkmark$  interaction with nucleus is neglected  $\bullet$

$\checkmark$  scattering on atomic  $e$  is important  $\bullet$

Four momentum transfer  $q = p - p'$

$$q_\mu = (T, \vec{q}), \quad q^2 = \vec{q}^2$$

energy and spatial momentum  
 transfer from neutrinos  
 to atomic electrons

Kouzakov,  
 Studenikin,  
 Voloshin,  
 2010; 2011

At small  $T$  electrons can be treated nonrelativistically

so that  $vii$  process is scattering of  $\mu_\nu$  on EMF of electrons

$$A = (A_0, \vec{A}) \quad A_0(\vec{q}) = \sqrt{4\pi\alpha} \rho(\vec{q})/\vec{q}^2 \quad \vec{A}(\vec{q}) = \sqrt{4\pi\alpha} \vec{j}(\vec{q})/\vec{q}^2$$

where

$$\rho(\vec{q}) = \sum_{a=1}^Z \exp(i\vec{q} \cdot \vec{r}_a) \quad \vec{j}(\vec{q}) = -\frac{i}{2m} \sum_{a=1}^Z \left[ \exp(i\vec{q} \cdot \vec{r}_a) \frac{\partial}{\partial \vec{r}_a} + \frac{\partial}{\partial \vec{r}_a} \exp(i\vec{q} \cdot \vec{r}_a) \right]$$

are Fourier transforms of  $e$  number and current density operators,  
Summation is performed over positions  $\vec{r}_a$  of all  $Z$  electrons in atom

Vertex function

$$\Lambda^i = \frac{\mu_\nu}{2m_e} \sigma^{ik} q_k$$



# Double differential $\nu$ - $e$ cross section

$$\frac{d^2\sigma_{(\mu)}}{dT dq^2} = 4\pi \alpha \frac{\mu_\nu^2}{q^2} \left[ \left(1 - \frac{T^2}{q^2}\right) S(T, q^2) + \left(1 - \frac{q^2}{4E_\nu^2}\right) R(T, q^2) \right]$$

$$\frac{d^2\sigma_{(\mu)}}{dT dq^2} = \left( \frac{d^2\sigma_{(\mu)}}{dT dq^2} \right)_{\parallel} + \left( \frac{d^2\sigma_{(\mu)}}{dT dq^2} \right)_{\perp}$$

Kouzakov,  
Studenikin,  
Voloshin,  
2010; 2011

where dynamical structure factor (Van Hove, 1954)

$$S(T, q^2) = \sum_n \delta(T - E_n + E_0) |\langle n | \rho(\vec{q}) | 0 \rangle|^2 \quad \text{and} \quad (\vec{j}_{\perp} \cdot \vec{q}) = 0$$

$$R(T, q^2) = \sum_n \delta(T - E_n + E_0) |\langle n | j_{\perp}(\vec{q}) | 0 \rangle|^2$$

sum is over all states  $|n\rangle$  of electron system,  $|0\rangle$  initial state

... dynamical structure factor

$$(\vec{j}_\perp \cdot \vec{q}) = 0$$

and

$$S(T, q^2) = \sum_n \delta(T - E_n + E_0) |\langle n | \rho(\vec{q}) | 0 \rangle|^2$$

$$R(T, q^2) = \sum_n \delta(T - E_n + E_0) |\langle n | j_\perp(\vec{q}) | 0 \rangle|^2$$

are related

$$S(T, q^2) = \frac{1}{\pi} \text{Im} F(T, q^2) , \quad R(T, q^2) = \frac{1}{\pi} \text{Im} L(T, q^2)$$

to  $\rho$ - $\rho$  and  $j$ - $j$  Green's functions

$$F(T, q^2) = \sum_n \frac{|\langle n | \rho(\vec{q}) | 0 \rangle|^2}{T - E_n + E_0 - i\epsilon} = \left\langle 0 \left| \rho(-\vec{q}) \frac{1}{T - H + E_0 - i\epsilon} \rho(\vec{q}) \right| 0 \right\rangle ,$$

$$L(T, q^2) = \sum_n \frac{|\langle n | j_\perp(\vec{q}) | 0 \rangle|^2}{T - E_n + E_0 - i\epsilon} = \left\langle 0 \left| j_\perp(-\vec{q}) \frac{1}{T - H + E_0 - i\epsilon} j_\perp(\vec{q}) \right| 0 \right\rangle$$

For single-differential inclusive cross section measured in experiment

$$\frac{d\sigma_{(\mu)}}{dT} = 4\pi \alpha \mu_\nu^2 \int_{T^2}^{4E_\nu^2} S(T, q^2) \frac{dq^2}{q^2}$$

$$R(T, q^2) = \frac{T^2}{q^2} S(T, q^2)$$

transversal contribution

practically for most  $q^2$

is negligible

SM electroweak contribution to cross section

$$\frac{d\sigma_{EW}}{dT} = \frac{G_F^2}{4\pi} (1 + 4 \sin^2 \theta_W + 8 \sin^4 \theta_W) \int_{T^2}^{4E_\nu^2} S(T, q^2) dq^2$$

nonrelativistic limit

$$\int_{T^2}^{4E_\nu^2} \Rightarrow \int_0^\infty$$

For free electron  $S_{(FE)}(T, q^2) = \delta(T - q^2/2m)$

$$\int_0^\infty S_{(FE)}(T, q^2) \frac{dq^2}{q^2} = \frac{1}{T}, \quad \int_0^\infty S_{(FE)}(T, q^2) dq^2 = 2m$$

$$\frac{d\sigma_{(\mu)}}{dT} = 4\pi \alpha \mu_\nu^2 \left( \frac{1}{T} - \frac{1}{E_\nu} \right) = \pi \frac{\alpha^2}{m^2} \left( \frac{\mu_\nu}{\mu_B} \right)^2 \left( \frac{1}{T} - \frac{1}{E_\nu} \right)$$

● ... for electron bound in atom ...

$$S(T, q^2) = \frac{m}{2pq} \left[ \theta \left( T - \frac{q^2}{2m} + \frac{pq}{m} \right) - \theta \left( T - \frac{q^2}{2m} - \frac{pq}{m} \right) \right]$$

( $\nu$ - $e$  scattering on free electrons)

free  
electron  
approximation  
is valid



*No important effect of Atomic Ionization  
on cross section in  $\mu$ , experiments  
(once all possible final electronic states accounted for)*

*M.Voloshin, 23 Aug 2010;*

*K.Kouzakov, A.Studenikin, 26 Nov 2010;*



*H.Wong et al, arXiv: 1001.2074 V3, 28 Nov 2010*

## GEMMA (2005-2008)

### Germanium Experiment on measurement of Magnetic Moment of Antineutrino

JINR (Dubna) + ITEP (Moscow) at Kalinin Nuclear Power Plant



$$\mu_{\nu} < 3.2 \times 10^{-11} \mu_B$$



...till 13 January 2010 and again since 23 August 2010  
best limit on  $\checkmark$  magnetic moment

A.Beda et al, Phys.Part.Nucl.Lett. 7 (2010) 406

result known since 2009:

A.Beda, E.Demidova, A.Starostin et al,  
arXiv:09.06.1926, June 10, 2009,

A.Beda, V.Brudanin, E.Demidova et al,  
in: "Particle Physics on the Eve of LHC",

ed. A.Studenikin, World Scientific (Singapore),

p.112, 2009 (13th Lomonosov Conference) [www.icas.ru](http://www.icas.ru)

## *GEMMA (2005-2012)*

*JINR (Dubna) + ITEP (Moscow) at Kalinin Nuclear Power Plant*

*Best world experimental limit 2012*



$$\mu_\nu < 2.9 \times 10^{-11} \mu_B$$

*A.Beda et al, Special issue on “Neutrino Physics”  
of Advances in High Energy Physics 2012, ID 350150*

*... further quite **realistic prospects** of the near future (V.Brudanin):*

$$\mu_\nu \sim 1 \times 10^{-11} \mu_B$$

3

*... a bit of  $\checkmark$  electromagnetic  
properties theory*

## 3.1 $\checkmark$ vertex function

The most general study of the  
massive neutrino vertex function

(including electric and magnetic  
form factors) in arbitrary  $R_\xi$  gauge  
in the context of the SM + SU(2)-singlet

$\gamma_R$  accounting for masses of particles  
in polarization loops





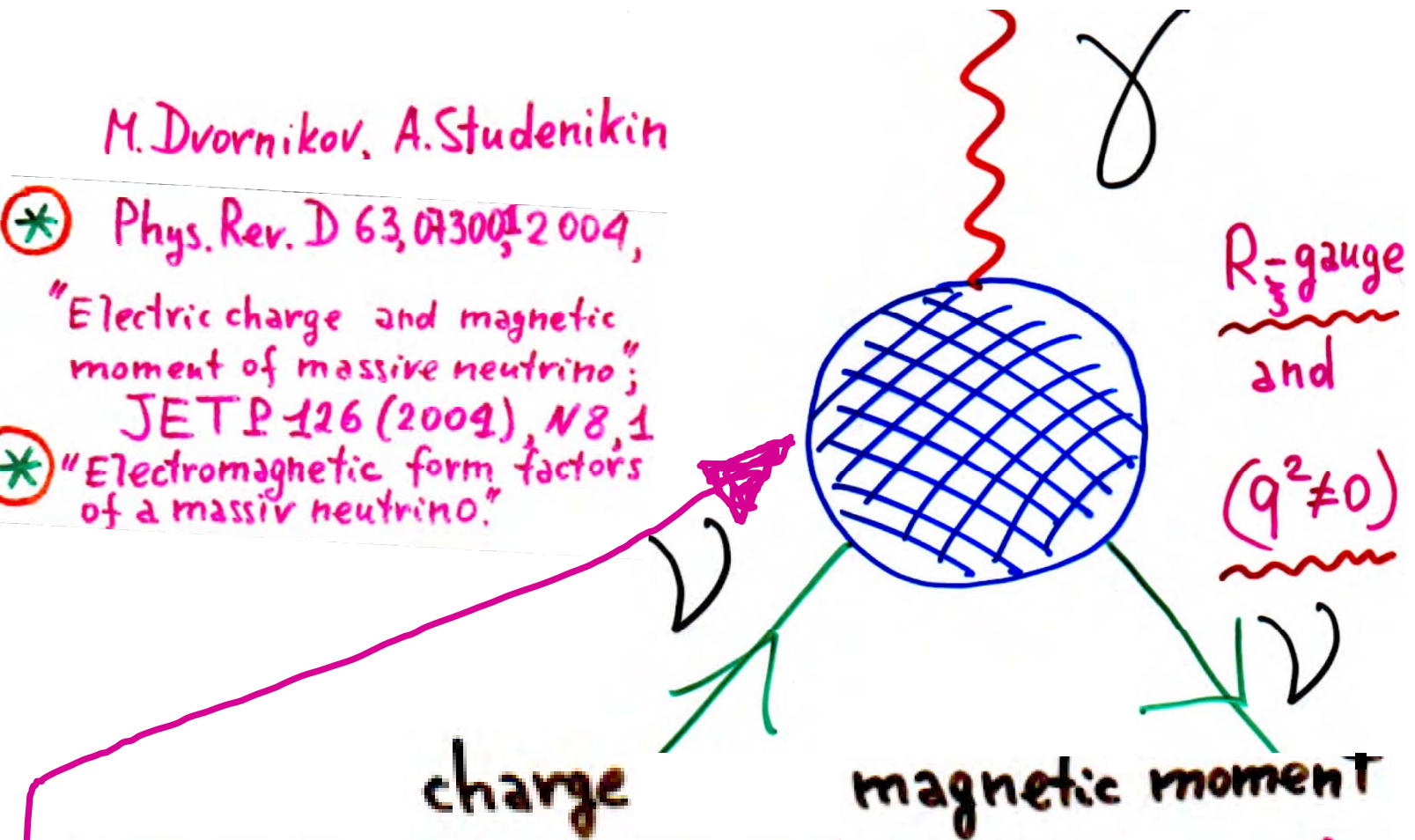
M. Dvornikov, A. Studenikin

\* Phys. Rev. D 63, 073001, 2001,

"Electric charge and magnetic moment of massive neutrino";

JETP 126 (2004), N 8, 1

\* "Electromagnetic form factors of a massive neutrino."



charge

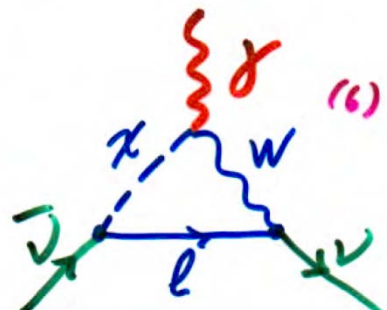
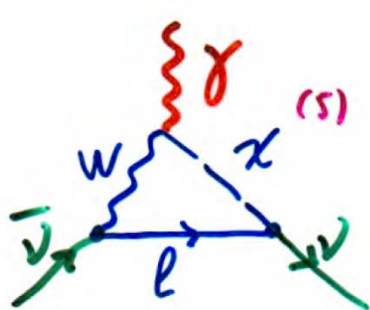
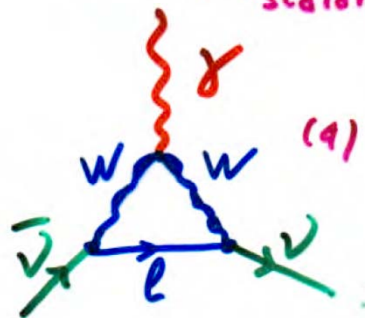
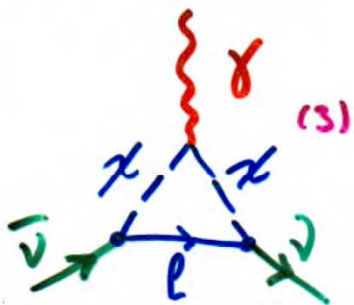
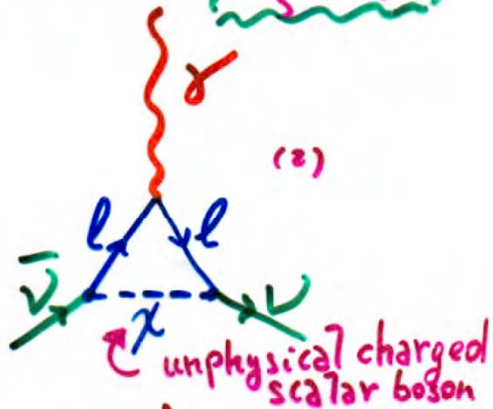
magnetic moment

$$\Delta_{\mu}(q) = \underbrace{f_Q(q^2)}_{\text{charge}} \gamma_{\mu} + \underbrace{f_M(q^2)}_{\text{magnetic moment}} i \sigma_{\mu\nu} q^{\nu} - \underbrace{f_E(q^2)}_{\text{electric moment}} i \sigma_{\mu\nu} q^{\nu} \gamma_5 - \underbrace{f_A(q^2)}_{\text{anapole moment}} (q^{\nu} \gamma_{\mu} - q_{\mu} \gamma^{\nu}) \gamma_5$$

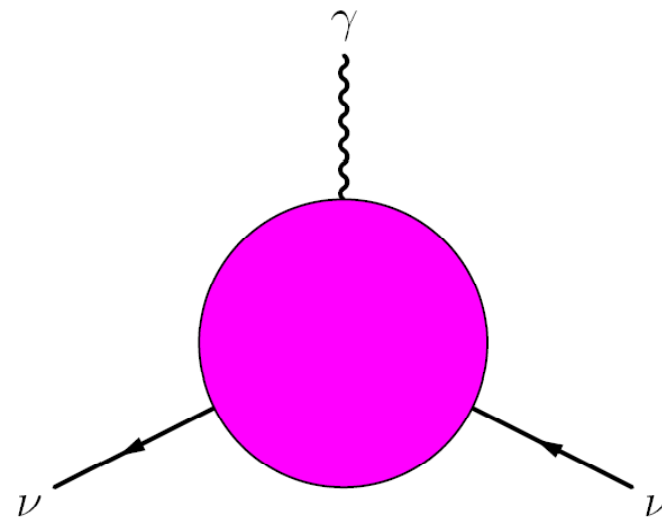
$$a = \left(\frac{m_e}{m_W}\right)^2$$

$$b = \left(\frac{m_\nu}{m_W}\right)^2$$

Proper vertices  $R_\xi$ -gauge



$$\Lambda_\mu(q) = \sum_{i=1}^{19} \Delta_\mu^i(q)$$



$$\Lambda_\mu(q)$$

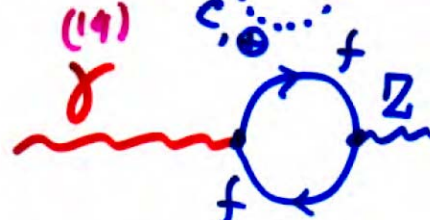
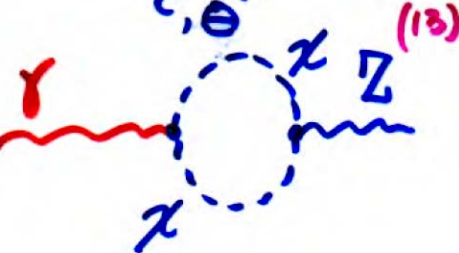
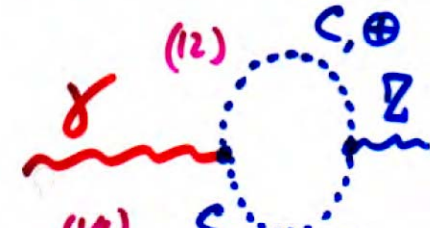
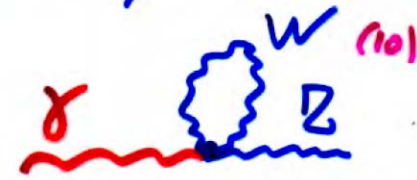
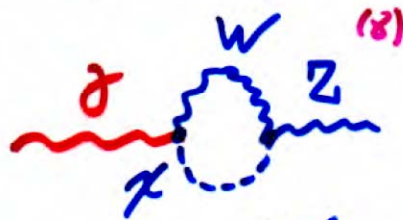
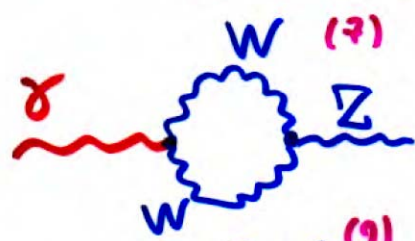
# Contributions of proper vertices diagrams

(dimensional-regularization scheme)

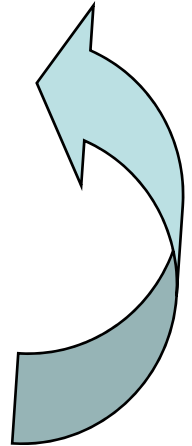
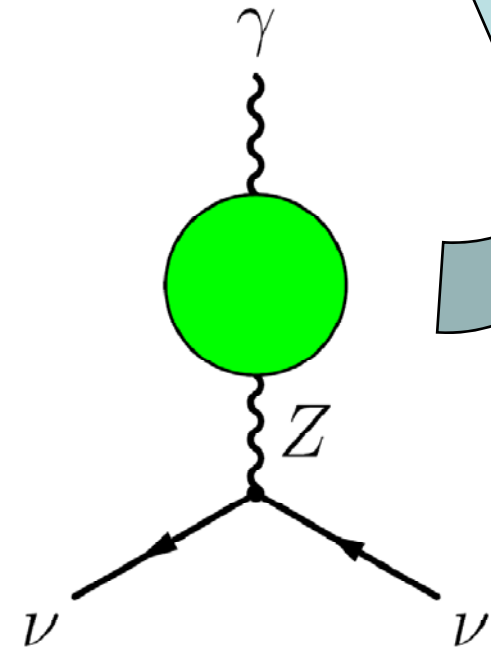
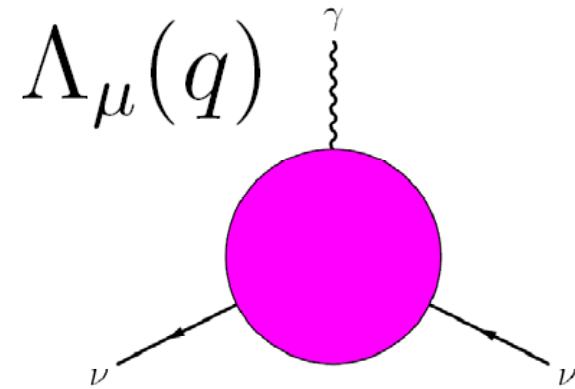
- $\Lambda_{\mu}^{(1)} = i \frac{eg^2}{2} \int \frac{d^N k}{(2\pi)^N} \left[ g^{\kappa\lambda} - (1-\alpha) \frac{k^{\kappa} k^{\lambda}}{k^2 - \alpha M_W^2} \right] \times \frac{\gamma_{\kappa}^L (\not{p}' - \not{k} + m_{\ell}) \gamma_{\mu} (\not{p} - \not{k} + m_{\ell}) \gamma_{\lambda}^L}{[(p' - k)^2 - m_{\ell}^2][(p - k)^2 - m_{\ell}^2][k^2 - M_W^2]},$
- $\Lambda_{\mu}^{(2)} = i \frac{eg^2}{2M_W^2} \int \frac{d^N k}{(2\pi)^N} \frac{(m_{\nu} P_L - m_{\ell} P_R)(\not{p}' - \not{k} + m_{\ell}) \gamma_{\mu} (\not{p} - \not{k} + m_{\ell})(m_{\ell} P_L - m_{\nu} P_R)}{[(p' - k)^2 - m_{\ell}^2][(p - k)^2 - m_{\ell}^2][k^2 - \alpha M_W^2]},$
- $\Lambda_{\mu}^{(3)} = i \frac{eg^2}{2M_W^2} \int \frac{d^N k}{(2\pi)^N} (2k - p - p')_{\mu} \frac{(m_{\nu} P_L - m_{\ell} P_R)(\not{k} + m_{\ell})(m_{\ell} P_L - m_{\nu} P_R)}{[(p' - k)^2 - \alpha M_W^2][(p - k)^2 - \alpha M_W^2][k^2 - m_{\ell}^2]},$
- $\Lambda_{\mu}^{(4)} = i \frac{eg^2}{2} \int \frac{d^N k}{(2\pi)^N} \gamma_{\kappa}^L (\not{k} + m_{\ell}) \gamma_{\lambda}^L \left[ \delta_{\beta}^{\kappa} - (1-\alpha) \frac{(p' - k)^{\kappa} (p' - k)_{\beta}}{(p' - k)^2 - \alpha M_W^2} \right] \left[ \delta_{\gamma}^{\lambda} - (1-\alpha) \frac{(p - k)^{\lambda} (p - k)_{\gamma}}{(p - k)^2 - \alpha M_W^2} \right]$   
 $\times \frac{\delta_{\mu}^{\beta} (2p' - p - k)_{\gamma} + g^{\beta\gamma} (2k - p - p')_{\mu} + \delta_{\mu}^{\gamma} (2p - p' - k)_{\beta}}{[(p' - k)^2 - M_W^2][(p - k)^2 - M_W^2][k^2 - m_{\ell}^2]},$
- $\Lambda_{\mu}^{(5)+(6)} = i \frac{eg^2}{2} \int \frac{d^N k}{(2\pi)^N}$   
 $\times \left\{ \frac{\gamma_{\beta}^L (\not{k} - m_{\ell})(m_{\ell} P_L - m_{\nu} P_R)}{[(p' - k)^2 - M_W^2][(p - k)^2 - \alpha M_W^2][k^2 - m_{\ell}^2]} \left[ \delta_{\mu}^{\beta} - (1-\alpha) \frac{(p' - k)^{\beta} (p' - k)_{\mu}}{(p' - k)^2 - \alpha M_W^2} \right] \right.$   
 $\left. - \frac{(m_{\nu} P_L - m_{\ell} P_R)(\not{k} - m_{\ell}) \gamma_{\beta}^L}{[(p' - k)^2 - \alpha M_W^2][(p - k)^2 - M_W^2][k^2 - m_{\ell}^2]} \left[ \delta_{\mu}^{\beta} - (1-\alpha) \frac{(p - k)^{\beta} (p - k)_{\mu}}{(p - k)^2 - \alpha M_W^2} \right] \right\}$

$$\Lambda_{\mu}^j(q) = \frac{g}{2 \cos \theta_w} \Pi_{\mu\nu}^{(j)}(q) \frac{1}{q^2 - M_Z^2} \times \left\{ g^{\nu\alpha} - (1 - \alpha_Z) \frac{q^{\nu} q^{\alpha}}{q^2 - \alpha_Z M_Z^2} \right\} \gamma_{\alpha}, j=7, \dots, 14$$

$\gamma$ -Z self-energy diagrams



$f = u, c, t, d, s, b$   
quarks



$\gamma$  - Z self-energy diagrams



# Direct calculations of complete set of one-loop contributions to $\nu$ vertex function in **minimally extended Standard Model**

for a *massive Dirac neutrino*:

*M.Dvornikov,  
A.Studenikin,  
PRD, 2004*

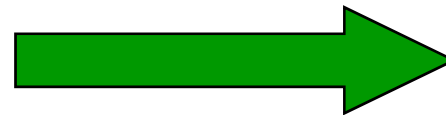
... in case **CP** conservation

●  $\Lambda_\mu(q) \longrightarrow f_Q(q^2), f_M(q^2), f_E(q^2), f_A(q^2)$

● **Electric charge**  $f_Q(0) = \mathbf{0}$  and is **gauge-independent**

● **Magnetic moment**  $f_M(0)$  is **finite and gauge-independent**

● **Gauge and  $q \times q$  dependence ...**





# Gauge and $q \times q$ dependence ...

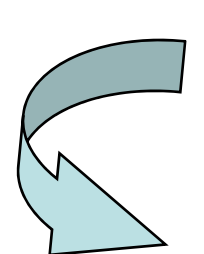
*Dvornikov,  
Studenikin,  
PRD 2004*

✓ magnetic moment  
●  $\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_\nu$

1.5000  
1.4998  
1.4995  
1.4994  
1.4992

$\alpha = 100$   
 $\alpha = 1$  ('t Hooft-Feynman)  
 $\alpha = 0.1$

$$\alpha = \frac{1}{\xi}$$



$$\bar{f}_M(t)$$

$$\bar{f}_M(t) = \sum_{i=1}^6 \bar{f}_M^{(i)}(t)$$

0 1 2 3 4 5

$t \times 10^{-4} M_W^2$

$$f_M(q^2) = \frac{eG_F}{4\pi^2\sqrt{2}} m_\nu \sum_{i=1}^6 \bar{f}_M^{(i)}(q^2)$$

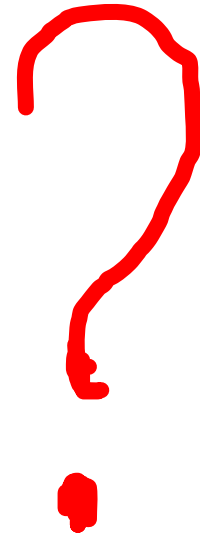
✓ dipole magnetic form factor

# Magnetic moment dependence

$$\mu_\nu = \mu_\nu(m_\nu)$$



on neutrino mass



3.2

# Calculation of $\nu$ magnetic moment (massive $\nu$ , arbitrary $R_\xi$ -gauge)

Dvornikov,  
Studenikin, PRD 2004

$$\Lambda_\mu(q) = f_Q(q^2) \gamma_\mu + f_M(q^2) i \sigma_{\mu\nu} q^\nu - f_E(q^2) \sigma_{\mu\nu} q^\nu \gamma_5 + f_A(q^2) (q^2 \gamma_\mu - q_\mu \not{q}) \gamma_5$$

*magnetic moment*

$$\mu(a, b, \alpha) = f_M(q^2 = 0)$$

two mass parameters

$$a = \left(\frac{m_\ell}{M_W}\right)^2$$

$$b = \left(\frac{m_\nu}{M_W}\right)^2$$

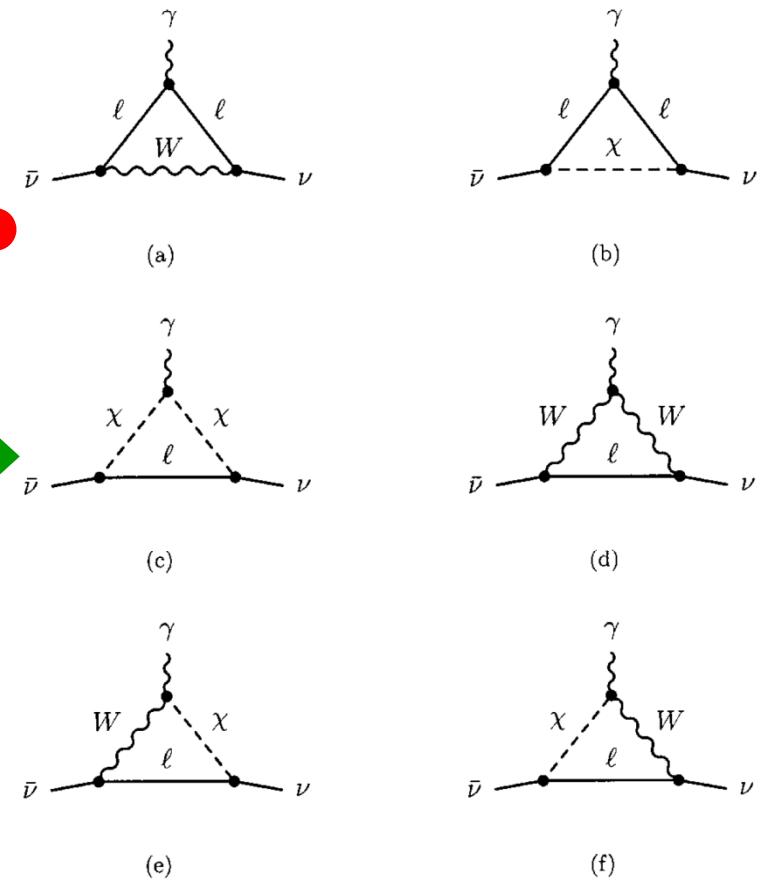
$$\mu(a, b, \alpha) = \sum_{i=1}^6 \mu^{(i)}(a, b, \alpha)$$

and gauge-fixing parameter

$$\alpha = \frac{1}{\xi}$$

$\xi = 0$  - unitary gauge,  $\xi = 1$  - 't Hooft-Feynman gauge

## Proper vertices







# magnetic moment

( for arbitrary neutrino mass, heavy neutrino... )



LEP data



only 3 light  $\nu$ s coupled to  $Z^0$ ,

for any additional neutrino

$$m_{\nu} \geq 45 \text{ Gev}$$



$$m_\nu \ll m_e \ll M_W$$

light  $\checkmark$

$$\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_e$$

$$\mu_\nu = \frac{eG_F}{4\pi^2\sqrt{2}} m_\nu \frac{3}{4(1-a)^3} (2 - 7a + 6a^2 - 2a^2 \ln a - a^3), \quad a = \left(\frac{m_e}{M_W}\right)^2$$

Dvornikov,  
Studenikin,  
*Phys.Rev.D* 69  
(2004) 073001;  
*JETP* 99 (2004) 254



$$m_e \ll m_\nu \ll M_W$$

intermediate  $\checkmark$



Gabral-Rosetti,  
Bernabeu, Vidal,  
Zepeda,  
*Eur.Phys.J C* 12  
(2000) 633

$$\mu_\nu = \frac{3eG_F}{8\pi^2\sqrt{2}} m_\nu \left\{ 1 + \frac{5}{18} b \right\}, \quad b = \left(\frac{m_\nu}{M_W}\right)^2$$

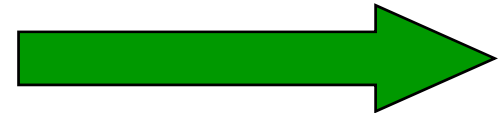


$$m_e \ll M_W \ll m_\nu$$

$$\mu_\nu = \frac{eG_F}{8\pi^2\sqrt{2}} m_\nu$$

heavy  $\checkmark$   
 $\sim 10^{-19} \mu_e \left(\frac{m_\nu}{1\text{eV}}\right)$

...  $\mu_\nu$  in case of mixing...



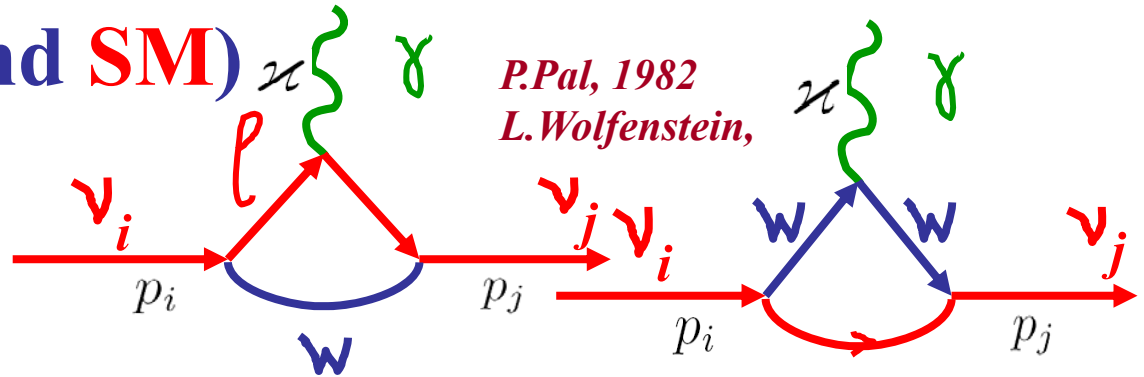
3.5

# Neutrino (beyond SM)

## dipole moments

(+ transition moments)

P.Pal, 1982  
L.Wolfenstein,



### Dirac neutrino

$$\left. \begin{matrix} \mu_{ij} \\ \epsilon_{ij} \end{matrix} \right\} = \frac{eG_F m_i}{8\sqrt{2}\pi^2} \left(1 \pm \frac{m_j}{m_i}\right) \sum_{l=e, \mu, \tau} f(r_l) U_{lj} U_{li}^*$$

$$r_l = \left(\frac{m_l}{m_W}\right)^2$$

- $m_e = 0.5 \text{ MeV}$
- $m_\mu = 105.7 \text{ MeV}$
- $m_\tau = 1.78 \text{ GeV}$
- $m_W = 80.2 \text{ GeV}$

$m_i, m_j \ll m_l, m_W$

$$f(r_l) \approx \frac{3}{2} \left(1 - \frac{1}{2} r_l\right), \quad r_l \ll 1$$

transition moments vanish because unitarity of  $U$  implies that its rows or columns represent orthogonal vectors

### Majorana neutrino only for

$$i \neq j$$

$$\mu_{ij}^M = 2\mu_{ij}^D \quad \text{and} \quad \epsilon_{ij}^M = 0$$

or

$$\mu_{ij}^M = 0 \quad \text{and} \quad \epsilon_{ij}^M = 2\epsilon_{ij}^D$$

transition moments are suppressed, Glashow-Iliopoulos-Maiani cancellation, for diagonal moments there is no GIM cancellation

... depending on relative CP phase of  $\nu_i$  and  $\nu_j$

The first nonzero contribution from **neutrino transition moments**

$$f_{r_l} \rightarrow -\cancel{\frac{3}{2}} + \frac{3}{4} \left(\frac{m_l}{m_W}\right)^2 \ll 1$$

*GIM cancellation*

$$\left. \begin{matrix} \mu_{ij} \\ \epsilon_{ij} \end{matrix} \right\} = \frac{3eG_F m_i}{32\sqrt{2}\pi^2} \left(1 \pm \frac{m_j}{m_i}\right) \left(\frac{m_\tau}{m_W}\right)^2 \sum_{l=e, \mu, \tau} \left(\frac{m_l}{m_\tau}\right)^2 U_{lj} U_{li}^*$$

$$\mu_B = \frac{e}{2m_e}$$

$$\left. \begin{matrix} \mu_{ij} \\ \epsilon_{ij} \end{matrix} \right\} = 4 \times 10^{-23} \mu_B \left(\frac{m_i \pm m_j}{1 \text{ eV}}\right) \sum_{l=e, \mu, \tau} \left(\frac{m_l}{m_\tau}\right)^2 U_{lj} U_{li}^*$$

... **neutrino radiative decay is very slow**

● **Dirac**  $\checkmark$  **diagonal ( $i=j$ ) magnetic moment**

$$\epsilon_{ii}^D = 0 \text{ for } CP\text{-invariant interactions}$$

$$\mu_{ii} = \frac{3eG_F m_i}{8\sqrt{2}\pi^2} \left(1 - \frac{1}{2} \sum_{l=e, \mu, \tau} r_l |U_{li}|^2\right) \approx 3.2 \times 10^{-19} \left(\frac{m_i}{1 \text{ eV}}\right) \mu_B$$

$r_l = \left(\frac{m_l}{m_W}\right)^2$

$$\mu_{ii}^M = \epsilon_{ii}^M = 0$$

*Lee, Shrock, Fujikawa, 1977*

● *no GIM cancellation*

●  $\mu_{ii}^D$  - to leading order - **independent on**  $U_{li}$  and  $m_{l=e, \mu, \tau}$

$$\mu_e^2 = \sum_{i=1,2,3} |U_{ie}|^2 \mu_{ii}^2$$

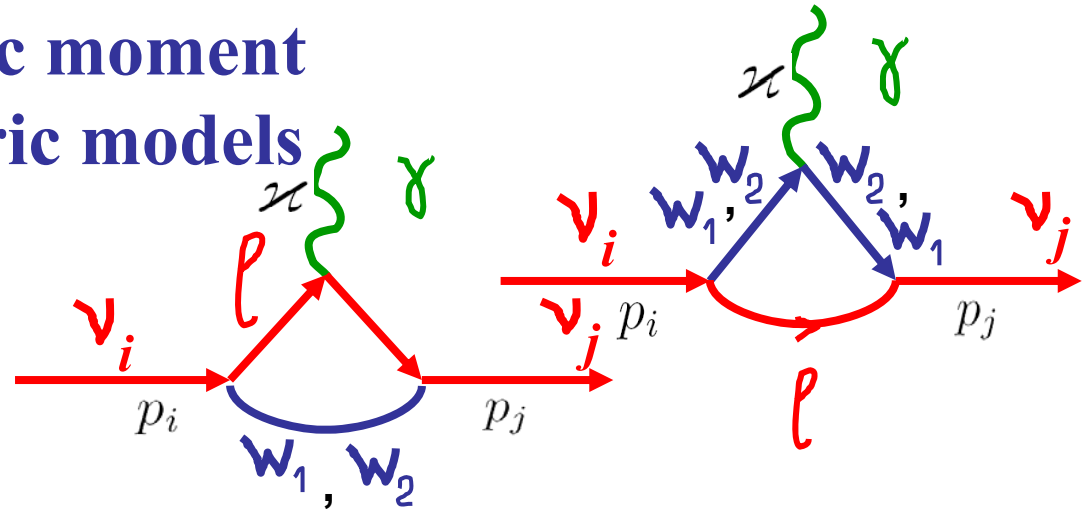
...possibility to measure fundamental  $\mu_{ii}^D$

$\mu_{ii}^D = 0$  for **massless**  $\checkmark$  (in the absence of **right-handed charged currents**)  $\rightarrow$

# 3.6 Neutrino magnetic moment in left-right symmetric models

$$SU_L(2) \times SU_R(2) \times U(1)$$

Gauge bosons mass states  $W_1 = W_L \cos \xi - W_R \sin \xi$   
 $W_2 = W_L \sin \xi + W_R \cos \xi$



with mixing angle  $\xi$  of gauge bosons  $W_{L,R}$  with pure  $(V \pm A)$  couplings

*Kim, 1976; Marciano, Sanda, 1977; Beg, Marciano, Ruderman, 1978*

$$\mu_{\nu l} = \frac{eG_F}{2\sqrt{2}\pi^2} \left[ m_l \left( 1 - \frac{m_{W_1}^2}{m_{W_2}^2} \right) \sin 2\xi + \frac{3}{4} m_{\nu l} \left( 1 + \frac{m_{W_1}^2}{m_{W_2}^2} \right) \right]$$

... charged lepton mass ...

... neutrino mass ...

*...the present status...*

*to have visible  $\mu \neq 0$*

*is not an easy task for*

*theoreticians*

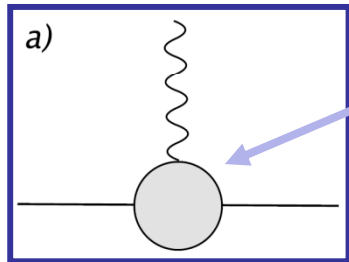
*and experimentalists*

### 3.3 Naïve relationship between the size of $m_\nu$ and $\mu_\nu$

... problem to get large  $\mu_\nu$  and still acceptable  $m_\nu$

If  $\mu_\nu$  is generated by physics beyond the SM at energy scale  $\Lambda$ ,

*P.Vogel e.a., 2006*

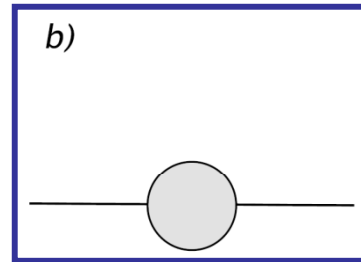


then

$$\mu_\nu \sim \frac{eG}{\Lambda}$$

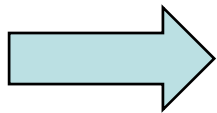
...combination of constants and loop factors...

contribution to  $m_\nu$  given by



, then

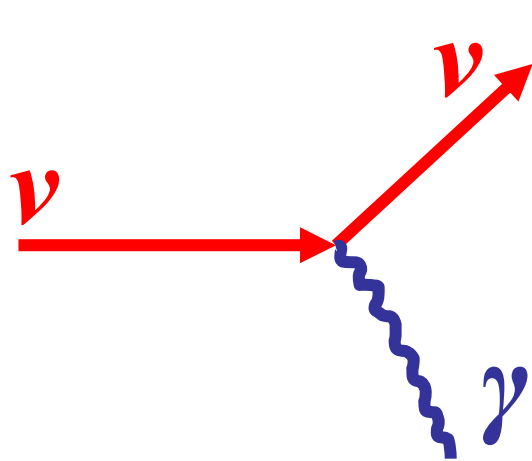
$$m_\nu \sim G\Lambda$$



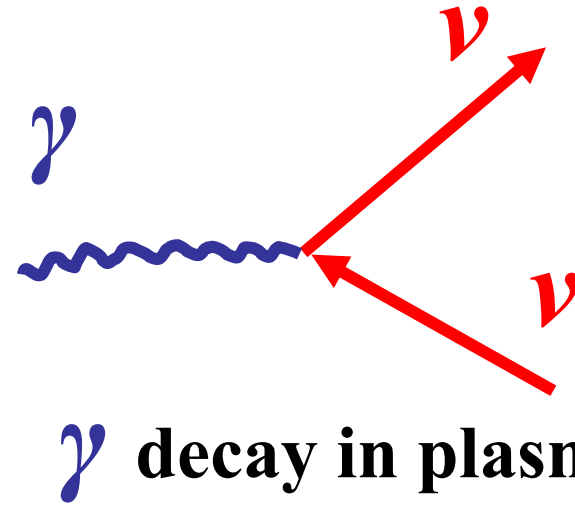
*Voloshin, 1988;  
Barr, Freire,  
Zee, 1990*

$$m_\nu \sim \frac{\Lambda^2}{2m_e} \frac{\mu_\nu}{\mu_B} \sim \frac{\mu_\nu}{10^{-18} \mu_B} [\Lambda(\text{TeV})]^2 \text{ eV}$$

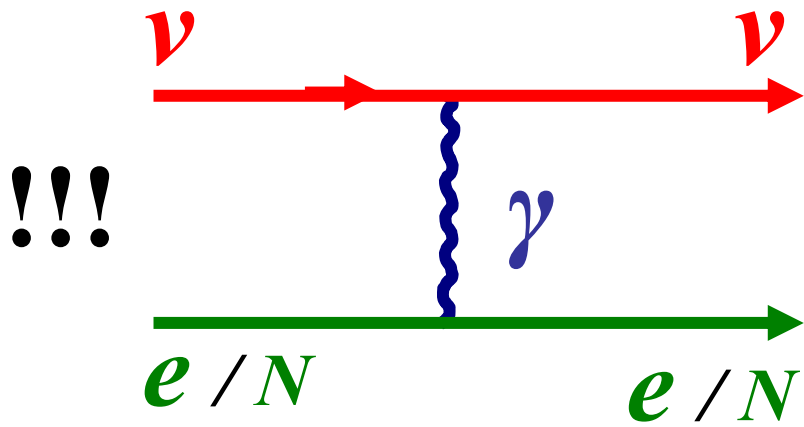
# Neutrino-photon couplings



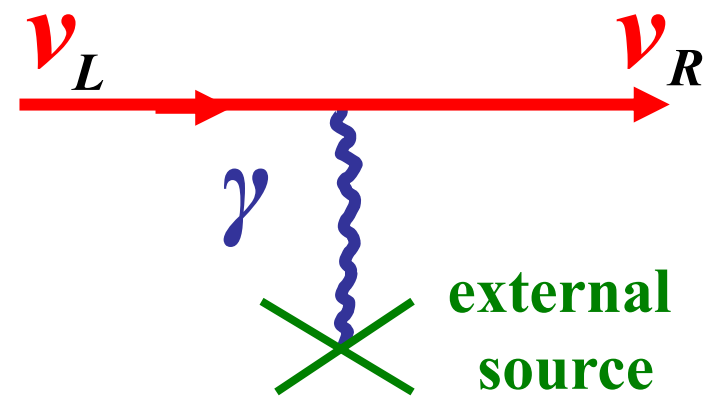
$\nu$  decay, Cherenkov radiation



$\gamma$  decay in plasma



Scattering



Spin precession



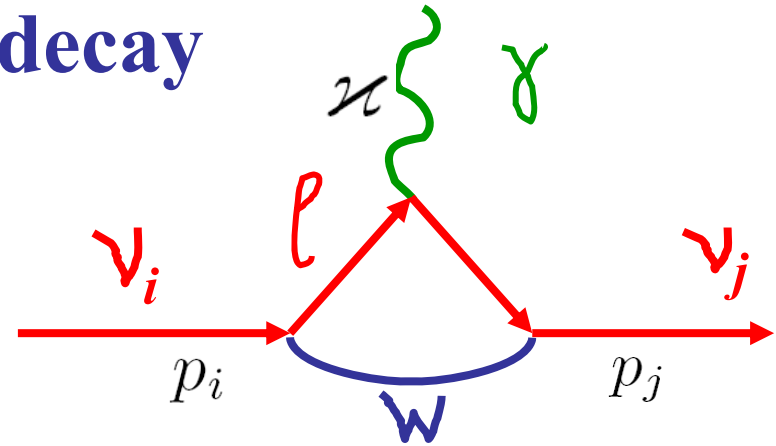
# 3.7

## Neutrino radiative decay

$$\nu_i \longrightarrow \nu_j + \gamma$$

$m_i > m_j$

$$L_{int} = \frac{1}{2} \bar{\psi}_i \sigma_{\alpha\beta} (\sigma_{ij} + \epsilon_{ij} \gamma_5) \psi_j F^{\alpha\beta} + h.c.$$



Radiative decay rate

*Petkov 1977; Zatsepin, Smirnov 1978;  
Bilenky, Petkov 1987; Pal, Wolfenstein 1982*

$$\Gamma_{\nu_i \rightarrow \nu_j + \gamma} = \frac{\mu_{eff}^2}{8\pi} \left( \frac{m_i^2 - m_j^2}{m_i^2} \right)^3 \approx 5 \left( \frac{\mu_{eff}}{\mu_B} \right)^2 \left( \frac{m_i^2 - m_j^2}{m_i^2} \right)^3 \left( \frac{m_i}{1 \text{ eV}} \right)^3 s^{-1}$$

$$\mu_{eff}^2 = |\mu_{ij}|^2 + |\epsilon_{ij}|^2$$

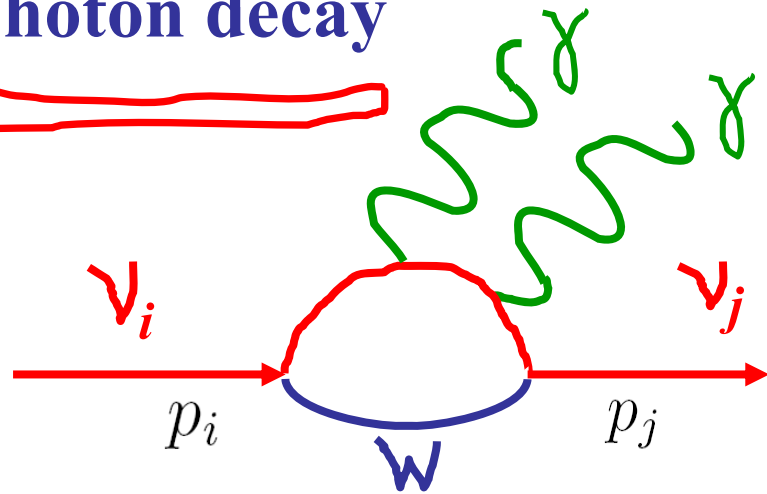
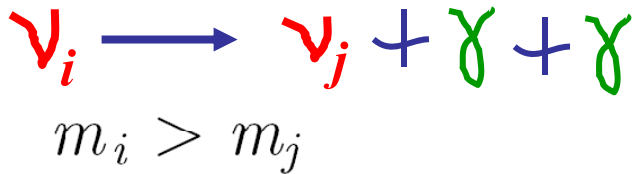
● Radiative decay has been constrained from absence of decay photons:

- 1) reactor  $\bar{\nu}_e$  and solar  $\nu_e$  fluxes,
- 2) SN 1987A  $\nu$  burst (all flavours),
- 3) spectral distortion of CMBR

*Raffelt 1999  
Kolb, Turner 1990;  
Ressell, Turner 1990*

**3.8**

**Neutrino radiative two-photon decay**



*fine structure constant*

$$\Gamma_{\nu_i \rightarrow \nu_j + \gamma + \gamma} \sim \frac{\alpha_{QED}}{4\pi} \Gamma_{\nu_i \rightarrow \nu_j + \gamma}$$

*... there is no GIM cancellation...*

$$f(r_l) \approx \frac{3}{2} \left( \cancel{1} - \frac{1}{2} \left( \frac{m_l}{m_W} \right)^2 \right) \rightarrow (m_i/m_l)^2$$

*Nieves, 1983; Ghosh, 1984*

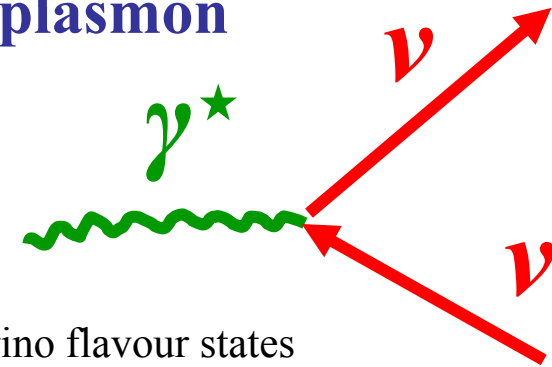
**... can be of interest for certain range of  $\nu$  masses...**

3.9

# The tightest astrophysical bound on $\mu_{\nu}$

G.Raffelt,  
PRL 1990

comes from cooling of **red giant** stars by plasmon decay  
 $\gamma^* \longrightarrow \nu \bar{\nu}$



$$L_{int} = \frac{1}{2} \sum_{a,b} \left( \mu_{a,b} \bar{\psi}_a \sigma_{\mu\nu} \psi_b + \epsilon_{a,b} \bar{\psi}_a \sigma_{\mu\nu} \gamma_5 \psi_b \right)$$

neutrino flavour states

Matrix element

$$\epsilon_{\alpha} k^{\alpha} = 0$$

$$|M|^2 = M_{\alpha\beta} p^{\alpha} p^{\beta}, \quad M_{\alpha\beta} = 4\mu^2 (2k_{\alpha} k_{\beta} - 2k^2 \epsilon_{\alpha}^* \epsilon_{\beta} - k^2 g_{\alpha,\beta}),$$

Decay rate

$$\Gamma_{\gamma \rightarrow \nu \bar{\nu}} = \frac{\mu^2}{24\pi} \frac{(\omega^2 - k^2)^2}{\omega}$$

= 0 in vacuum  $\omega = k$

In the classical limit



- like a massive particle with  $\omega^2 - k^2 = \omega_{pl}^2$

Energy-loss rate per unit volume

$$Q_{\mu} = g \int \frac{d^3k}{(2\pi)^3} \omega f_{BE} \Gamma_{\gamma \rightarrow \nu \bar{\nu}}$$

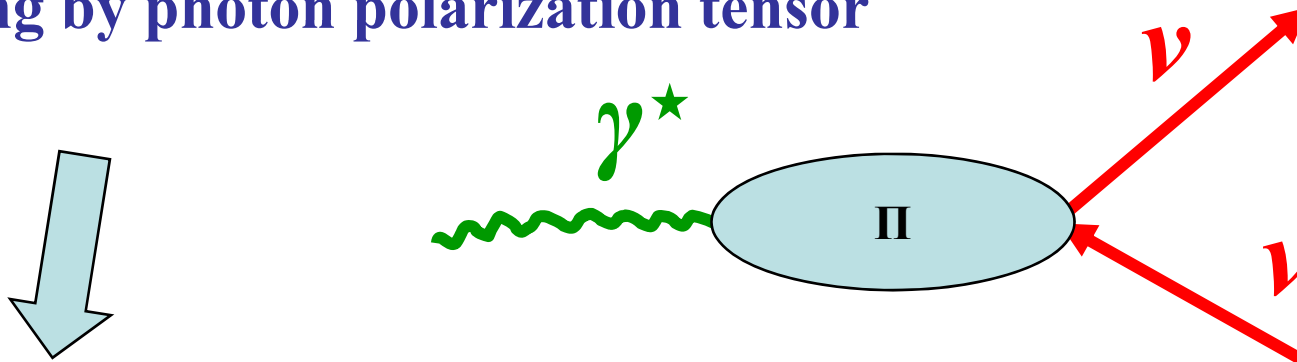
$$\mu^2 \rightarrow \sum_{a,b} (|\mu_{a,b}|^2 + |\epsilon_{a,b}|^2)$$

distribution function of plasmons

$$Q_\mu = g \int \frac{d^3k}{(2\pi)^3} \omega f_{BE} \Gamma_{\gamma \rightarrow \nu \bar{\nu}}$$

Magnetic moment **plasmon** decay  
 enhances the Standard Model photo-neutrino  
 cooling by photon polarization tensor

Energy-loss rate  
 per unit volume



more fast cooling of the star.

In order not to delay helium ignition (  $\leq 5\%$  in  $Q$  )



*... best  
 astrophysical  
 limit on  
 magnetic moment...*

$$\mu \leq 3 \times 10^{-12} \mu_B$$

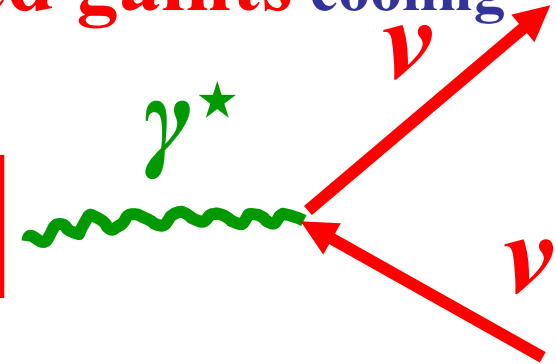
**G.Raffelt,  
 PRL 1990**

$$\mu^2 \rightarrow \sum_{a,b} (|\mu_{a,b}|^2 + |\epsilon_{a,b}|^2)$$

**3.10**

*Dobroliubov, Ignatiev (1990); Babu, Volkas (1992); Mohapatra, Nussinov (1992) ...*

● **Constraints on neutrino millicharge from red giants cooling**



Interaction Lagrangian

$$L_{int} = -iq_\nu \bar{\psi}_\nu \gamma^\mu \psi_\nu A^\mu$$

millicharge

Decay rate

$$\Gamma_{q_\nu} = \frac{q_\nu^2}{12\pi} \omega_{pl} \left( \frac{\omega_{pl}}{\omega} \right)$$

- $q_\nu \leq 2 \times 10^{-14} e$  ...to avoid helium ignition in low-mass **red giants**

*Halt, Raffelt, Weiss, PRL 1994*

- $q_\nu \leq 3 \times 10^{-17} e$  ... absence of anomalous energy-dependent dispersion of SN1987A **✓** signal, most model independent

- ... from “charge neutrality” of neutron...

$$q_\nu \leq 3 \times 10^{-21} e$$

# Astrophysics bounds on $\mu_\nu$

$$\mu_\nu(\text{astro}) < 10^{-10} - 10^{-12} \mu_B$$

Mostly derived from consequences of **helicity-state change** in astrophysical medium:

- available degrees of freedom in BBN,
- stellar cooling via plasmon decay,
- cooling of SN1987a.

Red Giant Lumin.  
 $\mu_\nu \leq 3 \cdot 10^{-12} \mu_B$   
G. Raffelt, D. Dearborn,  
J. Silk, 1989.

Bounds depend on

- modeling of astrophysical systems,
- on assumptions on the neutrino properties.

●●● Generic assumption:

- absence of other nonstandard interactions except for  $\mu_\nu$ .

A global treatment would be desirable, incorporating **oscillation** and **matter effects** as well as the complications due to interference and **competitions among various channels**

**Direct**

and  
influence

**Indirect**

of electromagnetic fields

on  $\nu$

through non-trivial  
neutrino electromagnetic  
properties (magnetic moment):

★ neutrino  
spin

★ spin-flavour  
oscillations...

★ different  $\nu\bar{\nu}$  processes

due to e.m. field influence on  
charged particles coupled  
to neutrinos


★ neutron beta-decay in  $B$

★ change of  $\nu$  oscillation pattern  
due to matter polarization under  
influence of external e.m. fields ...

# 4 $\nu$ spin and spin-flavour oscillations in $B_{\perp}$

Consider **two different neutrinos**:  $\nu_{eL}, \nu_{\mu R}, m_L \neq m_R$   
 with **magnetic moment interaction**

$$L \sim \bar{\nu} \sigma_{\lambda\rho} F^{\lambda\rho} \nu' = \bar{\nu}_L \sigma_{\lambda\rho} F^{\lambda\rho} \nu_R' + \bar{\nu}_R \sigma_{\lambda\rho} F^{\lambda\rho} \nu_L'.$$

Twisting magnetic field  $B = |B_{\perp}| e^{i\phi(t)}$   for solar  $\nu$  etc ...

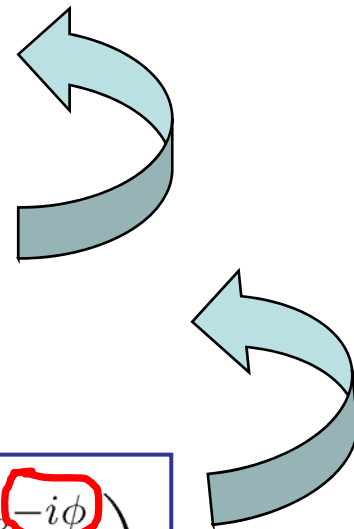
$\nu$  evolution equation

$$i \frac{d}{dt} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} = H \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$$

$$H = \begin{pmatrix} E_L & \mu_{e\mu} B e^{-i\phi} \\ \mu_{e\mu} B e^{+i\phi} & E_R \end{pmatrix} = \dots \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \tilde{H}$$



$$\tilde{H} = \begin{pmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta + \frac{V_{\nu e}}{2} & \mu_{e\mu} B e^{-i\phi} \\ \mu_{e\mu} B e^{+i\phi} & \frac{\Delta m^2}{4E} - \frac{V_{\nu e}}{2} \end{pmatrix}$$





... **Flavour oscillations**  $\longleftrightarrow$  **Spin oscillations...**

$$P_{\nu_e \nu_\mu} = \sin^2 2\theta \sin^2 \frac{\Delta m^2}{4E} z \longleftrightarrow P_{\nu_L \nu_R} = \sin^2 \beta \sin^2 \Omega z$$

$$\Omega^2 = (\mu_{e\mu} B)^2 + \left(\frac{\Delta_{LR}}{4E}\right)^2$$

$$\sin^2 2\theta$$



$$\frac{(\mu_{e\mu} B)^2}{(\mu_{e\mu} B)^2 + \left(\frac{\Delta_{LR}}{4E}\right)^2} = \sin^2 \beta$$

$$\frac{\Delta m^2}{4E}$$



$$\sqrt{(\mu_{e\mu} B)^2 + \left(\frac{\Delta_{LR}}{4E}\right)^2}$$

$$B = |\mathbf{B}_\perp| e^{i\phi(t)}$$

$$\Delta_{LR} = \frac{\Delta m^2}{2} (\cos 2\theta + 1) - 2EV_{\nu_e} + 2E\dot{\phi}$$

**Probability of  $\nu_{eL} \leftrightarrow \nu_{\mu R}$  oscillations in  $B = |\mathbf{B}_\perp| e^{i\phi(t)}$  and matter**

●  $P_{\nu_{eL}\nu_{\mu R}} = \sin^2\beta \sin^2\Omega z, \quad \sin^2\beta = \frac{(\mu_{e\mu} B)^2}{(\mu_{e\mu} B)^2 + \left(\frac{\Delta_{LR}}{4E}\right)^2}$

$$\Delta_{LR} = \frac{\Delta m^2}{2}(\cos 2\theta + 1) - 2EV_{\nu_e} + 2E\dot{\phi}$$

$$\Omega^2 = (\mu_{e\mu} B)^2 + \left(\frac{\Delta_{LR}}{4E}\right)^2$$

● **Resonance amplification of oscillations in matter:**

$$\Delta_{LR} \rightarrow 0$$



$$\sin^2\beta \rightarrow 1$$

*Akhmedov, 1988*  
*Lim, Marciano*

**In magnetic field**

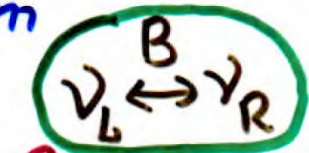
$$\nu_{eL} \nu_{\mu R}$$

$$i \frac{d}{dz} \nu_{eL} = -\frac{\Delta_{LR}}{4E} \nu_{eL} + \mu_{e\mu} B \nu_{\mu R}$$

$$i \frac{d}{dz} \nu_{\mu L} = \frac{\Delta_{LR}}{4E} \nu_{\mu L} + \mu_{e\mu} B \nu_{eR}$$

# Neutrino conversions and oscillations in magnetic field

●  $\otimes$   $\nu$   $\odot$  problem  ...for recent analysis see



Cisneros, 1971

\* { Voloshin, Vysotsky, Okun, 1986  
Barbieri, Fiorentini, 1988

$\odot$  twisting B { Smirnov, 1991  
Akhmedov, Petcov, Smirnov, 1993

J. Pulido, 2006,

TAUP-09; ●

A. Balantekin,

C. Volpe, 2005

...subdominant  
contribution to

LMA - MSW

solution...

●  $\otimes$  **Supernova**  $\nu_L \xleftrightarrow{B} \nu_R$   
● Dar, 1987

Fujikawa, Shrock, 1988

Voloshin, 1988

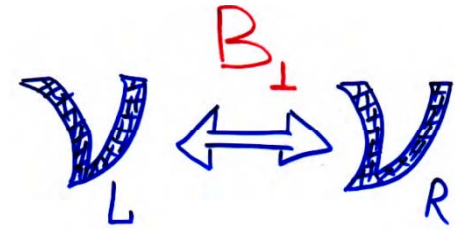


Spin-flavour oscillations in **early universe** - strong

→ population of  $\nu$  wrong-helicity states (r.h.) would  
accelerate expansion of universe (???)



# Criteria of significant importance of $\nu$ spin oscillations in $B_{\perp}$ :



1) amplitude of oscillations must be far from zero

$$B \geq B_{cr} = \frac{1}{2\tilde{\mu}} \left| \frac{\Delta m_{\nu}^2}{2E_{\nu}} - \sqrt{2}G_F n_{eff} + \dot{\phi} \right|$$

2)  $\nu$  path length in medium must be large

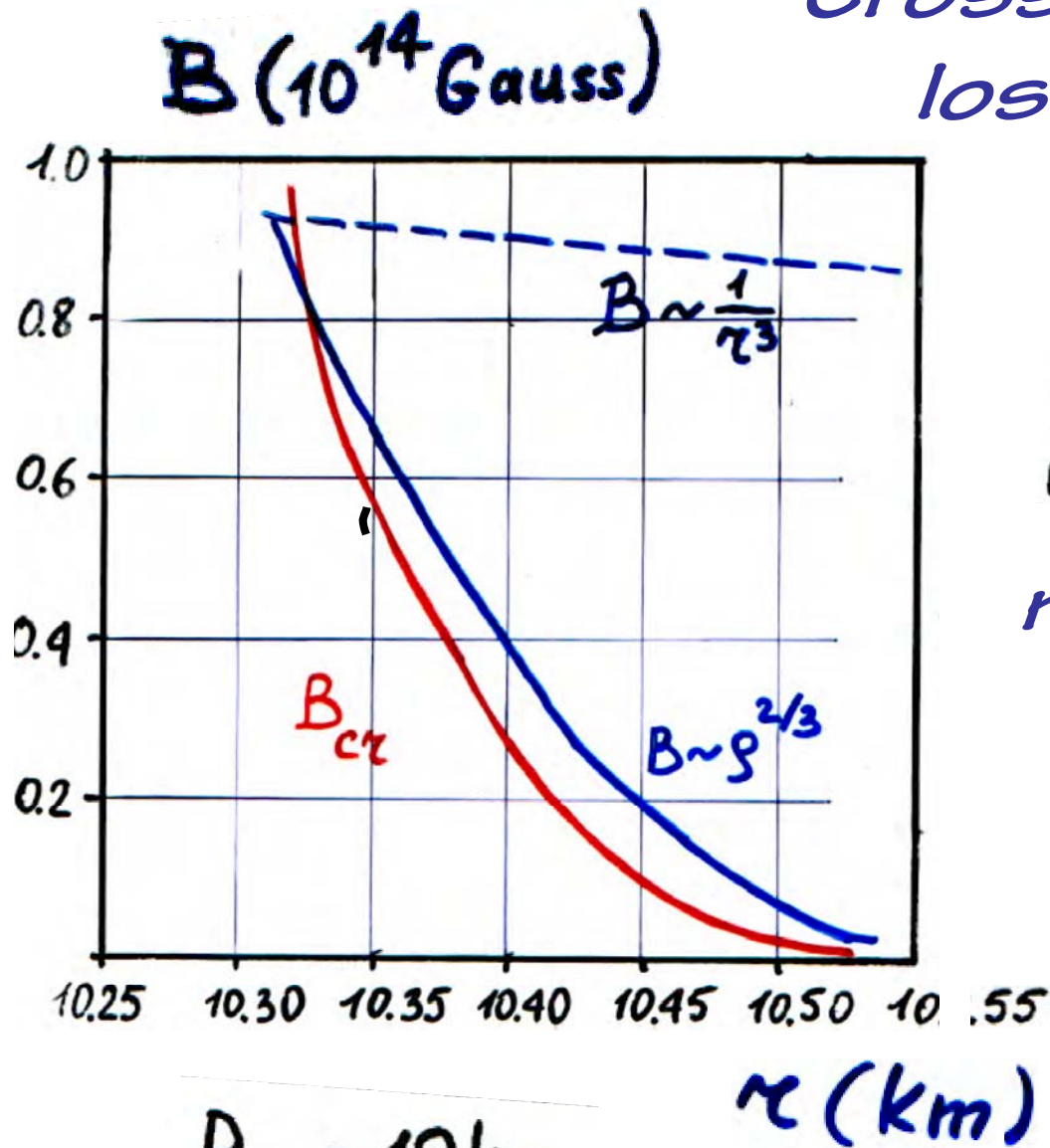
$$x \geq L_{eff}/2$$

**G.Likhachev, A.S.,  
JETP 81 (1995) 419**

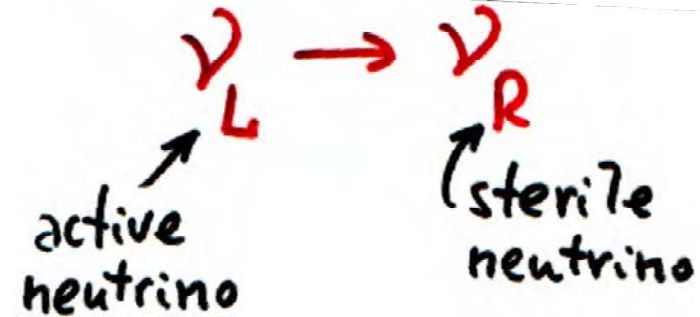
**A.S.,  
Phys.Atom.Nucl.  
67 (2004) 993**

$$L_{eff} = 2\pi \left[ \left( \frac{\Delta m_{\nu}^2}{2E} A - \sqrt{2}G_F n_{eff} + \dot{\phi} \right)^2 + (2\tilde{\mu}B)^2 \right]^{-1/2}$$

“Cross-boundary effect”:  
losses of 50% of  $\nu$



$R_{NS} \sim 10 \text{ km}$



near NS surface

A. Egorov,  
G. Likhachev,  
A. Studenikin,  
1995, 1997

JETP 81 (1995) 419;

A. Studenikin,  
Phys. Atom. Nucl.  
67 (2004) 993

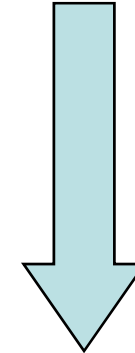
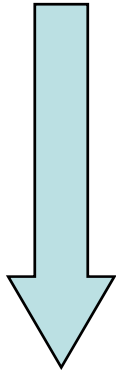
**Direct**

**and  
influence**

**Indirect**

**of electromagnetic fields**

**on**



**through non-trivial  
neutrino electromagnetic  
properties (magnetic moment):**

**due to e.m. field influence on  
charged particles coupled  
to neutrinos**

- ★ neutrino spin
- ★ spin-flavour oscillations...
- ★ different  $\nu\gamma$  processes

- ★ neutron beta-decay in **B**
- ★ change of  $\nu$  oscillation pattern due to matter polarization under influence of external **e.m. fields** ...

# $\beta$ -decay of neutron in magnetic field

{Birth of  $\gamma$  astrophysics in B}



- \* L. Korovina, " $\beta$ -decay of polarized neutron in magnetic field", Sov.Phys.J., # 6 (1964) 86
- \* I. Ternov, B. Lysov, L. Korovina, Mosc.Univ.Bull.,Phys.,Astron., #5 (1965) 58  
"On the theory of neutron  $\beta$ -decay in external magnetic field"
- \* J. Matese, R. O'Connell, "Neutron beta decay in a uniform magnetic field", Phys.Rev.180 (1969) 1289
- \* L. Fassio-Canuto, "Neutron beta decay in a strong magnetic field" Phys.Rev.187 (1969) 2141
- \* G. Greenstein, Nature 223 (1969) 938

\* Asymmetry in  $\tilde{\nu}$  emission

$$\frac{W(\theta)}{W_0} = \frac{1}{2} \int \sin \theta_{\tilde{\nu}} d\theta_{\tilde{\nu}} \left\{ 1 + \frac{2(\alpha^2 + \alpha)}{1 + 3\alpha^2} S_n \cos \theta_{\tilde{\nu}} \right. \\ \left. - 4.9 \frac{eB}{\Delta^2} \left( \frac{\alpha^2 - 1}{1 + 3\alpha^2} \cos \theta_{\tilde{\nu}} + \frac{2(\alpha^2 - \alpha)}{1 + 3\alpha^2} S_n \right) \right\}$$



astrophysical applications





K.Kouzakov, A.Studenikin  
Phys.Rev.C 72 (2005) 015502



“Bound-state beta-decay  
of neutron in strong  
magnetic field”

Usual (continuum - state)  $\beta$  decay  $n \rightarrow p + e^- + \bar{\nu}_e$

"Rare" (bound - state)  $\beta$  decay  $n \rightarrow (pe^-) + \bar{\nu}_e$

R. Daudel, M. Jean, and M. Lecoine, J. Phys. Radium **8**, 238 (1947)

$$\frac{w_b}{w_c} \cong 4.2 \times 10^{-6}$$

$$\tau_c \sim 15 \text{ min}$$

$$\tau_b \sim 7 \text{ years}$$

J.N. Bahcall, Phys. Rev. **124**, 495 (1961) [Dirac equation]

L.L. Nemenov, Sov. J. Nucl. Phys. **15**, 582 (1972) [Schrödinger equation]

X. Song, J. Phys. G: Nucl. Phys. **13**, 1023 (1987) [Bethe-Salpeter equation]

K.A. Kouzakov and A.I. Studenikin, Phys. Rev. C **72**, 015502 (2005)

<http://arxiv.org/hep-ph/0412134>

## Summary

First analysis of bound-state  $\beta$  decay in a strong magnetic field ( $B \sim 10^{13} - 10^{18}$  G)

✓  $w_b/w_c \sim 0.1 - 0.4$  in contrast to the field-free case, where  $w_b/w_c \sim 10^{-6}$

✓ A logarithmiclike behavior

$$w_b/w_c \propto \log_{10}(B/B_e) + b \quad (b > 0)$$

**Outlook:** Astrophysical applications?

3.12

✓ e.m. form factors are affected by matter and  $B$

\* magnetic moment  $\mu_\nu = \int \mu_\nu(B)$

Egorov, Studenikin, 1997

Borisov, Zhukovskiy, Karlin, Ternov, 1985

\* induced electric charge of  $\nu$  in magnetized matter

\*

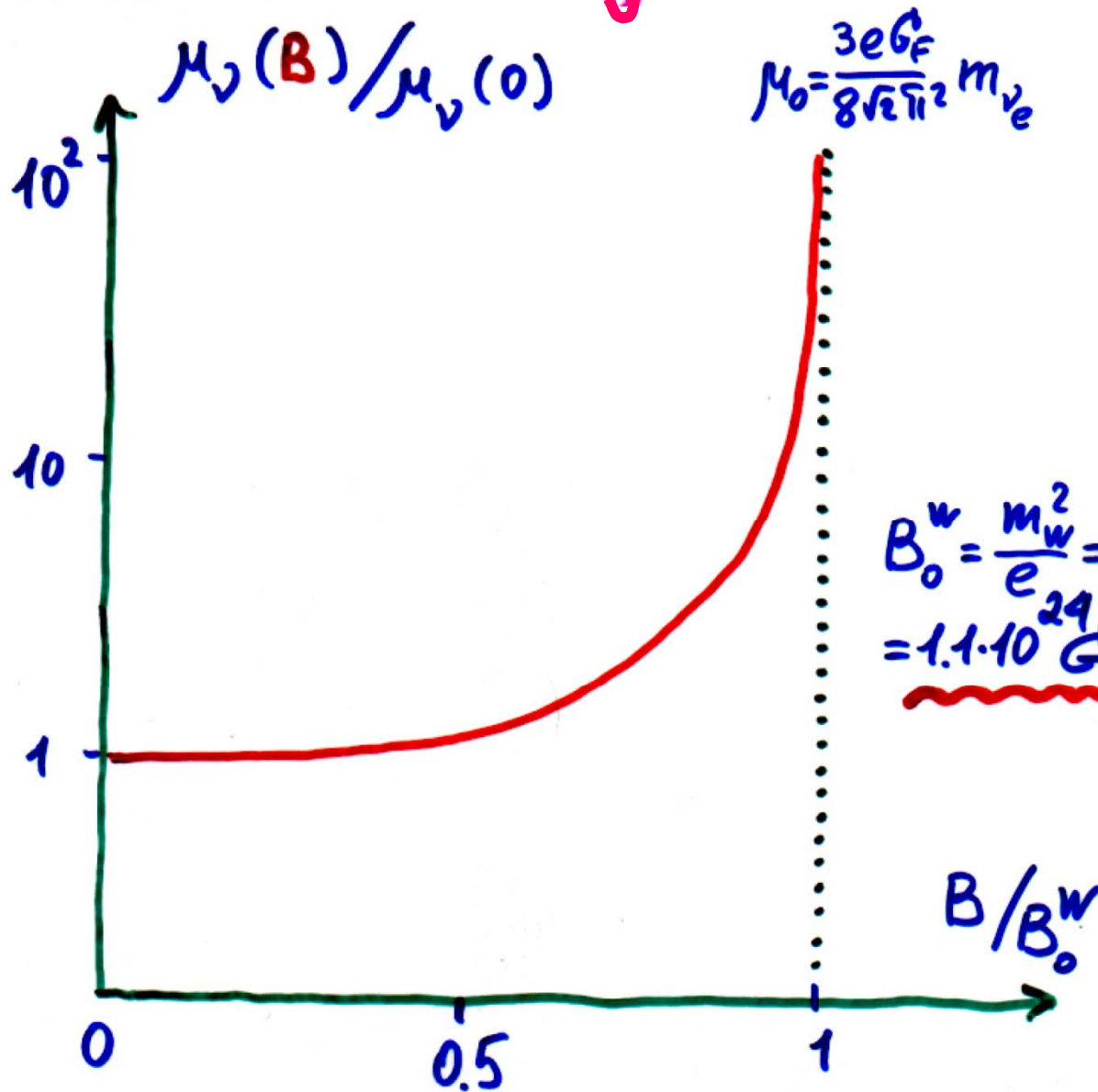
Oraevsky, Semikoz

Smorodinsky, 1986

Bhattacharaya, Ganguly, Konar, 2002

Nieves, 2003

# Neutrino magnetic moment

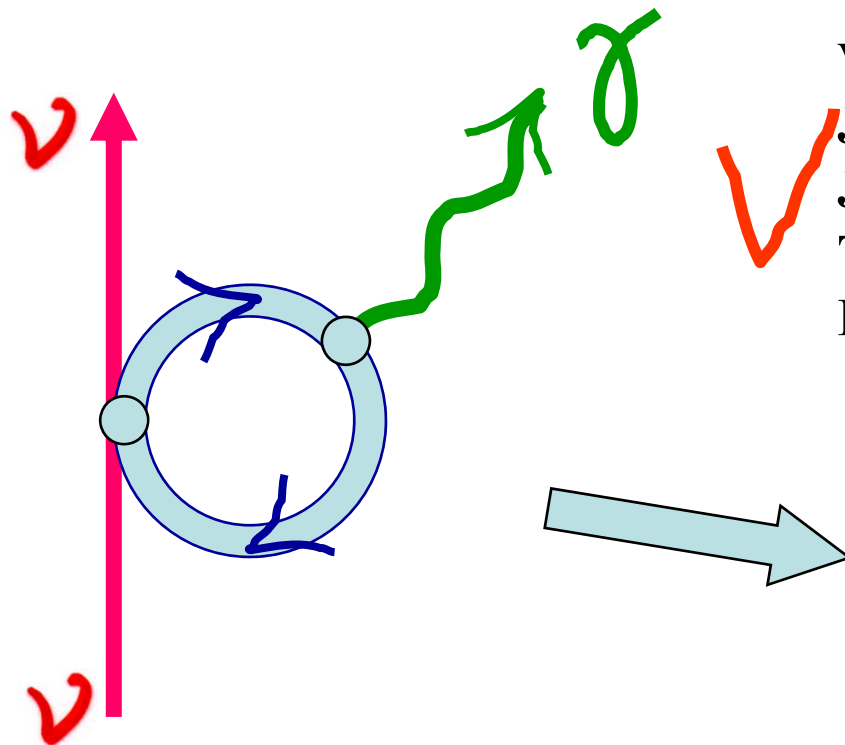


Borisov,  
 Zhukovskiy,  
 Kurilin,  
 Ternov, 1985;

Masood,  
 Perez Rojas,  
 Gaitan,  
 Rodrigues-Romo,  
 1999

# ✓ “effective electric charge” in magnetized plasma

- ✓  $\nu$ s do not couple with  $\gamma$ s in vacuum,  
... however, when
- ✓ in thermal medium ( $e^-$  and  $e^+$ )



V.Oraevsky, V.Semikoz, Ya.Smorodinsky,  
JETP Lett. 43 (1986) 709;  
J.Nieves, P.Pal, Phys.Rev.D 49 (1994) 1398;  
T.Altherr, P.Salati, Nucl.Phys.B421 (1994) 662;  
K.Bhattacharya, A.Ganguly, 2002

...different  $\nu\gamma$  interactions in  
astrophysical and cosmological media

# New mechanism of electromagnetic radiation

*Spin light of neutrino in matter*

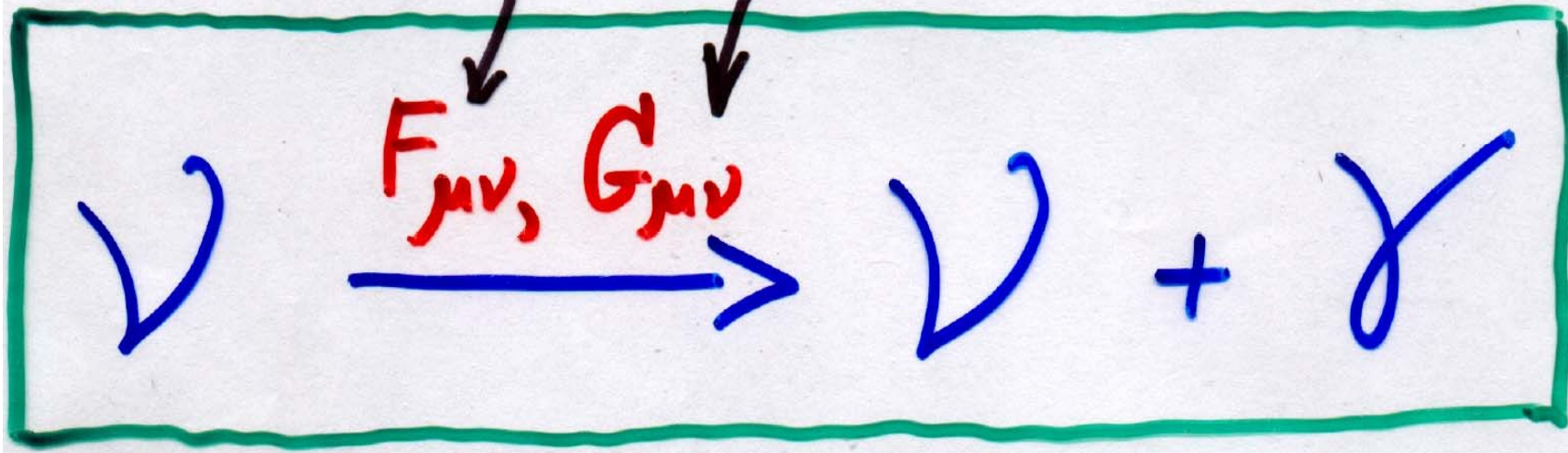


"Spin light of neutrino"

in matter and

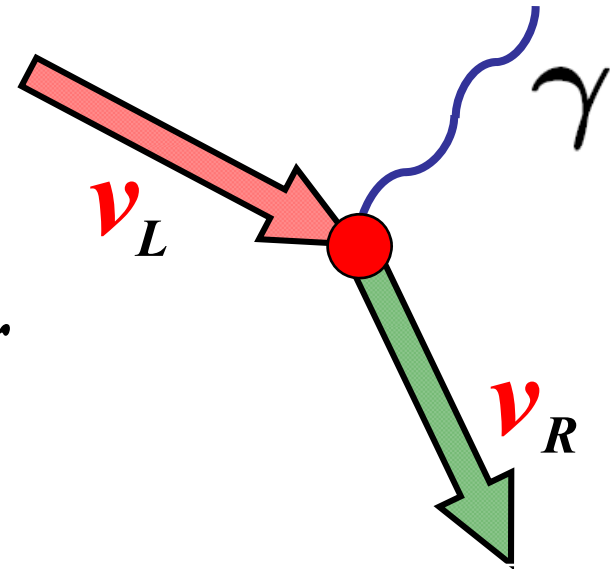
electromagnetic fields

SLν





## *Spin light of neutrino in matter*



- new mechanism of the electromagnetic process stimulated by the presence of matter, in which neutrino with **non-zero magnetic moment** emits light

*A.Lobanov, A.Studenikin, Phys.Lett. B 564 (2003) 27,  
Phys.Lett. B 601 (2004) 171*

*A.S., A.Ternov, Phys.Lett. B 608 (2005) 107*

*A.Grigoriev, A.S., A.Ternov, Phys.Lett. B 622 (2005) 199*

*A.S., J.Phys.A: Math.Gen. 39 (2006) 6769*

*A.S., J.Phys.A: Math.Theor. 41 (2008) 16402*



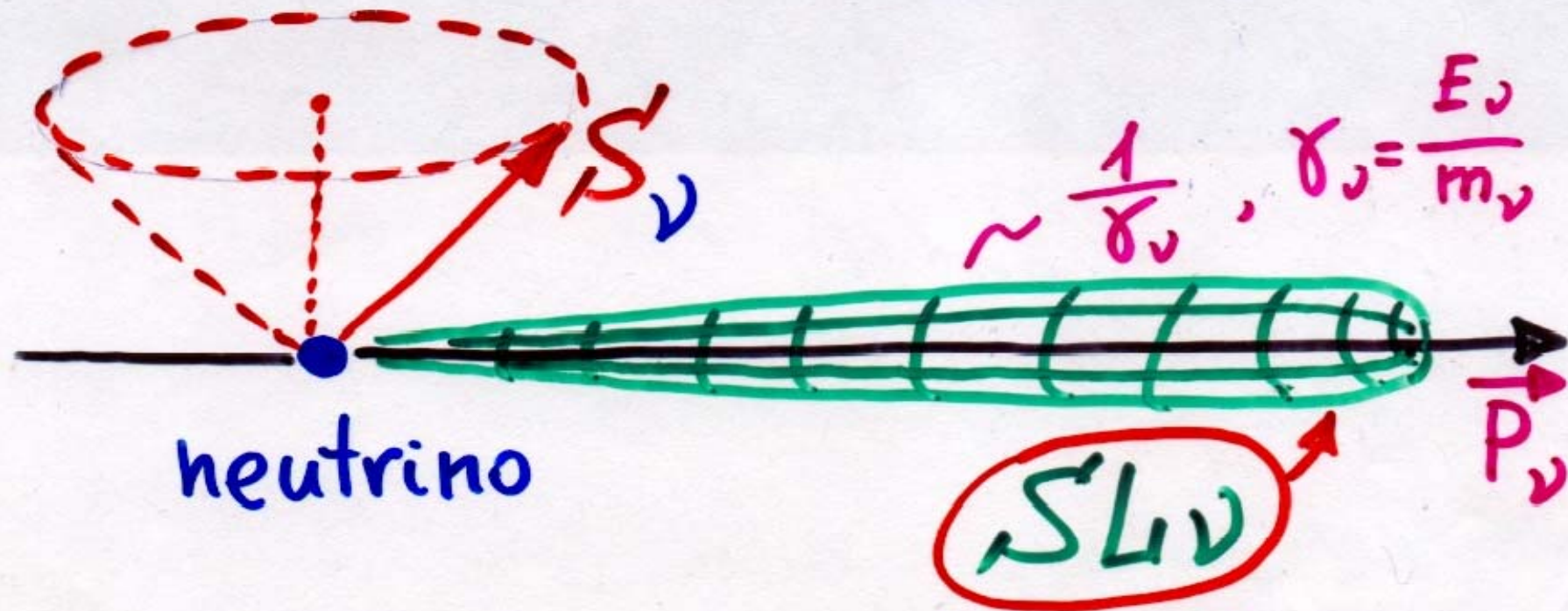
# Quasi-classical theory of spin light of neutrino in matter and gravitational field



A.Lobanov, A.Studenikin, Phys.Lett. B 564 (2003) 27,  
Phys.Lett. B 601 (2004) 171;

M.Dvornikov, A.Grigoriev, A.Studenikin, Int.J.Mod.Phys. D 14 (2005) 309

Neutrino spin precession in background environment





# spin evolution in presence of general external fields

M.Dvornikov, A.Studenikin,  
JHEP 09 (2002) 016

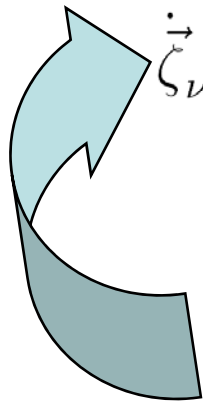
*General types non-derivative interaction with external fields*

$$\begin{aligned}
-\mathcal{L} = & g_s s(x) \bar{\nu} \nu + g_p \pi(x) \bar{\nu} \gamma^5 \nu + g_v V^\mu(x) \bar{\nu} \gamma_\mu \nu + g_a A^\mu(x) \bar{\nu} \gamma_\mu \gamma^5 \nu + \\
& + \frac{g_t}{2} T^{\mu\nu} \bar{\nu} \sigma_{\mu\nu} \nu + \frac{g'_t}{2} \Pi^{\mu\nu} \bar{\nu} \sigma_{\mu\nu} \gamma^5 \nu,
\end{aligned}$$

scalar, pseudoscalar, vector, axial-vector,  
tensor and pseudotensor fields:

$$\begin{aligned}
s, \pi, V^\mu = & (V^0, \vec{V}), A^\mu = (A^0, \vec{A}), \\
T_{\mu\nu} = & (\vec{a}, \vec{b}), \Pi_{\mu\nu} = (\vec{c}, \vec{d})
\end{aligned}$$

*Relativistic equation (quasiclassical) for spin vector:*



$$\begin{aligned}
\dot{\vec{\zeta}}_\nu = & 2g_a \left\{ A^0 [\vec{\zeta}_\nu \times \vec{\beta}] - \frac{m_\nu}{E_\nu} [\vec{\zeta}_\nu \times \vec{A}] - \frac{E_\nu}{E_\nu + m_\nu} (\vec{A} \vec{\beta}) [\vec{\zeta}_\nu \times \vec{\beta}] \right\} \\
& + 2g_t \left\{ [\vec{\zeta}_\nu \times \vec{b}] - \frac{E_\nu}{E_\nu + m_\nu} (\vec{\beta} \vec{b}) [\vec{\zeta}_\nu \times \vec{\beta}] + [\vec{\zeta}_\nu \times [\vec{a} \times \vec{\beta}]] \right\} + \\
& + 2ig'_t \left\{ [\vec{\zeta}_\nu \times \vec{c}] - \frac{E_\nu}{E_\nu + m_\nu} (\vec{\beta} \vec{c}) [\vec{\zeta}_\nu \times \vec{\beta}] - [\vec{\zeta}_\nu \times [\vec{d} \times \vec{\beta}]] \right\}.
\end{aligned}$$

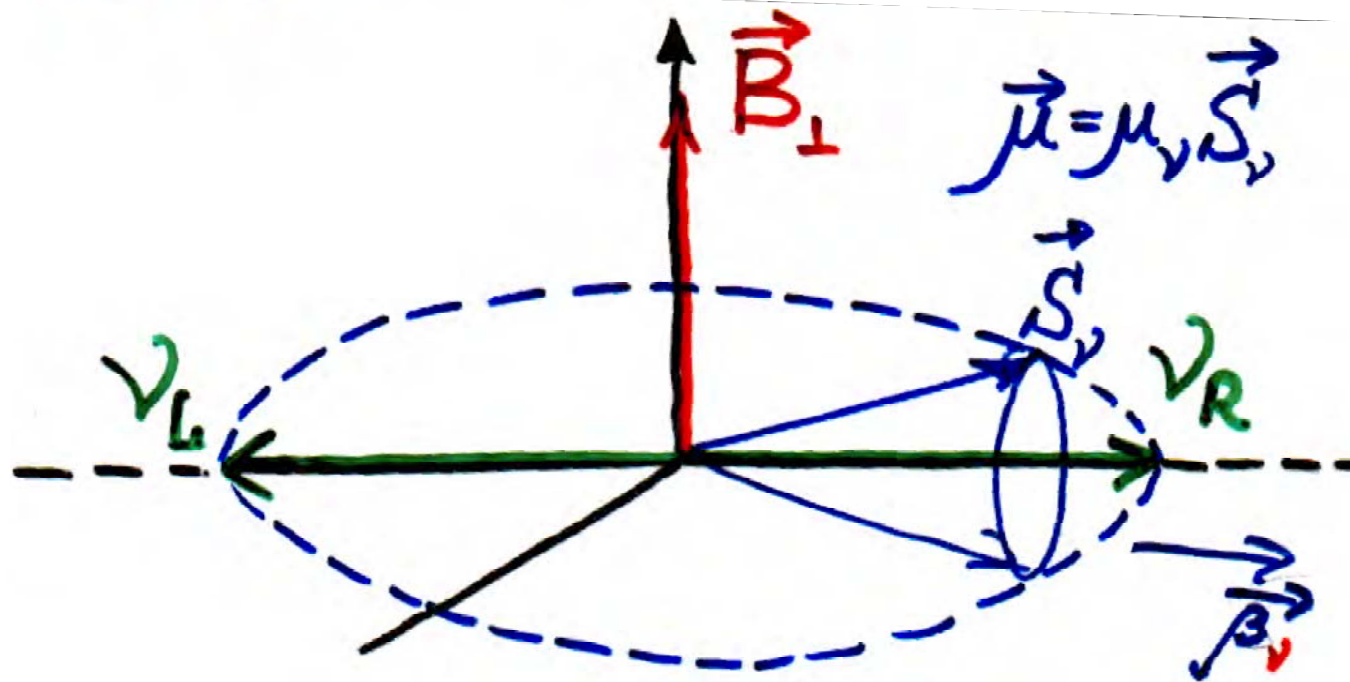
● *Neither  $S$  nor  $\pi$  nor  $V$  contributes to spin evolution*

● **Electromagnetic interaction**

$$T_{\mu\nu} = F_{\mu\nu} = (\vec{E}, \vec{B})$$

● **SM weak interaction**

$$\begin{aligned}
G_{\mu\nu} = & (-\vec{P}, \vec{M}) & \vec{M} = \gamma(A^0 \vec{\beta} - \vec{A}) \\
& & \vec{P} = -\gamma[\vec{\beta} \times \vec{A}],
\end{aligned}$$



$$\frac{d\vec{S}}{dt} = 2\mu_B [\vec{S} \times \underline{\underline{\vec{B}}}] + 2\mu_B [\vec{S} \times \vec{G}]$$

electromagnetic  
interaction with  
e.m. field

---

weak interaction  
with matter

---

# New mechanism of electromagnetic radiation

? Why **Spin Light** of neutrino  $SL\nu$  of electron  $SLe$  in matter.

Analogies with:

\* classical electrodynamics

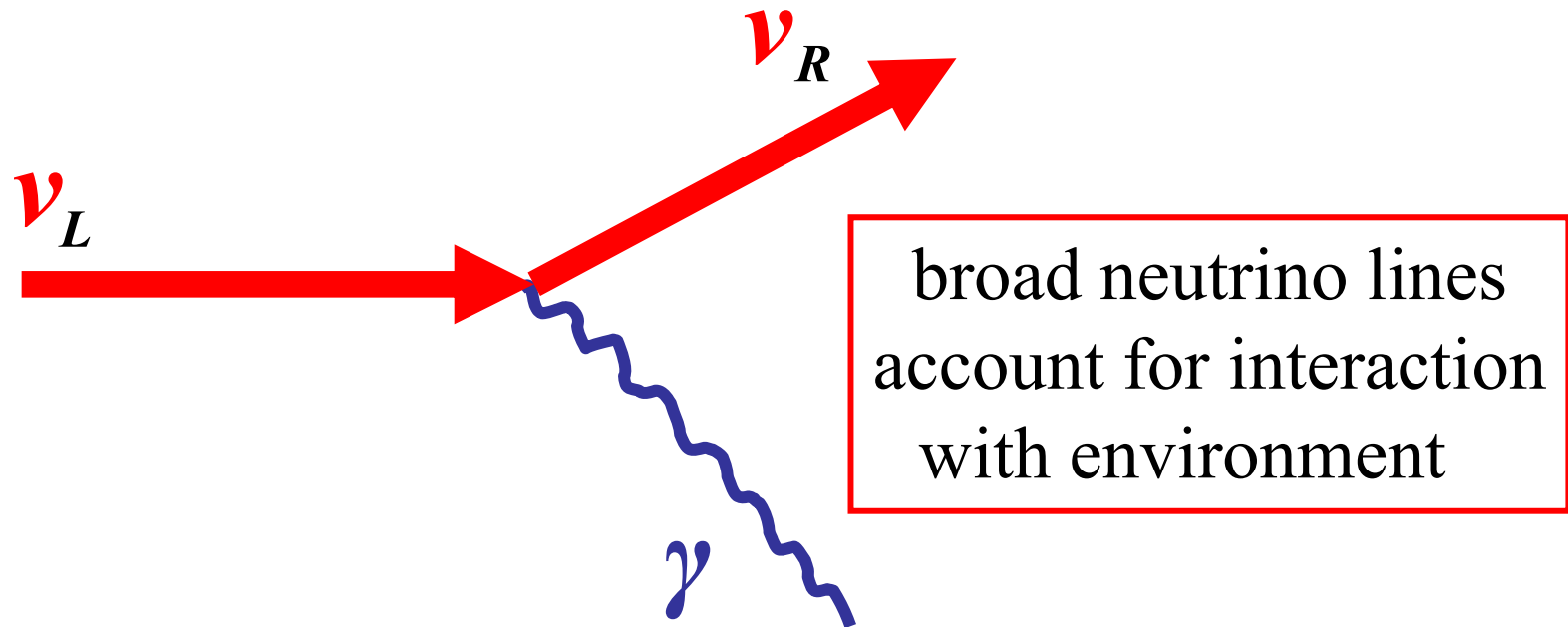
an object with charge  $Q=0$  and

magnetic moment  $\vec{m} = \frac{1}{2} \sum_i e_i [\vec{r}_i \times \vec{v}_i] \neq 0$

$$\overset{\text{cl. el.}}{I} = \frac{2}{3} \ddot{\vec{m}}^2$$

← magnetic dipole radiation power

## Neutrino – photon couplings (II)



“Spin light of neutrino in matter”

... within the quantum treatment based on  
method of exact solutions ...



$\nu$  and  $e$

in matter being treated within  
«**method of exact solutions**»  
of quantum wave equations

A.Studenikin, A.Ternov,  
**Phys.Lett.B 608** (2005) 107;  
**hep-ph/0410297**,  
“Neutrino quantum states in matter”;  
**hep-ph/0410296**,  
“Generalized Dirac-Pauli equation  
and neutrino quantum states in  
matter”

A.Grigoriev, A.Studenikin,  
A.Ternov,  
**Phys.Lett.B 608 622** (2005) 199

A.Studenikin, **J.Phys.A: Math.Theor. 41**  
(2008) 16402, “Method of wave equations  
exact solutions in studies of neutrino and  
electron interactions in dense matter”;

**Ann. Fond. de Broglie 31** (2006) 289,  
“Neutrinos and electrons in background  
matter: a new approach”

**J.Phys.A: Math.Gen.39** (2006) 6769

I.Balantsev, Yu.Popov, A.Studenikin,  
**J.Phys.A: Math.Theor. 44** (2011) 255301,  
“On a problem of relativistic particles  
motion in a strong magnetic field and dense  
matter”

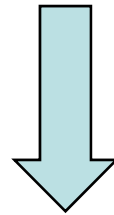
●  $\nu$  *energy quantization  
in rotating matter...*

# Method of exact solutions

Modified **Dirac equations** for  $\nu$  (and  $e$ )  
(containing the correspondent effective matter potentials)

+

**exact solutions** (particles wave functions)



a basis for investigation of different phenomena which  
can proceed when **neutrinos** and **electrons** move in  
dense media

(**astrophysical** and **cosmological** environments).

«method of exact solutions»

# Interaction of particles in external electromagnetic fields ( **Furry representation** in quantum electrodynamics )

Potential of electromagnetic field

$$A_\mu(x) = A_\mu^q(x) + A_\mu^{ext}(x),$$

evolution operator

$$U_F(t_1, t_2) = T \exp \left[ -i \int_{t_1}^{t_2} j^\mu(x) A_\mu^q(x) dx \right],$$

quantized part  
of potential

charged particles **current**

$$j_\mu(x) = \frac{e}{2} [\bar{\Psi}_F \gamma_\mu, \Psi_F],$$

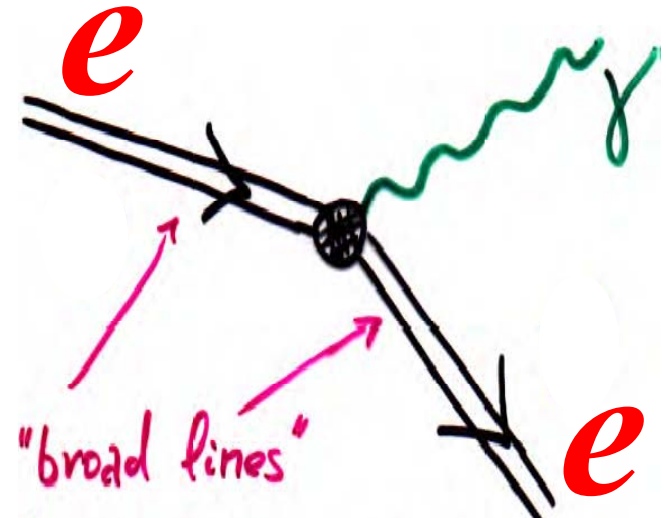
**Dirac equation** in external classical (non-quantized) field  $A_\mu^{ext}(x)$

$$\left\{ \gamma^\mu \left( i\partial_\mu - eA_\mu^{ext}(x) \right) - m_e \right\} \Psi_F(x) = 0$$



...beyond perturbation series expansion,  
**strong fields and non linear effects...**

$B_\perp$   
 $e \rightarrow e + \gamma$   
synchrotron radiation





# Modified Dirac equation for neutrino in matter

Addition to the vacuum neutrino Lagrangian

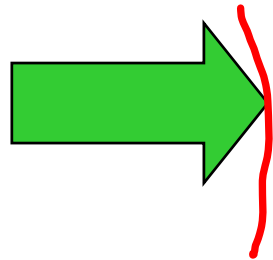
$$\Delta L_{eff} = \Delta L_{eff}^{CC} + \Delta L_{eff}^{NC} = -f^\mu \left( \bar{\nu} \gamma_\mu \frac{1 + \gamma^5}{2} \nu \right)$$

matter  
current

where

$$f^\mu = \frac{G_F}{\sqrt{2}} \left( (1 + 4 \sin^2 \theta_W) j^\mu - \lambda^\mu \right)$$

matter  
polarization



$$\left\{ i\gamma_\mu \partial^\mu - \frac{1}{2} \gamma_\mu (1 + \gamma_5) f^\mu - m \right\} \Psi(x) = 0$$

It is supposed that there is a macroscopic amount of electrons in the scale of a neutrino de Broglie wave length. Therefore, **the interaction of a neutrino with the matter (electrons) is coherent.**

L.Chang, R.Zia,'88; J.Panteleone,'91; K.Kiers, N.Weiss, M.Tytgat,'97-'98; P.Manheim,'88; D.Nötzold, G.Raffelt,'88; J.Nieves,'89; V.Oraevsky, V.Semikoz, Ya.Smorodinsky,89; W.Naxton, W-M.Zhang'91; M.Kachelriess,'98; A.Kusenko, M.Postma,'02.

**A.Studenikin, A.Ternov, hep-ph/0410297;  
*Phys.Lett.B* 608 (2005) 107**

This is the most general equation of motion of a neutrino in which the effective potential accounts for both the **charged** and **neutral-current** interactions with the background matter and also for the possible effects of the matter **motion and polarization.**

## Neutrino wave function in matter (II)

$$\Psi_{\varepsilon, \mathbf{p}, s}(\mathbf{r}, t) = \frac{e^{-i(E_\varepsilon t - \mathbf{p}\mathbf{r})}}{2L^{\frac{3}{2}}} \begin{pmatrix} \sqrt{1 + \frac{m}{E_\varepsilon - \alpha m}} \sqrt{1 + s \frac{p_3}{p}} \\ s \sqrt{1 + \frac{m}{E_\varepsilon - \alpha m}} \sqrt{1 - s \frac{p_3}{p}} e^{i\delta} \\ s\varepsilon\eta \sqrt{1 - \frac{m}{E_\varepsilon - \alpha m}} \sqrt{1 + s \frac{p_3}{p}} \\ \varepsilon\eta \sqrt{1 - \frac{m}{E_\varepsilon - \alpha m}} \sqrt{1 - s \frac{p_3}{p}} e^{i\delta} \end{pmatrix}$$

**A.Studenikin, A.Ternov, hep-ph/0410297;**  
***Phys.Lett.B* 608 (2005) 107;**

$$\eta = \text{sign}(1 - s\alpha \frac{m}{p}), \delta = \arctan(p_2/p_1)$$

**A.Grigoriev, A.Studenikin, A.Ternov,**  
***Phys.Lett.B* 622 (2005) 199**

$$E_\varepsilon - \alpha m = \varepsilon \sqrt{\mathbf{p}^2 \left(1 - s\alpha \frac{m}{p}\right)^2 + m^2}$$

The quantity  $\varepsilon = \pm 1$  splits the solutions into the two branches that in the limit of vanishing matter density,  $\alpha \rightarrow 0$ , reproduce the positive and negative-frequency solutions, respectively.

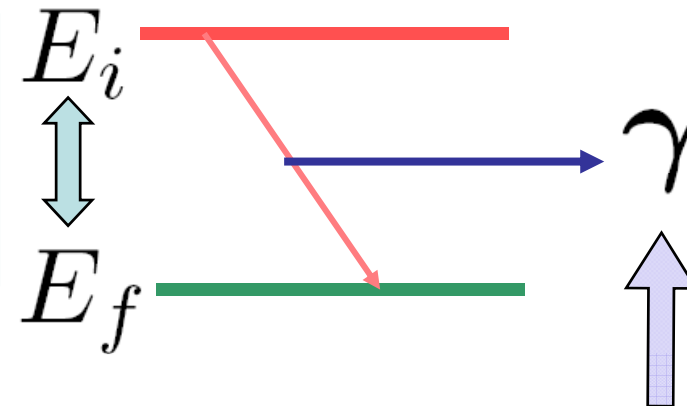
# Quantum theory of spin light of neutrino (I)

Quantum treatment of *spin light of neutrino* in matter

shows that this process originates from the two subdivided phenomena:



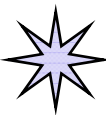
the **shift** of the neutrino **energy levels** in the presence of the background matter, which is different for the two opposite **neutrino helicity states**,



$$E = \sqrt{\mathbf{p}^2 \left(1 - s\alpha \frac{m}{p}\right)^2 + m^2} + \alpha m$$

$$s = \pm 1$$

the radiation of the photon in the process of the neutrino transition from the **“excited” helicity state** to the **low-lying helicity state** in matter



A.Studenikin, A.Ternov, Phys.Lett.B 608 (2005) 107;

A.Grigoriev, A.Studenikin, A.Ternov, Phys.Lett.B 622 (2005) 199;

Grav. & Cosm. 14 (2005) 132;

**neutrino-spin self-polarization effect in the matter**

A.Lobanov, A.Studenikin, Phys.Lett.B 564 (2003) 27;

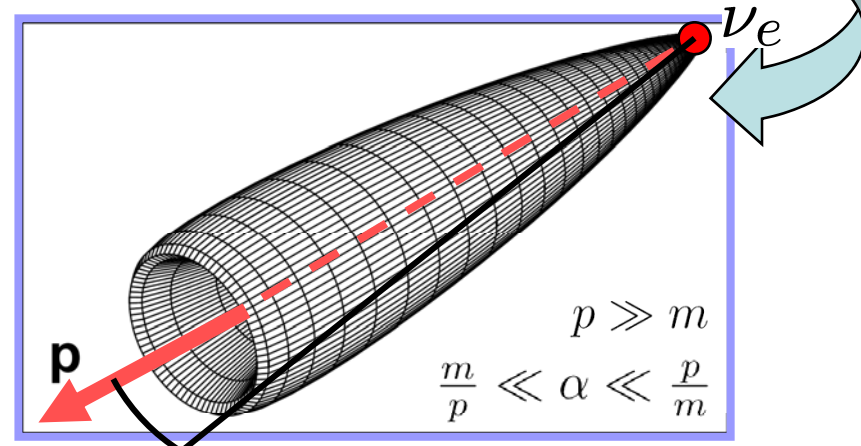
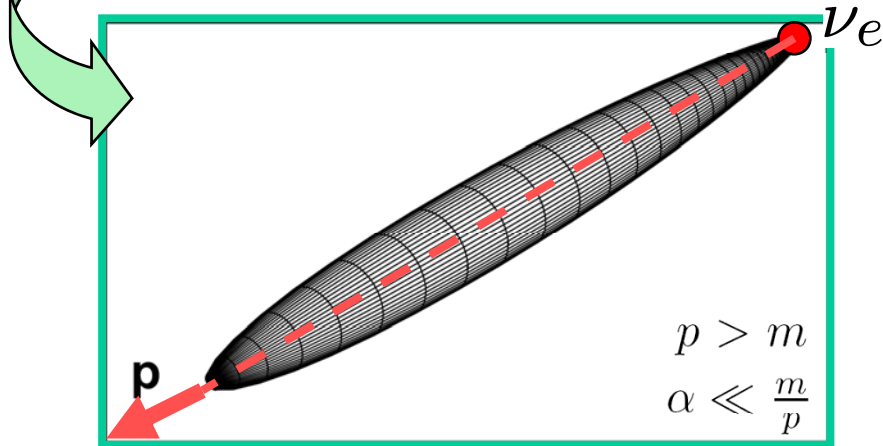
Phys.Lett.B 601 (2004) 171

# Spatial distribution of radiation power

From the angular distribution of

$$SL\nu$$

$$I = \mu^2 \int_0^\pi \omega^4 [(\tilde{\beta}\tilde{\beta}' + 1)(1 - y \cos \theta) - (\tilde{\beta} + \tilde{\beta}')(\cos \theta - y)] \frac{\sin \theta}{1 + \tilde{\beta}'y} d\theta$$



for  $p/m = 5$  and  $\alpha = 0.01$

$$n \approx 10^{35} \text{ cm}^{-3}$$

neutrino  
momentum

mass

matter density

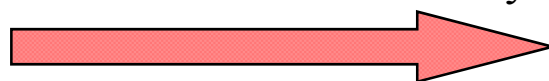
$$\cos \theta_{max} \approx 1 - \frac{2}{3} \alpha \frac{m}{p}$$

maximum in  
radiation power  
distribution

for  $p/m = 10^3$  and  $\alpha = 100$

$$n \approx 10^{39} \text{ cm}^{-3}$$

increase of matter density



projector-like distribution

cap-like distribution

It is possible to have

$$\tau = \frac{1}{\Gamma_{SL\nu}} \ll \text{age of the Universe ?}$$

For ultra-relativistic  $\checkmark$

with momentum  $p \sim 10^{20} eV$

and magnetic moment  $\mu \sim 10^{-10} \mu_B$

in very dense matter  $n \sim 10^{40} cm^{-3}$

from

$$\Gamma_{SL\nu} = 4\mu^2 \alpha^2 m_\nu^2 p$$

$$p \gg m_{plasmon}$$

recently also  
discussed by  
A.Kuznetsov,  
N.Mikheev, 2006

A.Lobanov, A.S., PLB 2003; PLB 2004

A.Grigoriev, A.S., PLB 2005

A.Grigoriev, A.S., A.Ternov, PLB 2005

$$\alpha m_\nu = \frac{1}{2\sqrt{2}} G_F n (1 + \sin^2 \theta_W)$$

it follows that

$$\tau = \frac{1}{\Gamma_{SL\nu}} = 1.5 \times 10^{-8} s$$

# Kinematics

$$\nu_i \rightarrow \nu_j + \gamma$$

SL $\nu$

Initial and final neutrino energies:

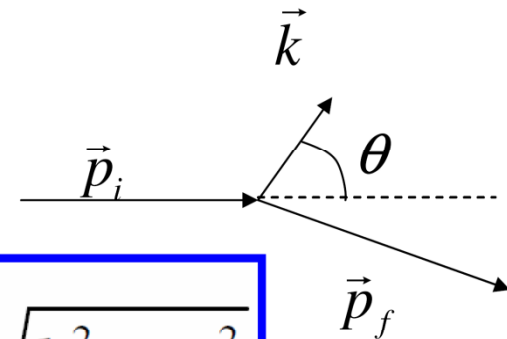
$$E_{i,f} = \sqrt{(p_{i,f} - s_{i,f}\tilde{n})^2 + m^2} + \tilde{n}$$

$$\tilde{n} = \alpha m$$

Energy and momentum conservation laws:

$$E_i = E_f + \omega$$

$$\vec{p}_i = \vec{p}_f + \vec{k}$$



$$\sqrt{(p_i - s_i\tilde{n})^2 + m^2} = \sqrt{(p_f - s_f\tilde{n})^2 + m^2} + \sqrt{k^2 + m_\gamma^2}$$

$$s_i = -1$$

$$s_f = +1$$

$$m_\gamma = \sqrt{2\alpha}(3\sqrt{\pi n})^{1/3}$$

Threshold  
condition

$$\tilde{n}p_i > \frac{m_\gamma^2}{4}$$

A.Lobanov, A.Studenikin, PLB 2003; PLB 2004  
A.Grigoriev, A.Studenikin, PLB 2005  
A.Grigoriev, A.Studenikin, A.Ternov, PLB 2005  
A.Kuznetsov, N.Mikheev, 2006

# Total rate and power of the Spin Light

SL $\nu$

- **SL $\nu$  without plasma influence**

$$\Gamma = 4\mu^2 \tilde{n}^2 (\tilde{n} + p)$$

$$I = \frac{4}{3} \mu^2 \tilde{n}^2 (3\tilde{n}^2 + 4p\tilde{n} + p^2)$$

$$m_\gamma \rightarrow 0$$

- **Far from the threshold**

$$\Gamma = 4\mu^2 p \tilde{n}^2 (1 + 6\lambda + 4\lambda \ln \lambda)$$

$$I = \frac{4}{3} \mu^2 p^2 \tilde{n}^2 \left( 1 - 6\lambda - 57\lambda \frac{\tilde{n}}{p} - 12\lambda \frac{\tilde{n}}{p} \ln \lambda \right)$$

$$\lambda = \frac{m_\gamma^2}{4\tilde{n}p}$$

$$\lambda \ll 1$$

- **Approaching the threshold**

$$\Gamma \sim (1 - \lambda)$$

$$I \sim (1 - \lambda)$$

$$\lambda \rightarrow 1$$

# The effect of plasmon mass on spin light of neutrino in dense matter

A.Grigoriev<sup>a</sup>, A.Lokhov<sup>b</sup>, A.Studenikin<sup>b,c</sup> <sup>1</sup>, A.Ternov<sup>d</sup>

<sup>a</sup>*Skobeltsyn Institute of Nuclear Physics, Moscow State University, 119991 Moscow, Russia*

<sup>b</sup>*Department of Theoretical Physics, Moscow State University, 119991 Moscow, Russia*

<sup>c</sup>*Joint Institute for Nuclear Research, 141980 Dubna, Russia*

<sup>d</sup>*Department of Theoretical Physics, Moscow Institute for Physics and Technology, 141700 Dolgoprudny, Russia*

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## Abstract

We develop the theory of spin light of neutrino in matter ( $SL\nu$ ) and include the effect of plasma influence on the emitted photon. We use the special technique based on exact solutions of particles wave equations in matter to perform all the relevant calculations, and track how the plasmon mass enters the process characteristics including the neutrino energy spectrum,  $SL\nu$  rate and power. The new feature it induces is the existence of the process threshold for which we have found the exact expression and the dependence of the rate and power on this threshold condition. The  $SL\nu$  spatial distribution accounting for the above effects has been also obtained. These results might be of interest in connection with the recently reported hints of ultra-high energy neutrinos  $E = 1 \div 10$  PeV observed by IceCube.

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## 1. Introduction

Neutrino physics in matter and external electromagnetic fields is a rather longstanding research field nevertheless still having advances and providing some interesting predictions for various phenomena. A broad spectrum of issues here are connected with possible electromagnetic properties of neutrino (for more details refer to [1]). The recent studies of neutrino electromagnetic properties revealed a new mechanism of electromagnetic radiation by a neutrino propagating in dense matter that has been proposed in [2]. This type of electromagnetic radiation was called the spin light of neutrino in matter ( $SL\nu$ ). In a quasi-classical treatment this radiation originates due to neutrino electromagnetic moment precession in dense background matter. The quantum theory of this phenomena has been developed in [3, 4].

A new convenient and elegant way of description of neutrino interaction processes in matter has been proposed and developed in a series of papers [3, 5] (see also [4]). The elaborated method is based on the use of the exact solutions of the modified Dirac equation

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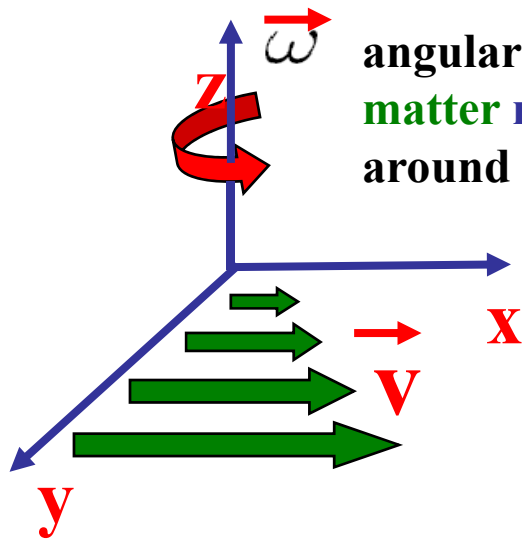
<sup>1</sup>studentik@srd.sinp.msu.ru



# Neutrino energy quantization in matter



*A. Grigoriev, A. Savochkin, A. Studenikin,  
Russ. Phys. J. 50 (2007) 845*

*A. Studenikin,  
J. Phys. A: Math. Theor. 41 (2008) 164047*






angular speed of  
**matter rotation**  
around **OZ**

Consider  moving in **rotating medium**  
composed of neutrons (generalization s.f.):

 wave function 


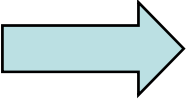
$$\left\{ i\gamma_\mu \partial^\mu - \frac{1}{2} \gamma_\mu (1 + \gamma_5) f^\mu - m \right\} \Psi(x) = 0$$

*A. Ternov,  
A. Studenikin,  
Phys. Lett. B 608 (2005) 107;  
A. Grigoriev, A.S., A. Ternov,  
Phys. Lett. B 622 (2005) 199*

where **matter potential**  $f^\mu = -G(n, n\mathbf{v})$ ,  $\mathbf{v} = (\omega y, 0, 0)$ ,  $\rho = Gn\omega$   $G = \frac{G_F}{\sqrt{2}}$   
**neutron number density**  **speed of matter**  **angular speed of rotation** 

 **energy spectrum**

$$\tilde{p}_0 = \sqrt{p_3^2 + 2\rho N + Gn}, \quad N = 0, 1, 2, \dots$$

 **circular orbits**  **trapping inside dense stars**

... consistent model of a rotating matter with account for  $\checkmark$  mass  
*I.Balantsev, Yu.Popov, A.Studenikin,*  
*Nuov.Cim.B 32 (2009) 53,*  
*arXiv: 0906.2391,*  
*J.Phys.A: Math.Theor. 44 (2011)*  
*255301*

$$\left\{ i\gamma_\mu \partial^\mu - \frac{1}{2} \gamma_\mu (1 + \gamma_5) f^\mu - m \right\} \Psi(x) = 0$$

$$f^\mu = -G(n, n\mathbf{v}), \quad \mathbf{v} = (-\omega y, \omega x, 0)$$

## Energy spectra

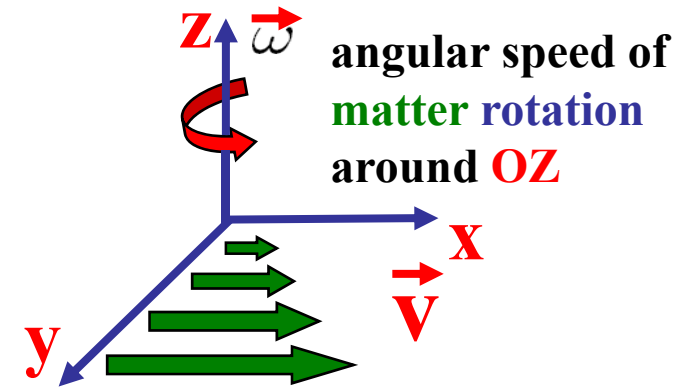
$$p_0 = \sqrt{m^2 + p_3^2 + 4N\rho - Gn} \quad \text{for } \checkmark$$

$$\tilde{p}_0 = \sqrt{m^2 + p_3^2 + 4N\rho + Gn} \quad \text{for } \checkmark$$

$$N = 0, 1, 2, \dots \quad \rho = Gn\omega$$

**One example:** consider antineutrino  $\bar{\nu}$  in rotating neutron matter, then energy of transversal motion

$$\tilde{p}_{\perp} = \sqrt{2\rho N} \quad \rho = Gn\omega$$



Quantum number  $N$  also determines **radius** of antineutrino quasi-classical orbit in moving matter:

$$R = \sqrt{\frac{2N}{Gn\omega}} \rightarrow \text{binding orbits inside a Neutron Star !?}$$

**NS:**

$$\begin{aligned} R_{NS} &= 10 \text{ km} \\ n &= 10^{37} \text{ cm}^{-3} \\ \omega &= 2\pi \times 10^3 \text{ s}^{-1} \end{aligned}$$

for this set

radius of trajectory

$$R = \sqrt{\frac{2N}{Gn\omega}} < R_{NS} = 10 \text{ km}$$

if  $N \leq N_{max} = 10^{10}$ ,  $\bar{\nu}$  with  $N \leq 10^{10}$  can be bound inside the star

thus,  $\bar{\nu}$  with energy  $\tilde{p}_0 \sim 1 \text{ eV}$  can be bound inside NS  
 $N \gg 1$  and  $p_3 = 0$

# Millicharged magnetic $\nu$ in rotating magnetized matter

Ilya Balatsev, Ilya Tokarev, A.S., 2012,  
 Phys.Atom.Nucl.,  
 Phys.El.Part.&Atom.Nucl., arXiv:1209.

Modified Dirac equation for  $\nu$  wave function

$$\left( \gamma_\mu (p^\mu + q_0 A^\mu) - \frac{1}{2} \gamma_\mu (c_l + \gamma_5) f^\mu - \frac{i}{2} \mu \sigma_{\mu\nu} F^{\mu\nu} - m \right) \Psi(x) = 0$$

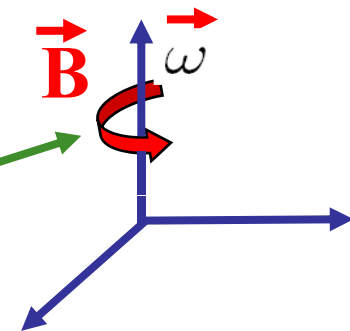
external magnetic field

matter potential

$$V_m = \frac{1}{2} \gamma_\mu (c_l + \gamma_5) f^\mu$$

rotating matter

$$f^\mu = -Gn_n(1, -\epsilon y \omega, \epsilon x \omega, 0)$$



✓ *energy is quantized*  
*in magnetized and rotating matter*

$$G = \frac{G_F}{\sqrt{2}}$$

$$p_0 = \sqrt{p_3^2 + 2N|2Gn_n\omega - \epsilon q_\nu B| + m^2} - Gn_n - q\phi$$

$$N = 0, 1, 2, \dots$$

*scalar potential*  
*of electric field*

*... similar to Landau levels*  
*in magnetic field ...*

## Effective Lorentz force

$$\mathbf{F}_{eff} = q_{eff}\mathbf{E}_{eff} + q_{eff}[\boldsymbol{\beta} \times \mathbf{B}_{eff}]$$

$$q_{eff}\mathbf{E}_{eff} = q_m\mathbf{E}_m + q\mathbf{E}$$

$$q_{eff}\mathbf{B}_{eff} = |q_m B_m + \epsilon q B| \mathbf{e}_z$$

*matter induced 'charge'*

$$q_m = -G$$

*matter 'electric' field*

$$\mathbf{E}_m = -\nabla n_n$$

*matter induced 'magnetic' field*

$$\mathbf{B}_m = 2n_n\boldsymbol{\omega}$$

*... consequences ...*

*binding orbits inside a Neutron Star !?*

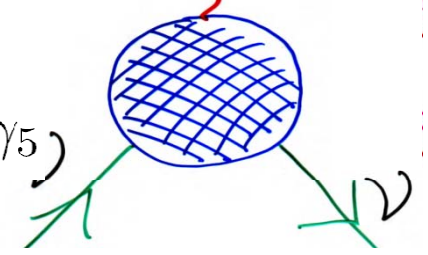
*Conclusion*

# ✓ *e.m. vertex function* $\Rightarrow$ 4 form factors

**charge**      **dipole magnetic and electric**

- $$\Lambda_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2)i\sigma_{\mu\nu}q^\nu + f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)(q^2\gamma_\mu - q_\mu\not{q})\gamma_5$$

**anapole**



EM properties  $\Rightarrow$  a way to distinguish **Dirac** and **Majorana** ✓

- Standard Model with  $\nu_R$  ( $m_\nu \neq 0$ ):  $\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e} \sim 3 \cdot 10^{-19} \mu_B \left(\frac{m_{\nu_e}}{1\text{eV}}\right)$

- In extensions of SM  $\Rightarrow$

enhancement of magnetic moment ✓, even

electrically millicharged ✓

- Limits from reactor  $\nu$ -e scattering experiments (2012):

$$\mu_\nu < 2.9 \times 10^{-11} \mu_B$$

A.Beda et al.  
(GEMMA Coll.)

- Limits from astrophysics, star cooling (1990):

$$\mu \leq 3 \times 10^{-12} \mu_B$$

G.Raffelt

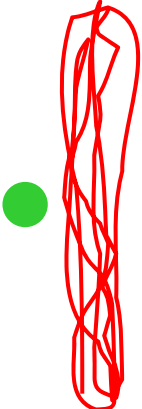


$\mu_{\nu}$  is presently *most probably* “known to be in the range”


$$10^{-20} \mu_B \leq \mu_{\nu} \leq 10^{-11} \mu_B$$

$\mu_{\nu}$  provides a tool for exploration possible physics  
beyond the **Standard Model**

**Due to smallness of neutrino-mass-induced magnetic moments,**


$$\mu_{ii} \approx 3.2 \times 10^{-19} \left( \frac{m_i}{1 \text{ eV}} \right) \mu_B$$

**any indication for non-trivial electromagnetic properties of  $\nu$ , that could be obtained within reasonable time in the future, would give evidence for interactions **beyond extended Standard Model****



*Bruno Pontecorvo*  
*was a staff member*  
*of Faculty of Physics of MSU*  
*and headed*  
*Department of Elementary*  
*Particle Physics*

Бруно Понтекорво

